

The University of the State of New York  
REGENTS HIGH SCHOOL EXAMINATION

**THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS**

**COURSE III**

**Thursday, June 20, 2002 — 1:15 to 4:15 p.m., only**

**Notice . . .**

Scientific calculators must be available to all students taking this examination.

The formulas that you may need to answer some questions in this examination are found on page 2. The last page of the booklet is the answer sheet. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of the answer sheet.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. The answer sheet cannot be accepted if you fail to sign this declaration.

**DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.**

## Formulas

### Pythagorean and Quotient Identities

$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 & \tan A &= \frac{\sin A}{\cos A} \\ \tan^2 A + 1 &= \sec^2 A & \cot A &= \frac{\cos A}{\sin A} \\ \cot^2 A + 1 &= \csc^2 A\end{aligned}$$

### Functions of the Sum of Two Angles

$$\begin{aligned}\sin(A + B) &= \sin A \cos B + \cos A \sin B \\ \cos(A + B) &= \cos A \cos B - \sin A \sin B \\ \tan(A + B) &= \frac{\tan A + \tan B}{1 - \tan A \tan B}\end{aligned}$$

### Functions of the Difference of Two Angles

$$\begin{aligned}\sin(A - B) &= \sin A \cos B - \cos A \sin B \\ \cos(A - B) &= \cos A \cos B + \sin A \sin B \\ \tan(A - B) &= \frac{\tan A - \tan B}{1 + \tan A \tan B}\end{aligned}$$

### Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

### Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos A$$

### Functions of the Double Angle

$$\begin{aligned}\sin 2A &= 2 \sin A \cos A \\ \cos 2A &= \cos^2 A - \sin^2 A \\ \cos 2A &= 2 \cos^2 A - 1 \\ \cos 2A &= 1 - 2 \sin^2 A \\ \tan 2A &= \frac{2 \tan A}{1 - \tan^2 A}\end{aligned}$$

### Functions of the Half Angle

$$\begin{aligned}\sin \frac{1}{2}A &= \pm \sqrt{\frac{1 - \cos A}{2}} \\ \cos \frac{1}{2}A &= \pm \sqrt{\frac{1 + \cos A}{2}} \\ \tan \frac{1}{2}A &= \pm \sqrt{\frac{1 - \cos A}{1 + \cos A}}\end{aligned}$$

### Area of Triangle

$$K = \frac{1}{2}ab \sin C$$

### Standard Deviation

$$\text{S.D.} = \sqrt{\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2}$$

Part I

Answer 30 questions from this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet. Where applicable, answers may be left in terms of  $\pi$  or in radical form. [60]

1 What is the amplitude of the graph of the equation  $y = 3 \sin 2x$ ?

2 A translation maps  $P(4,1)$  to  $P'(2,-1)$ . What are the coordinates of  $Q'$ , the image of  $Q(1,3)$  under the same translation?

3 Express in simplest form in terms of  $i$ :

$$5\sqrt{-25} - 3\sqrt{-100}$$

4 If  $g(x) = \left(\frac{1}{64}\right)^x$ , find  $g\left(-\frac{1}{3}\right)$ .

5 Express  $45^\circ$  in radian measure.

6 If  $\log_4 x = 2$ , find  $x$ .

7 If  $f(x) = 2x + 1$  and  $g(x) = x^2$ , find  $(g \circ f)(2)$ .

8 If 0.00326 is written in scientific notation as  $3.26 \times 10^n$ , find the value of  $n$ .

9 If the domain of  $f(x) = x^2 + 1$  is limited to  $\{0,1,2,3\}$ , what is the maximum value of the range?

10 Express in simplest form:  $\frac{x - \frac{9}{x}}{2 + \frac{6}{x}}$

11 In  $\triangle ABC$ ,  $m\angle A = 30$ ,  $m\angle B = 65$ , and  $BC = 10$ . Find  $AC$  to the nearest tenth.

12 Evaluate:  $\sum_{k=0}^3 \frac{k}{2}$

13 Solve for  $r$ :  $\frac{1}{r} = \frac{1}{2} + \frac{1}{3}$

14 Solve for the *negative* value of  $x$ :  $|2x - 3| + 1 = 17$

*Directions (15–35):* For *each* question chosen, write on the separate answer sheet the *numeral* preceding the word or expression that best completes the statement or answers the question.

15 Which expression is equivalent to  $\cos 120^\circ$ ?

- (1)  $\cos 60^\circ$  (3)  $-\sin 60^\circ$   
(2)  $\cos 30^\circ$  (4)  $-\sin 30^\circ$

16 In the distance formula, rate varies inversely with time. If rate is doubled, time is

- (1) decreased by 2 (3) halved  
(2) increased by 2 (4) doubled

17 For which value or values of  $n$  is the expression  $\frac{n-1}{2n+4}$  undefined?

- (1) 1, only (3) 1 and  $-2$   
(2)  $-2$ , only (4) 0

18 What is the reciprocal of  $3 - \sqrt{5}$ ?

- (1)  $\frac{3 - \sqrt{5}}{4}$  (3)  $\frac{3 + \sqrt{5}}{14}$   
(2)  $\frac{3 + \sqrt{5}}{4}$  (4)  $\frac{3 + \sqrt{5}}{14}$

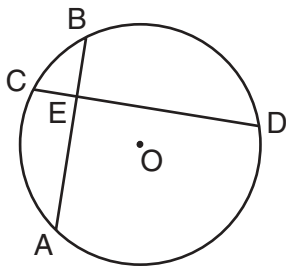
19 The graph of  $xy = -6$  lies in

- (1) Quadrant I, only  
(2) Quadrant II, only  
(3) Quadrants I and III  
(4) Quadrants II and IV

20 The expression  $\sec^2 \theta - \tan^2 \theta$  is equal to

- (1) 1 (3)  $\sin^2 \theta$   
(2) 0 (4)  $\frac{1}{\cos^2 \theta}$

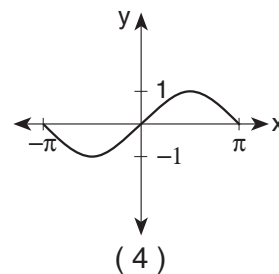
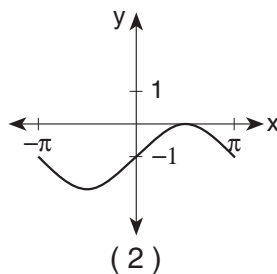
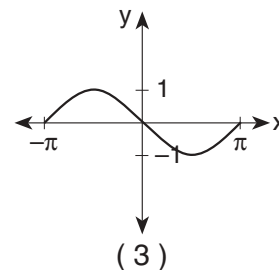
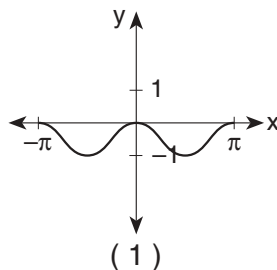
- 21 In the accompanying diagram of circle  $O$ , chords  $\overline{AB}$  and  $\overline{CD}$  intersect at  $E$  and  $m\widehat{AC}:m\widehat{CB}:m\widehat{BD}:m\widehat{DA} = 4:2:6:8$ .



What is  $m\angle DEB$ ?

- (1) 36 (3) 100  
 (2) 90 (4) 126
- 22 The roots of the equation  $2x^2 + 6x + 5 = 0$  are  
 (1) imaginary  
 (2) real and irrational  
 (3) real, rational, and unequal  
 (4) real, rational, and equal
- 23 In  $\triangle ABC$ ,  $m\angle A = 30$ ,  $a = 12$ , and  $b = 10$ . Which type of triangle is  $\triangle ABC$ ?  
 (1) acute (3) obtuse  
 (2) isosceles (4) right
- 24 What is the value of  $(5i^3)^3$ ?  
 (1)  $-125i$  (3)  $-15i$   
 (2)  $125i$  (4)  $15i$
- 25 What is a value of  $\cos\left(\text{Arc tan } \frac{2}{3}\right)$ ?  
 (1)  $\frac{\sqrt{13}}{3}$  (3) 5  
 (2)  $\frac{3\sqrt{13}}{13}$  (4) 13
- 26 If  $\sin \theta$  is less than 0 and  $\sec \theta$  is greater than 0, in which quadrant does the terminal side of  $\theta$  lie?  
 (1) I (3) III  
 (2) II (4) IV
- 27 Which equation is equivalent to  $1 - \frac{6}{t^2} = \frac{1}{t}$ ?  
 (1)  $(t - 3)(t + 2) = 0$  (3)  $(2t + 1)(3t - 1) = 0$   
 (2)  $(t - 2)(t + 3) = 0$  (4)  $(2t - 1)(3t + 1) = 0$

- 28 Which graph represents the function  $f(x) = -\sin x$  in the interval  $-\pi \leq x \leq \pi$ ?



- 29 The expression  $\csc A \sin 2A$  is equivalent to  
 (1)  $2 \sin A$  (3)  $2 \cos A$   
 (2) 2 (4)  $2 \cot A$
- 30 If a fair six-sided die is tossed five times, what is the probability of getting *exactly* three even numbers?  
 (1)  $\frac{1}{32}$  (3)  $\frac{10}{32}$   
 (2)  $\frac{3}{32}$  (4)  $\frac{3}{5}$
- 31 The sides of a parallelogram are 6 and 8, and the included angle is  $150^\circ$ . What is the area of the parallelogram?  
 (1) 24 (3)  $24\sqrt{3}$   
 (2) 48 (4)  $48\sqrt{2}$
- 32 What is the solution set of the inequality  $x^2 - 3x - 10 > 0$ ?  
 (1)  $\{x \mid -2 < x < 5\}$  (3)  $\{x \mid x < -5 \text{ or } x > 2\}$   
 (2)  $\{x \mid -5 < x < 2\}$  (4)  $\{x \mid x < -2 \text{ or } x > 5\}$
- 33 What is the third term in the expansion of  $(2x - y)^4$ ?  
 (1)  $-16x^3y$  (3)  $-24x^2y^2$   
 (2)  $16x^3y$  (4)  $24x^2y^2$

34 In a circle, an arc of length 5 is subtended by a central angle of  $\frac{5}{3}$  radians. What is the radius of the circle?

- (1)  $\frac{25}{3}$  (3) 3  
 (2)  $\frac{3}{25}$  (4) 5

35 What is the inverse of the function  $y = 3x - 2$ ?

- (1)  $y = 3x + 2$  (3)  $y = \frac{x-2}{3}$   
 (2)  $y = \frac{x+2}{3}$  (4)  $y = \frac{1}{3}x - 2$

Answers to the following questions are to be written on paper provided by the school.

**Part II**

Answer four questions from this part. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Calculations that may be obtained by mental arithmetic or the calculator do not need to be shown. [40]

36 *a* On the same set of axes, sketch and label the graphs of the equations  $y = -3 \cos x$  and  $y = \sin 2x$  in the interval  $-\pi \leq x \leq \pi$ . [8]

*b* Using the graphs sketched in part *a*, determine all values of  $x$  in the interval  $-\pi \leq x \leq \pi$  that satisfy the equation  $-3 \cos x = \sin 2x$ . [2]

37 *a* Solve for  $x$  and express the roots in simplest  $a + bi$  form:

$$x + \frac{5}{x} = 2 \quad [5]$$

*b* Express in simplest form:

$$\frac{x^2 - x - 56}{x^2 - 4} \cdot \frac{x^2 + x - 2}{8x - x^2} \div \frac{x^2 + 6x - 7}{x^2 + 2x - 8} \quad [5]$$

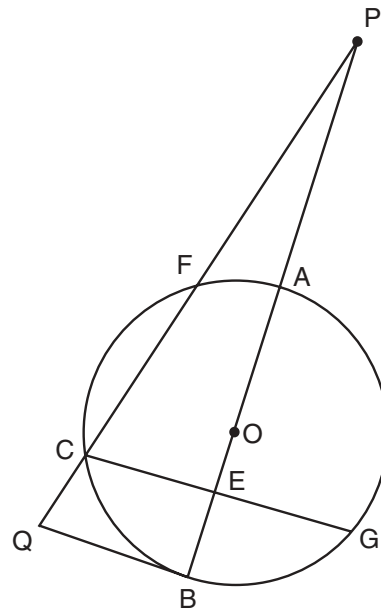
38 *a* On graph paper, sketch the graph of the equation  $y = 3^{-x}$  in the interval  $-2 \leq x \leq 2$ . [4]

*b* On the same set of axes, reflect the graph drawn in part *a* in the  $y$ -axis and label it *b*. [4]

*c* Write the equation of the graph drawn in part *b*. [2]

39 Find, to the nearest ten minutes or nearest tenth of a degree, all values of  $x$  in the interval  $0^\circ \leq x < 360^\circ$  that satisfy the equation  $6 \cos^2 x - 5 \sin x - 5 = 0$ . [10]

40 In the accompanying diagram of circle  $O$ , secant  $\overline{PFCQ}$ , secant  $\overline{PAOEB}$ , tangent  $\overline{QB}$ , and chord  $\overline{CEG}$  are drawn;  $m\widehat{BC}:m\widehat{CF}:m\widehat{FA} = 7:8:3$ ; and  $m\angle AEG = 95$ .



Find:

- a*  $m\widehat{CF}$  [2]  
*b*  $m\widehat{AG}$  [2]  
*c*  $m\angle P$  [2]  
*d*  $m\angle FCG$  [2]  
*e*  $m\angle FQB$  [2]

41 *a* Mr. Truong gave his 25 final grades according to the following chart:

$x_i$	$f_i$
75	3
80	2
85	6
90	7
95	5
100	2

- (1) Find the standard deviation of this set of grades to the *nearest tenth*. [4]
- (2) What percentage of the grades fall *outside* one standard deviation of the mean? [2]

*b* During the school year, Michele receives four report cards. The probability that she will get an A in mathematics on any one report card is  $\frac{4}{5}$ . What is the probability that she will get an A in mathematics on *at least* three of the four report cards? [4]

42 *a* Two forces of 25 pounds and 38 pounds act on a body at an angle of  $74.5^\circ$ . Find, to the *nearest tenth of a pound*, the magnitude of the resultant force. [6]

*b* Using the answer found in part *a*, find the angle between the resultant and the larger force to the *nearest tenth of a degree*. [4]

The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

SEQUENTIAL MATH – COURSE III

Thursday, June 20, 2002 — 1:15 to 4:15 p.m., only

Part I Score	.....
Part II Score	.....
Total Score	.....
Rater's Initials:	.....

ANSWER SHEET

Student ..... Sex:  Male  Female Grade .....

Teacher ..... School .....

Your answers to Part I should be recorded on this answer sheet.

Part I

Answer 30 questions from this part.

- |          |          |          |          |
|----------|----------|----------|----------|
| 1 .....  | 11 ..... | 21 ..... | 31 ..... |
| 2 .....  | 12 ..... | 22 ..... | 32 ..... |
| 3 .....  | 13 ..... | 23 ..... | 33 ..... |
| 4 .....  | 14 ..... | 24 ..... | 34 ..... |
| 5 .....  | 15 ..... | 25 ..... | 35 ..... |
| 6 .....  | 16 ..... | 26 ..... |          |
| 7 .....  | 17 ..... | 27 ..... |          |
| 8 .....  | 18 ..... | 28 ..... |          |
| 9 .....  | 19 ..... | 29 ..... |          |
| 10 ..... | 20 ..... | 30 ..... |          |

Your answers for Part II should be placed on paper provided by the school.

The declaration below should be signed when you have completed the examination.

I do hereby affirm, at the close of this examination, that I had no unlawful knowledge of the questions or answers prior to the examination and that I have neither given nor received assistance in answering any of the questions during the examination.

\_\_\_\_\_  
Signature

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Tear Here

Tear Here