

The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

COURSE III

Wednesday, August 13, 2003 — 8:30 to 11:30 a.m., only

Notice . . .

Scientific calculators must be available to all students taking this examination.

The formulas that you may need to answer some questions in this examination are found on page 2. The last page of the booklet is the answer sheet. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of the answer sheet.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. The answer sheet cannot be accepted if you fail to sign this declaration.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Formulas

Pythagorean and Quotient Identities

$$\begin{array}{ll} \sin^2 A + \cos^2 A = 1 & \tan A = \frac{\sin A}{\cos A} \\ \tan^2 A + 1 = \sec^2 A & \cot A = \frac{\cos A}{\sin A} \\ \cot^2 A + 1 = \csc^2 A & \end{array}$$

Functions of the Sum of Two Angles

$$\begin{array}{l} \sin(A + B) = \sin A \cos B + \cos A \sin B \\ \cos(A + B) = \cos A \cos B - \sin A \sin B \\ \tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \end{array}$$

Functions of the Difference of Two Angles

$$\begin{array}{l} \sin(A - B) = \sin A \cos B - \cos A \sin B \\ \cos(A - B) = \cos A \cos B + \sin A \sin B \\ \tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B} \end{array}$$

Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Functions of the Double Angle

$$\begin{array}{l} \sin 2A = 2 \sin A \cos A \\ \cos 2A = \cos^2 A - \sin^2 A \\ \cos 2A = 2 \cos^2 A - 1 \\ \cos 2A = 1 - 2 \sin^2 A \\ \tan 2A = \frac{2 \tan A}{1 - \tan^2 A} \end{array}$$

Functions of the Half Angle

$$\begin{array}{l} \sin \frac{1}{2}A = \pm \sqrt{\frac{1 - \cos A}{2}} \\ \cos \frac{1}{2}A = \pm \sqrt{\frac{1 + \cos A}{2}} \\ \tan \frac{1}{2}A = \pm \sqrt{\frac{1 - \cos A}{1 + \cos A}} \end{array}$$

Area of Triangle

$$K = \frac{1}{2}ab \sin C$$

Standard Deviation

$$\text{S.D.} = \sqrt{\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2}$$

Part I

Answer 30 questions from this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet. Where applicable, answers may be left in terms of π or in radical form. [60]

1 Express $\frac{7\pi}{18}$ radians in degree measure.

2 Evaluate: $\sum_{k=0}^4 (3k - 5)$

3 Solve for x : $\sqrt{2x + 7} - 5 = 0$

4 In $\triangle ABC$, $m\angle A = 35$, $m\angle B = 82$, and side $a = 4$ inches. Find the length of side b to the nearest tenth of an inch.

5 Find all values of x that satisfy the equation $|2x + 3| = 11$.

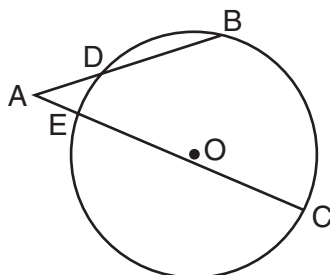
6 Find the value of the expression $2x^0 + x^{\frac{1}{3}}$ when $x = 125$.

7 Find the value of x that satisfies the equation $\log_3 x = 4$.

8 Solve for x : $\frac{2}{x} + \frac{3}{5x} = 1$

9 Solve for x : $4^{3x+1} = 8^{4x}$

10 In the accompanying diagram, secant \overline{AB} intersects circle O at D , secant \overline{AC} intersects circle O at E , $AE = 4$, $AC = 24$, and $AB = 16$. Find AD .



11 The probability that Caitlin gets an A on a mathematics test is $\frac{3}{4}$. Find the probability that she earns an A on *exactly* two of three mathematics tests.

12 Express $\frac{\frac{a}{b}}{\frac{1}{a} - \frac{1}{b}}$ in simplest form.

13 Two tangents are drawn to a circle from external point P . If the minor arc has a measure of 100° , find $m\angle P$.

14 The speed of a gear varies inversely as the number of teeth it has. If a gear with 24 teeth rotates at a speed of 45 revolutions per minute, how many revolutions per minute will a gear with 36 teeth make?

Directions (15–35): For *each* question chosen, write on the separate answer sheet the *numeral* preceding the word or expression that best completes the statement or answers the question.

15 What is the amplitude of the graph of $y = 4 \sin \frac{1}{2}x$?
 (1) 4π (3) $\frac{1}{2}$
 (2) 2π (4) 4

16 For which value of x is the function $f(x) = \frac{1}{1 + \cos x}$ undefined?
 (1) 0 (3) π
 (2) $\frac{\pi}{2}$ (4) $\frac{3\pi}{2}$

17 If $f(x) = \sin 2x + \cos x$, what is $f\left(\frac{\pi}{4}\right)$?
 (1) $1 + \frac{\sqrt{2}}{2}$ (3) $\sqrt{2}$
 (2) $\frac{1 + \sqrt{3}}{2}$ (4) 0

18 For all values of θ for which the expression is defined, $\frac{\csc \theta}{\sec \theta}$ is equivalent to

- (1) $\cos \theta$ (3) $\cot \theta$
 (2) $\sin \theta$ (4) $\tan \theta$

19 If the vertex angle of an isosceles triangle measures 30° and each leg measures 10 centimeters, what is the area of the triangle?

- (1) 25 cm^2 (3) $25\sqrt{3} \text{ cm}^2$
 (2) 50 cm^2 (4) $50\sqrt{3} \text{ cm}^2$

20 If the domain of $f(x) = 2x + 3$ is $\{-3 < x \leq 0\}$, which number is *not* in the range?

- (1) -1 (3) 3
 (2) 0 (4) 6

21 If $\cos \theta > 0$ and $\csc \theta < 0$, in which quadrant does the terminal side of θ lie?

- (1) I (3) III
 (2) II (4) IV

22 What is the product of the complex numbers $2 + 3i$ and $1 - 2i$, expressed in simplest terms?

- (1) $8 - i$ (3) $-4 - i$
 (2) 8 (4) -4

23 If side $a = 16$, side $b = 20$, and $m\angle A = 30^\circ$, how many distinct triangles can be constructed?

- (1) one acute triangle, only
 (2) two triangles
 (3) one obtuse triangle, only
 (4) no triangles

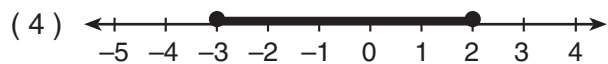
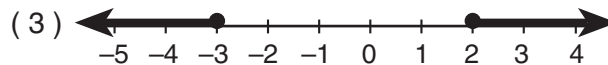
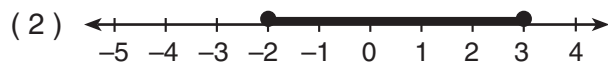
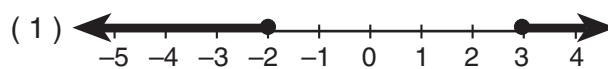
24 The graph of the equation $x^2 + 2y^2 = 5$ is

- (1) a circle (3) a parabola
 (2) an ellipse (4) a hyperbola

25 What is the value of $\tan \left(\text{Arc cos } \frac{\sqrt{3}}{2} \right)$?

- (1) 1 (3) $\sqrt{3}$
 (2) $\frac{1}{2}$ (4) $\frac{\sqrt{3}}{3}$

26 Which graph represents the solution of the inequality $x^2 - x - 6 \geq 0$?



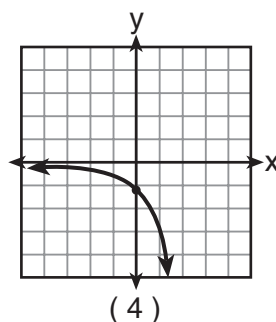
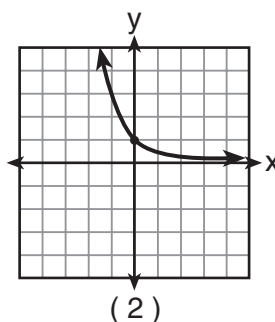
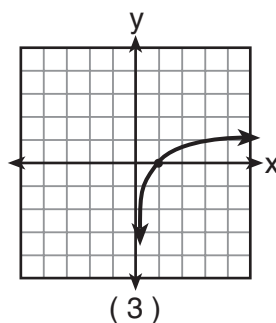
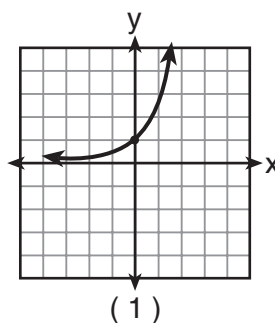
27 Which expression is equivalent to i^{37} ?

- (1) 1 (3) i
 (2) -1 (4) $-i$

28 What are the coordinates of the image of $(2, -5)$ after a counterclockwise rotation of 90° about the origin?

- (1) $(-2, 5)$ (3) $(-5, -2)$
 (2) $(2, 5)$ (4) $(5, 2)$

29 Which graph represents the inverse of the equation $y = 3^x$?



30 Expressed as a function of a positive acute angle, $\cot(-120)^\circ$ is equivalent to

- (1) $-\tan 60^\circ$ (3) $-\cot 30^\circ$
 (2) $\cot 60^\circ$ (4) $\cot 30^\circ$

31 A test was given to 120 students, and the scores approximated a normal distribution. If the mean score was 72 with a standard deviation of 7, approximately what percent of the scores were 65 or higher?

- (1) 50% (3) 76%
 (2) 68% (4) 84%

32 The roots of the equation $2x^2 + 3x - 5 = 0$ are

- (1) real, rational, and unequal
 (2) real, rational, and equal
 (3) real, irrational, and unequal
 (4) imaginary

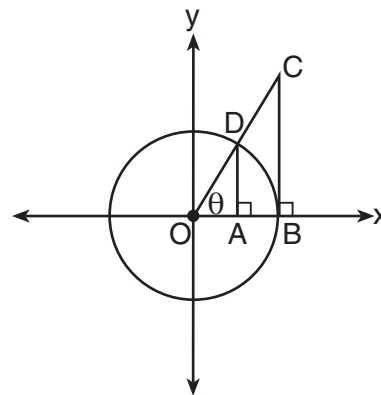
33 The expression $\frac{3 + 5\sqrt{3}}{4 - 2\sqrt{3}}$ is equivalent to

- (1) $\frac{-9 + 7\sqrt{3}}{2}$ (3) $\frac{-18 + 14\sqrt{3}}{4}$
 (2) $\frac{21 + 13\sqrt{3}}{2}$ (4) $\frac{42 - 26\sqrt{3}}{4}$

34 What is the sum of the coefficients in the expansion of $(a + b)^5$?

- (1) 5 (3) 32
 (2) 16 (4) 40

35 The accompanying diagram shows unit circle O , with radius $OD = 1$.



Which line segment has a length equivalent to $\tan \theta$?

- (1) \overline{AD} (3) \overline{OA}
 (2) \overline{BC} (4) \overline{OB}

Answers to the following questions are to be written on paper provided by the school.

Part II

Answer four questions from this part. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Calculations that may be obtained by mental arithmetic or the calculator do not need to be shown. [40]

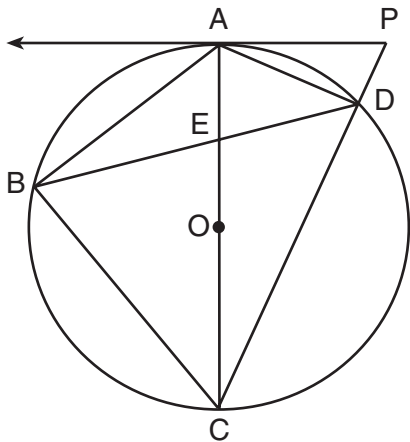
36 *a* On the same set of axes, sketch and label the graphs of $y = 3 \sin x$ and $y = \cos 2x$ in the interval $0 \leq x \leq 2\pi$. [8]

b Based on the graphs drawn in part *a*, determine all values of x in the interval $0 \leq x \leq 2\pi$ that satisfy the equation $3 \sin x - \cos 2x = 4$. [2]

37 *a* Two forces of 35 pounds and 70 pounds act on a body. The angle between the two forces is 40° . Find the magnitude of the resultant force to the nearest tenth of a pound. [6]

b Using the answer found in part *a*, determine, to the nearest degree, the angle between the resultant and the larger force. [4]

- 38 In the accompanying diagram of circle O , \overrightarrow{PA} is tangent to the circle at A ; \overline{PDC} is a secant; diameter \overline{AEOC} intersects chord \overline{BD} at E ; chords \overline{AB} , \overline{BC} , and \overline{DA} are drawn; $m\widehat{DA} = 46$; and $m\widehat{BC}$ is 32 more than $m\widehat{AB}$.



Find:

- a $m\widehat{AB}$ [2]
 b $m\angle BAC$ [2]
 c $m\angle P$ [2]
 d $m\angle DEC$ [2]
 e $m\angle PDA$ [2]
- 39 a Given the equation $y = 2^x$.
- (1) On graph paper, sketch and label the graph of the equation $y = 2^x$ in the interval $-3 \leq x \leq 3$. [2]
 - (2) On the same set of axes, reflect the graph drawn in part $a(1)$ in the line $y = x$ and label it c . [2]
 - (3) What is the equation of the graph drawn in part $a(2)$? [2]
- b Using logarithms, solve for x to the nearest hundredth: $5^x = 1,325$ [4]
- 40 Find all values of θ in the interval $0^\circ \leq \theta < 360^\circ$ that satisfy the equation $3 \cos 2\theta = 7 \cos \theta$. Express your answer to the nearest tenth of a degree or nearest ten minutes. [10]

- 41 a Solve for x and express the roots in simplest $a + bi$ form: $9x + \frac{2}{x} = -6$ [6]

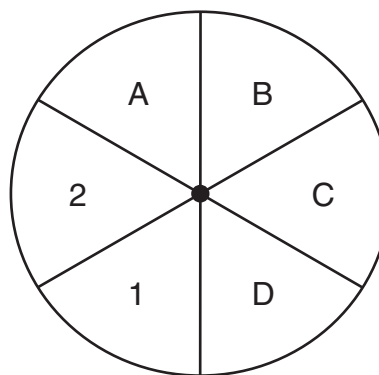
b Given: $f(x) = \sqrt{2x + 5}$ and $g(x) = 6x - 3$

Find:

(1) $g(f(10))$ [2]

(2) $(f \circ g)(x)$ [2]

- 42 a A spinner is divided into six equal sections and labeled as shown in the accompanying diagram.



- (1) Determine the probability of getting a letter in one spin. [1]
- (2) Determine the probability of getting *no* letters in three spins. [2]
- (3) Determine the probability of getting *at least* two letters in three spins. [3]

- b The table below shows the scores that a class of students received on their latest review quiz.

Score	Frequency
95	6
90	7
85	8
80	4

Find the standard deviation of these scores to the nearest tenth. [4]

The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

SEQUENTIAL MATH – COURSE III

Wednesday, August 13, 2003 — 8:30 to 11:30 a.m., only

Part I Score
Part II Score
Total Score
Rater's Initials:

ANSWER SHEET

Student Sex: Male Female Grade

Teacher School

Your answers to Part I should be recorded on this answer sheet.

Part I

Answer 30 questions from this part.

- | | | | |
|----------|----------|----------|----------|
| 1 | 11 | 21 | 31 |
| 2 | 12 | 22 | 32 |
| 3 | 13 | 23 | 33 |
| 4 | 14 | 24 | 34 |
| 5 | 15 | 25 | 35 |
| 6 | 16 | 26 | |
| 7 | 17 | 27 | |
| 8 | 18 | 28 | |
| 9 | 19 | 29 | |
| 10 | 20 | 30 | |

Your answers for Part II should be placed on paper provided by the school.

The declaration below should be signed when you have completed the examination.

I do hereby affirm, at the close of this examination, that I had no unlawful knowledge of the questions or answers prior to the examination and that I have neither given nor received assistance in answering any of the questions during the examination.

Signature

Tear Here

Tear Here

Tear Here

Tear Here