The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

GEOMETRY (Common Core)

Friday, June 17, 2016 — 1:15 to 4:15 p.m., only

Student Name: _________________________________________________________

School Name: _______________________________________________________________

Print your name and the name of your school on the lines above.

A separate answer sheet for Part I has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 36 questions. You must answer all questions in this examination. Write your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. All work should be written in pen, except for graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will not be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice...
A graphing calculator, a straightedge (ruler), and a compass must be available for you to use while taking this examination.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.
Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet.

1. A student has a rectangular postcard that he folds in half lengthwise. Next, he rotates it continuously about the folded edge. Which three-dimensional object below is generated by this rotation?

![Diagram of possible objects](image)

(1) Cone (2) Cylinder (3) Pyramid (4) Cube

2. A three-inch line segment is dilated by a scale factor of 6 and centered at its midpoint. What is the length of its image?

(1) 9 inches (2) 2 inches (3) 15 inches (4) 18 inches

Use this space for computations.
3 Kevin’s work for deriving the equation of a circle is shown below.

\[ x^2 + 4x = -(y^2 - 20) \]

**STEP 1** \[ x^2 + 4x = -y^2 + 20 \]

**STEP 2** \[ x^2 + 4x + 4 = -y^2 + 20 - 4 \]

**STEP 3** \( (x + 2)^2 = -y^2 + 20 - 4 \)

**STEP 4** \( (x + 2)^2 + y^2 = 16 \)

In which step did he make an error in his work?

(1) Step 1  (3) Step 3
(2) Step 2  (4) Step 4

4 Which transformation of \( \overline{OA} \) would result in an image parallel to \( \overline{OA} \)?

(1) a translation of two units down
(2) a reflection over the \( x \)-axis
(3) a reflection over the \( y \)-axis
(4) a clockwise rotation of 90° about the origin
Using the information given below, which set of triangles can not be proven similar?

6 A company is creating an object from a wooden cube with an edge length of 8.5 cm. A right circular cone with a diameter of 8 cm and an altitude of 8 cm will be cut out of the cube. Which expression represents the volume of the remaining wood?

\[
\begin{align*}
(1) \quad & (8.5)^3 - \pi(8)^2(8) \\
(2) \quad & (8.5)^3 - \pi(4)^2(8) \\
(3) \quad & (8.5)^3 - \frac{1}{3} \pi(8)^2(8) \\
(4) \quad & (8.5)^3 - \frac{1}{3} \pi(4)^2(8)
\end{align*}
\]

7 Two right triangles must be congruent if

(1) an acute angle in each triangle is congruent
(2) the lengths of the hypotenuses are equal
(3) the corresponding legs are congruent
(4) the areas are equal
8 Which sequence of transformations will map $\triangle ABC$ onto $\triangle A'B'C'$?

(1) reflection and translation
(2) rotation and reflection
(3) translation and dilation
(4) dilation and rotation

9 In parallelogram $ABCD$, diagonals $\overline{AC}$ and $\overline{BD}$ intersect at $E$. Which statement does not prove parallelogram $ABCD$ is a rhombus?

(1) $\overline{AC} \cong \overline{DB}$
(2) $\overline{AB} \cong \overline{BC}$
(3) $\overline{AC} \perp \overline{DB}$
(4) $\overline{AC}$ bisects $\angle DCB$. 
10 In the diagram below of circle $O$, $\overline{OB}$ and $\overline{OC}$ are radii, and chords $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$ are drawn.

Which statement must always be true?

(1) $\angle BAC \cong \angle BOC$

(2) $m\angle BAC = \frac{1}{2} m\angle BOC$

(3) $\triangle BAC$ and $\triangle BOC$ are isosceles.

(4) The area of $\triangle BAC$ is twice the area of $\triangle BOC$.

11 A 20-foot support post leans against a wall, making a 70° angle with the ground. To the nearest tenth of a foot, how far up the wall will the support post reach?

(1) 6.8  (3) 18.7

(2) 6.9  (4) 18.8

12 Line segment $NY$ has endpoints $N(-11,5)$ and $Y(5,-7)$. What is the equation of the perpendicular bisector of $\overline{NY}$?

(1) $y + 1 = \frac{4}{3}(x + 3)$

(2) $y + 1 = -\frac{3}{4}(x + 3)$

(3) $y - 6 = \frac{4}{3}(x - 8)$

(4) $y - 6 = -\frac{3}{4}(x - 8)$
13 In \( \triangle RST \) shown below, altitude \( SU \) is drawn to \( RT \) at \( U \).

\[
\text{If } SU = h, \ UT = 12, \text{ and } RT = 42, \text{ which value of } h \text{ will make } \triangle RST \text{ a right triangle with } \angle RST \text{ as a right angle?}
\]

(1) \(6\sqrt{3}\) \hspace{1cm} (3) \(6\sqrt{14}\)

(2) \(6\sqrt{10}\) \hspace{1cm} (4) \(6\sqrt{35}\)

14 In the diagram below, \( \triangle ABC \) has vertices \( A(4,5), B(2,1), \text{ and } C(7,3) \).

What is the slope of the altitude drawn from \( A \) to \( BC \)?

(1) \(\frac{2}{5}\) \hspace{1cm} (3) \(-\frac{1}{2}\)

(2) \(\frac{3}{2}\) \hspace{1cm} (4) \(-\frac{5}{2}\)
15 In the diagram below, $\triangle ERM \sim \triangle JTM$.

Which statement is always true?

1. $\cos J = \frac{RM}{RE}$
2. $\cos R = \frac{JM}{JT}$
3. $\tan T = \frac{RM}{EM}$
4. $\tan E = \frac{TM}{JM}$

16 On the set of axes below, rectangle $ABCD$ can be proven congruent to rectangle $KLMN$ using which transformation?

1. rotation
2. translation
3. reflection over the $x$-axis
4. reflection over the $y$-axis
17 In the diagram below, \( \overline{DB} \) and \( \overline{AF} \) intersect at point \( C \), and \( \overline{AD} \) and \( \overline{FBE} \) are drawn.

If \( AC = 6 \), \( DC = 4 \), \( FC = 15 \), \( m \angle D = 65^\circ \), and \( m \angle CBE = 115^\circ \), what is the length of \( \overline{CB} \)?

(1) 10 
(2) 12 
(3) 17 
(4) 22.5

18 Seawater contains approximately 1.2 ounces of salt per liter on average. How many gallons of seawater, to the nearest tenth of a gallon, would contain 1 pound of salt?

(1) 3.3 
(2) 3.5 
(3) 4.7 
(4) 13.3
19 Line segment $EA$ is the perpendicular bisector of $ZT$, and $ZE$ and $TE$ are drawn.

Which conclusion can \textit{not} be proven?

(1) $EA$ bisects angle $ZET$.
(2) Triangle $EZT$ is equilateral.
(3) $EA$ is a median of triangle $EZT$.
(4) Angle $Z$ is congruent to angle $T$.

20 A hemispherical water tank has an inside diameter of 10 feet. If water has a density of 62.4 pounds per cubic foot, what is the weight of the water in a full tank, to the \textit{nearest pound}?

(1) 16,336 \hspace{1cm} (3) 130,690
(2) 32,673 \hspace{1cm} (4) 261,381
21 In the diagram of \( \triangle ABC \), points \( D \) and \( E \) are on \( AB \) and \( CB \), respectively, such that \( AC \parallel DE \).

If \( AD = 24 \), \( DB = 12 \), and \( DE = 4 \), what is the length of \( AC \)?

(1) 8 \hspace{1cm} (3) 16

(2) 12 \hspace{1cm} (4) 72

22 Triangle \( RST \) is graphed on the set of axes below.

How many square units are in the area of \( \triangle RST \)?

(1) \( 9\sqrt{3} + 15 \) \hspace{1cm} (3) 45

(2) \( 9\sqrt{5} + 15 \) \hspace{1cm} (4) 90
23 The graph below shows \( \overline{AB} \), which is a chord of circle \( O \). The coordinates of the endpoints of \( \overline{AB} \) are \( A(3,3) \) and \( B(3,-7) \). The distance from the midpoint of \( \overline{AB} \) to the center of circle \( O \) is 2 units.

What could be a correct equation for circle \( O \)?

(1) \((x - 1)^2 + (y + 2)^2 = 29\)
(2) \((x + 5)^2 + (y - 2)^2 = 29\)
(3) \((x - 1)^2 + (y - 2)^2 = 25\)
(4) \((x - 5)^2 + (y + 2)^2 = 25\)

24 What is the area of a sector of a circle with a radius of 8 inches and formed by a central angle that measures 60°?

(1) \(\frac{8\pi}{3}\)
(2) \(\frac{16\pi}{3}\)
(3) \(\frac{32\pi}{3}\)
(4) \(\frac{64\pi}{3}\)
Part II

Answer all 7 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [14]

25 Describe a sequence of transformations that will map \( \triangle ABC \) onto \( \triangle DEF \) as shown below.
26 Point $P$ is on segment $AB$ such that $AP:PB$ is 4:5. If $A$ has coordinates $(4,2)$, and $B$ has coordinates $(22,2)$, determine and state the coordinates of $P$. 
27 In $\triangle CED$ as shown below, points $A$ and $B$ are located on sides $CE$ and $ED$, respectively. Line segment $AB$ is drawn such that $AE = 3.75, AC = 5, EB = 4.5$, and $BD = 6$.

Explain why $\overline{AB}$ is parallel to $\overline{CD}$. 
28 Find the value of \( R \) that will make the equation \( \sin 73^\circ = \cos R \) true when \( 0^\circ < R < 90^\circ \). Explain your answer.
29 In the diagram below, Circle 1 has radius 4, while Circle 2 has radius 6.5. Angle $A$ intercepts an arc of length $\pi$, and angle $B$ intercepts an arc of length $\frac{13\pi}{8}$.

Dominic thinks that angles $A$ and $B$ have the same radian measure. State whether Dominic is correct or not. Explain why.
30 A ladder leans against a building. The top of the ladder touches the building 10 feet above the ground. The foot of the ladder is 4 feet from the building. Find, to the nearest degree, the angle that the ladder makes with the level ground.
31 In the diagram below, radius $OA$ is drawn in circle $O$. Using a compass and a straightedge, construct a line tangent to circle $O$ at point $A$. [Leave all construction marks.]
Part III

Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

32 A barrel of fuel oil is a right circular cylinder where the inside measurements of the barrel are a diameter of 22.5 inches and a height of 33.5 inches. There are 231 cubic inches in a liquid gallon. Determine and state, to the nearest tenth, the gallons of fuel that are in a barrel of fuel oil.
33 Given: Parallelogram $ABCD$, $EFG$, and diagonal $DFB$

Prove: $\triangle DEF \sim \triangle BGF$
34 In the diagram below, \( \triangle A'B'C' \) is the image of \( \triangle ABC \) after a transformation.

Describe the transformation that was performed.

Explain why \( \triangle A'B'C' \sim \triangle ABC \).
Part IV

Answer the 2 questions in this part. Each correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

35 Given: Quadrilateral $ABCD$ with diagonals $AC$ and $BD$ that bisect each other, and $\angle 1 \equiv \angle 2$

![Diagram of quadrilateral ABCD with diagonals AC and BD that bisect each other, and angles 1 and 2 labeled.

Prove: $\triangle ACD$ is an isosceles triangle and $\triangle AEB$ is a right triangle.
A water glass can be modeled by a truncated right cone (a cone which is cut parallel to its base) as shown below.

![Diagram of a truncated right cone](image)

The diameter of the top of the glass is 3 inches, the diameter at the bottom of the glass is 2 inches, and the height of the glass is 5 inches.

The base with a diameter of 2 inches must be parallel to the base with a diameter of 3 inches in order to find the height of the cone. Explain why.

Question 36 is continued on the next page.
Question 36 continued

Determine and state, in inches, the height of the larger cone.

Determine and state, to the nearest tenth of a cubic inch, the volume of the water glass.
### High School Math Reference Sheet

1 inch = 2.54 centimeters  
1 meter = 39.37 inches  
1 mile = 5280 feet  
1 mile = 1760 yards  
1 mile = 8 fluid ounces  
1 cup = 16 ounces  
1 pint = 0.454 kilogram  
1 kilogram = 2.2 pounds  
1 quart = 2 cups  
1 gallon = 4 quarts  
1 gallon = 3.785 liters  
1 liter = 0.264 gallon  
1 meter = 39.37 inches  
1 pound = 16 ounces  
1 meter = 1.609 kilometers  
1 mile = 2000 pounds  
1 mile = 1.609 kilometers  
1 mile = 1760 yards  
1 mile = 5280 feet  
1 mile = 0.62 mile  
1 mile = 1760 yards  
1 mile = 1.609 kilometers  
1 mile = 8 fluid ounces  
1 cup = 8 fluid ounces  
1 liter = 1000 cubic centimeters

<table>
<thead>
<tr>
<th>Shape</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangle</td>
<td>$A = \frac{1}{2}bh$</td>
</tr>
<tr>
<td>Parallelogram</td>
<td>$A = bh$</td>
</tr>
<tr>
<td>Circle</td>
<td>$A = \pi r^2$</td>
</tr>
<tr>
<td>Circle</td>
<td>$C = \pi d$ or $C = 2\pi r$</td>
</tr>
<tr>
<td>General Prisms</td>
<td>$V = Bh$</td>
</tr>
<tr>
<td>Cylinder</td>
<td>$V = \pi r^2h$</td>
</tr>
<tr>
<td>Sphere</td>
<td>$V = \frac{4}{3}\pi r^3$</td>
</tr>
<tr>
<td>Cone</td>
<td>$V = \frac{1}{3}\pi r^2h$</td>
</tr>
<tr>
<td>Pyramid</td>
<td>$V = \frac{1}{3}Bh$</td>
</tr>
</tbody>
</table>

### Pythagorean Theorem

$$a^2 + b^2 = c^2$$

### Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

### Arithmetic Sequence

$$a_n = a_1 + (n - 1)d$$

### Geometric Sequence

$$a_n = a_1r^{n-1}$$

### Geometric Series

$$S_n = \frac{a_1 - a_1r^n}{1 - r}$$ where $r \neq 1$

### Radians

1 radian $= \frac{180}{\pi}$ degrees

### Degrees

1 degree $= \frac{\pi}{180}$ radians

### Exponential Growth/Decay

$$A = A_0 e^{k(t - t_0)} + B_0$$
Scrap Graph Paper — This sheet will not be scored.
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