

The University of the State of New York  
REGENTS HIGH SCHOOL EXAMINATION

# ALGEBRA II

Monday, June 24, 2024 — 9:15 a.m. to 12:15 p.m., only

## MODEL RESPONSE SET

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Question 25

25 Given  $x$  is a real number, write the expression in simplest  $a + bi$  form:

$$(x + 2i)(3 - 2xi) + 2x^2i$$

$$3x \cancel{- 2x^2i} + 6i - 4xi^2 \cancel{+ 2x^2i}$$

$$3x + 6i - 4xi^2$$

$$3x + 6i + 4x$$

$$\boxed{7x + 6i}$$

$$i^1 = i$$

$$i^2 = -1$$

$$i^3 = -i$$

$$i^4 = 1$$

**Score 2:** The student gave a complete and correct response.

**Question 25**

25 Given  $x$  is a real number, write the expression in simplest  $a + bi$  form:

$$(x + 2i)(3 - 2xi) + 2x^2i$$

	$x$	$2i$
$3$	$3x$	$6i$
$-2xi$	$-2x^2i$	$-4xi^2$

$$3x + 6i - 2x^2i + 4x$$

$$7x - \cancel{2x^2i} + 6i + \cancel{2x^2i}$$

$6i + 7x$
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**Score 2:** The student gave a complete and correct response.

**Question 25**

25 Given  $x$  is a real number, write the expression in simplest  $a + bi$  form:

$$(x + 2i)(3 - 2xi) + 2x^2i$$

$$3x - \cancel{2x^2i} + 6i - 4xi^2 + \cancel{2x^2i}$$

$$3x + 6i - 4xi^2$$

**Score 1:** The student did not express the result in simplest  $a + bi$  form.

**Question 25**

25 Given  $x$  is a real number, write the expression in simplest  $a + bi$  form:

$$(x + 2i)(3 - 2xi) + 2x^2i$$

27

$$3x - 2x^2i + 6i - 4xi^2 + 2x^2i$$

$$3x + 6i - 4xi^2$$

$$3x + 6i - 4x$$

$$\boxed{-x + 6i}$$

**Score 1:** The student incorrectly evaluated  $i^2$ .

**Question 25**

25 Given  $x$  is a real number, write the expression in simplest  $a + bi$  form:

$$(x + 2i)(3 - 2xi) + 2x^2i$$

$$3x - \cancel{2x^2} + 6i + 2xi^2 + \cancel{2x^2i}$$

$$3x + 6i - 2xi^2$$

$$1 + 7i^3$$

**Score 0:** The student made multiple errors.

**Question 25**

25 Given  $x$  is a real number, write the expression in simplest  $a + bi$  form:

$$(x + 2i)(3 - 2xi) + 2x^2i$$

$$(5+2i)(3-2(5)i) + 2(5)^2i;$$

$$35 - 44i + 2(25)i$$

$$35 - 44i + 50i$$

$$35 + 6i$$

**Score 0:** The student did not show enough relevant course-level work to receive any credit by evaluating the expression for  $x = 5$ .

**Question 26**

26 Solve  $3.8e^{1.5t} = 16$  algebraically for  $t$  to the *nearest hundredth*.

$$\frac{3.8e^{1.5t}}{3.8} = \frac{16}{3.8}$$

$$e^{1.5t} = 4.210526316$$

$$\ln e^{1.5t} = \ln 4.210526316$$

$$\frac{1.5t \ln e}{\ln e} = \frac{\ln 4.210526316}{\ln e}$$

$$\frac{1.5t}{1.5} = \frac{1.437587656}{1.5}$$

$$t = 0.96$$

**Score 2:** The student gave a complete and correct response.



**Question 26**

26 Solve  $3.8e^{1.5t} = 16$  algebraically for  $t$  to the nearest hundredth.

3.8

$$e^{1.5t} = 4.210526316$$

$$\frac{1.5t \ln e = \ln 4.210526316}{1.5}$$

$$= .95839$$

$$\approx \boxed{.96}$$

**Score 2:** The student gave a complete and correct response.

**Question 26**

26 Solve  $3.8e^{1.5t} = 16$  algebraically for  $t$  to the *nearest hundredth*.

$$\begin{aligned}3.8e^{1.5t} &= 16 \\ \frac{\ln 3.8e^{1.5t}}{\ln 3.8} &= \frac{\ln 16}{\ln 3.8} \\ e^{1.5t} &= 2.07684 \\ \ln e^{1.5t} &= \ln 2.07684 \\ \frac{1.5t \ln e}{1.5} &= \frac{\ln 2.07684}{1.5} \\ t &= 0.487232 \\ t &= 0.49\end{aligned}$$

**Score 1:** The student incorrectly applied the natural log.

**Question 26**

26 Solve  $3.8e^{1.5t} = 16$  algebraically for  $t$  to the nearest hundredth.

$$\begin{aligned}3.8e^{1.5(t)} &= 16 \\3.8e^{1.5(2)} &= 76.325 \\3.8e^{1.5(1)} &= 17.03041 \\3.8e^{1.5(.9)} &= 14.658 \\3.8e^{1.5(.959)} &= 16.014\end{aligned}$$

$$t \approx .96$$

**Score 1:** The student used a method other than algebraic.

Question 26

26 Solve  $3.8e^{1.5t} = 16$  algebraically for  $t$  to the nearest hundredth.

$$\frac{16}{3.8} = \frac{3.8e^{1.5t}}{3.8}$$

$$4.21 = e^{1.5t}$$

$$t = 1.03$$

**Score 0:** The student did not show enough relevant course-level work to receive any credit.

**Question 26**

26 Solve  $3.8e^{1.5t} = 16$  algebraically for  $t$  to the *nearest hundredth*.

$$1.5t \frac{\log 38}{\log 38} = \frac{\log 16}{\log 38}$$

$$1.5t = 2.07$$

**Score 0:** The student incorrectly applied the logarithm and did not solve for  $t$ .

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**Question 27**

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**27** In an attempt to get the student body's opinion of a new dress code, members of the statistics class surveyed the students of the first period computer science class. Explain a statistical bias in the method of data collection.

One statistical bias is surveying students of a first period class. This is because one class in the morning does not represent every student as people can have different schedules

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**Score 2:** The student gave a complete and correct response.

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**Question 27**

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**27** In an attempt to get the student body's opinion of a new dress code, members of the statistics class surveyed the students of the first period computer science class. Explain a statistical bias in the method of data collection.

The people who take Computer Science might all have similar opinions and in order to obtain better results they would need to ask different types of classes.

**Score 2:** The student gave a complete and correct response.

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**Question 27**

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**27** In an attempt to get the student body's opinion of a new dress code, members of the statistics class surveyed the students of the first period computer science class. Explain a statistical bias in the method of data collection.

They only collect data from computer science class.

**Score 1:** The student did not sufficiently explain the bias.



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**Question 27**

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**27** In an attempt to get the student body's opinion of a new dress code, members of the statistics class surveyed the students of the first period computer science class. Explain a statistical bias in the method of data collection.

The surveyers only surveyed one class which most likely does not take up even half of the student body. There was no randomness.

**Score 1:** The student gave an incomplete explanation of the bias.

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**Question 27**

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**27** In an attempt to get the student body's opinion of a new dress code, members of the statistics class surveyed the students of the first period computer science class. Explain a statistical bias in the method of data collection.

the survey taken was not on  
a large enough scale

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**Score 0:** The student did not satisfy the criteria for one or more credits.

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**Question 27**

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**27** In an attempt to get the student body's opinion of a new dress code, members of the statistics class surveyed the students of the first period computer science class. Explain a statistical bias in the method of data collection.

they went to find out how the first period class is dressed.

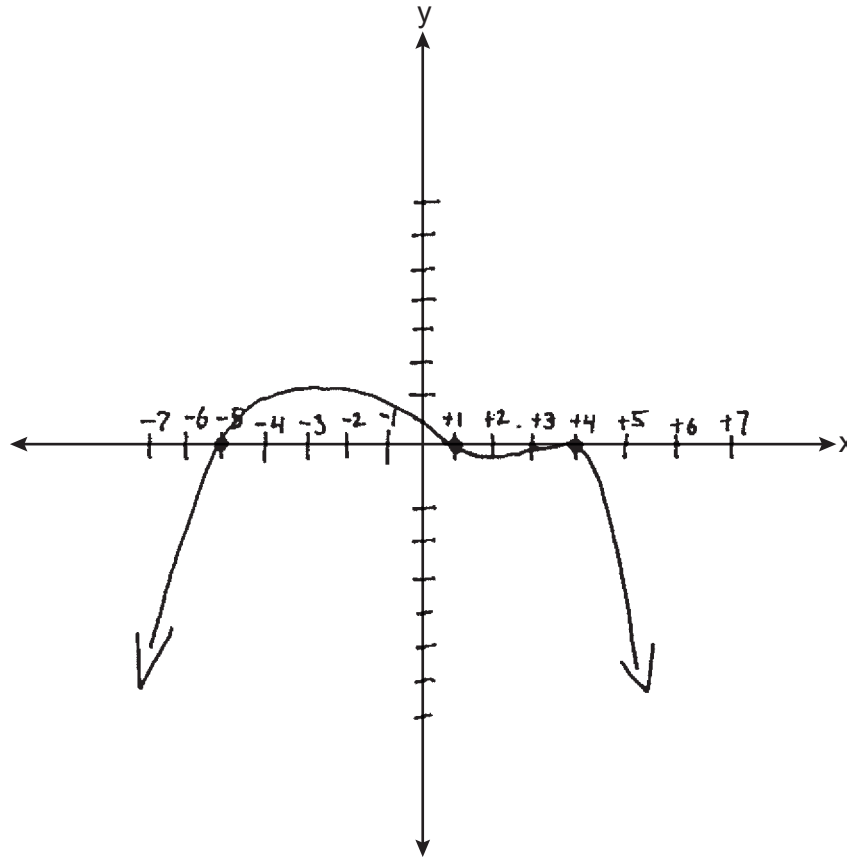
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**Score 0:** The student did not show enough relevant course-level work to receive any credit.

**Question 28**

**28** Sketch a graph of polynomial  $P(x)$ , given the criteria below:

- $P(x)$  has zeros only at  $-5$ ,  $1$ , and  $4$
- As  $x \rightarrow \infty$ ,  $P(x) \rightarrow -\infty$
- As  $x \rightarrow -\infty$ ,  $P(x) \rightarrow -\infty$

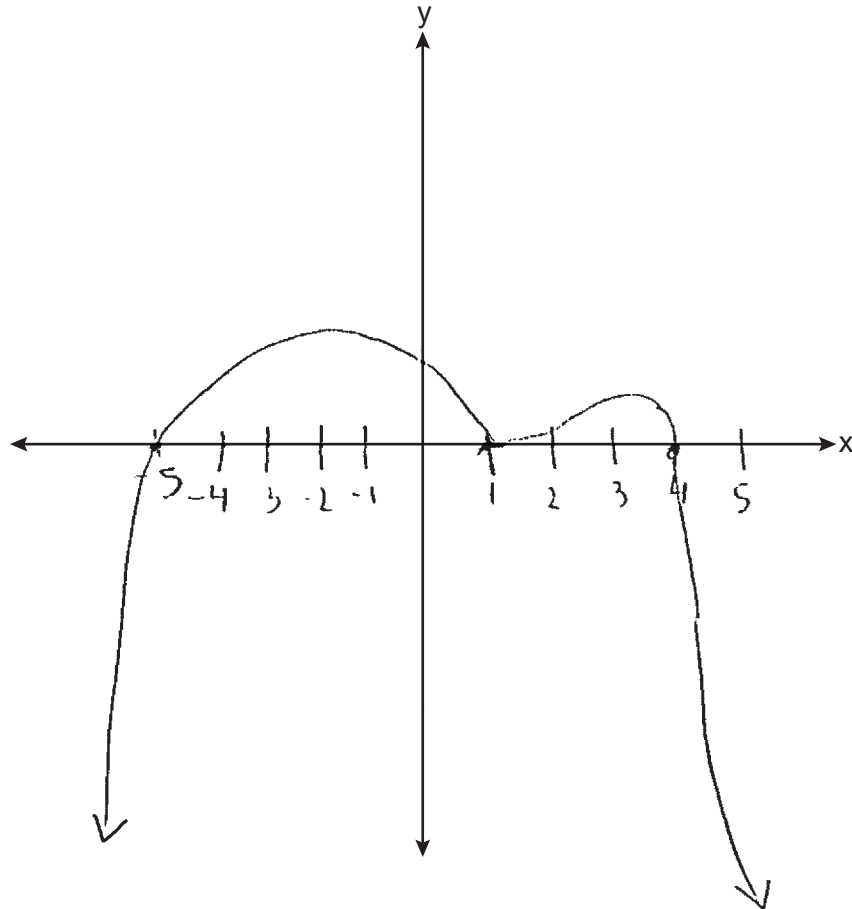


**Score 2:** The student gave a complete and correct response.

**Question 28**

**28** Sketch a graph of polynomial  $P(x)$ , given the criteria below:

- $P(x)$  has zeros only at  $-5$ ,  $1$ , and  $4$
- As  $x \rightarrow \infty$ ,  $P(x) \rightarrow -\infty$
- As  $x \rightarrow -\infty$ ,  $P(x) \rightarrow -\infty$



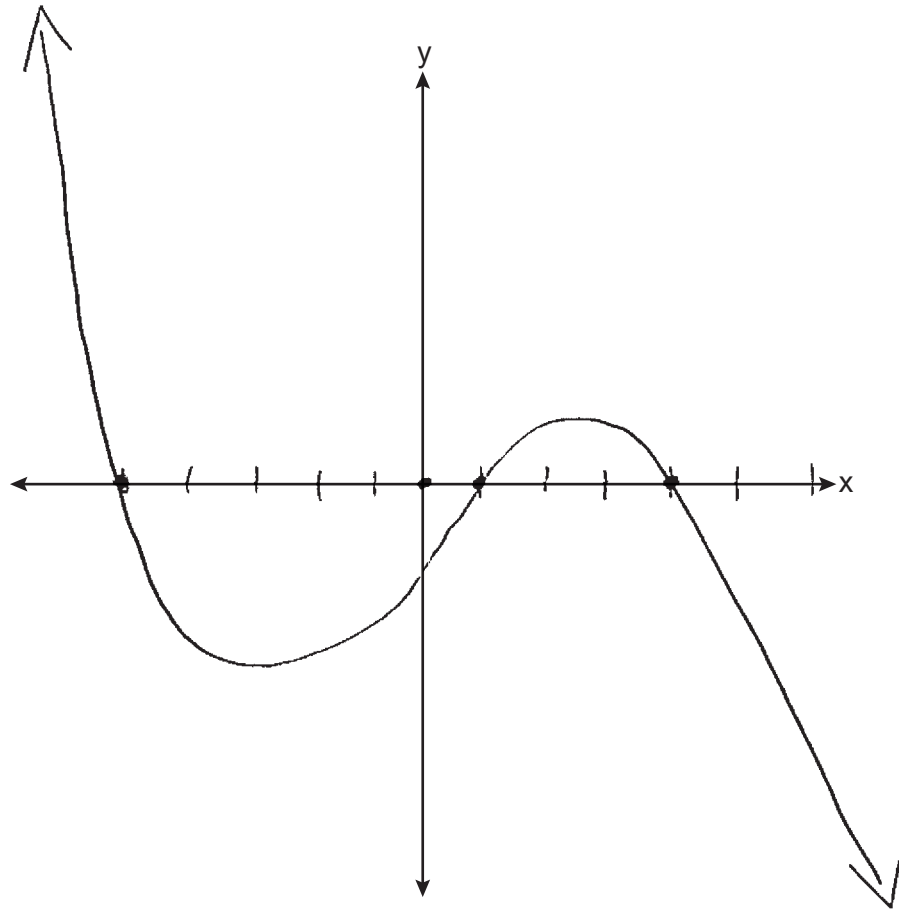
**Score 2:** The student gave a complete and correct response.

Question 28

28 Sketch a graph of polynomial  $P(x)$ , given the criteria below:

- $P(x)$  has zeros only at -5, 1, and 4
- As  $x \rightarrow \infty$ ,  $P(x) \rightarrow -\infty$
- As  $x \rightarrow -\infty$ ,  $P(x) \rightarrow -\infty$

$x^3$

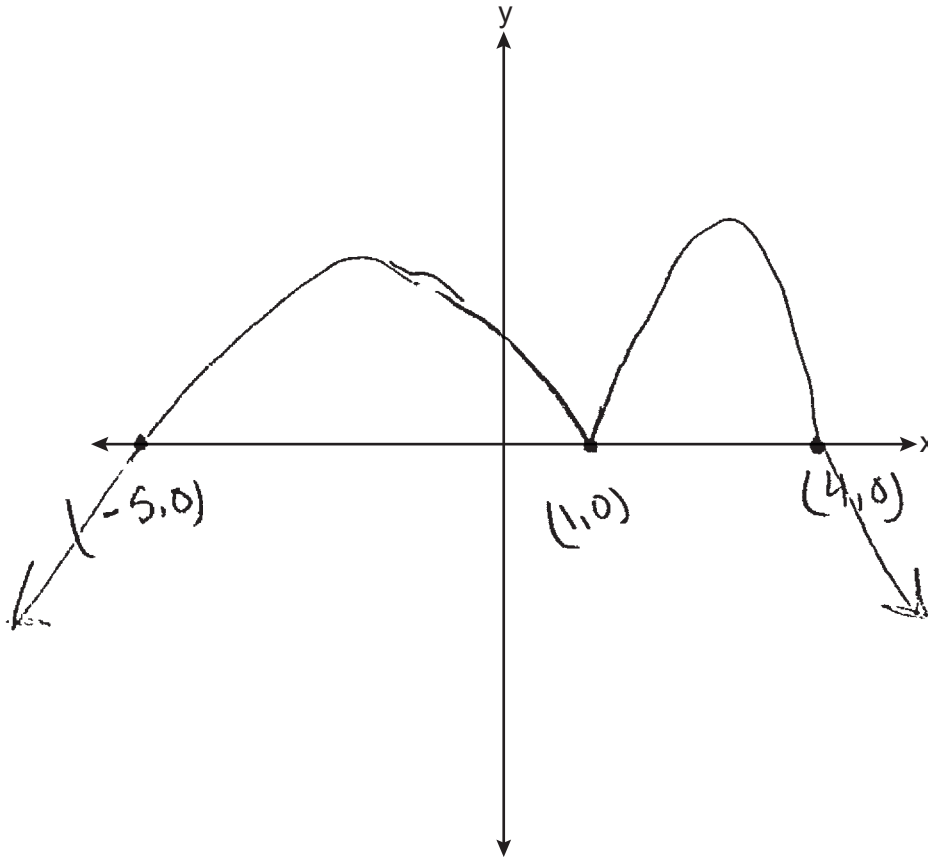


**Score 1:** The student incorrectly graphed the end behavior.

**Question 28**

**28** Sketch a graph of polynomial  $P(x)$ , given the criteria below:

- $P(x)$  has zeros only at  $-5$ ,  $1$ , and  $4$
- As  $x \rightarrow \infty$ ,  $P(x) \rightarrow -\infty$
- As  $x \rightarrow -\infty$ ,  $P(x) \rightarrow -\infty$

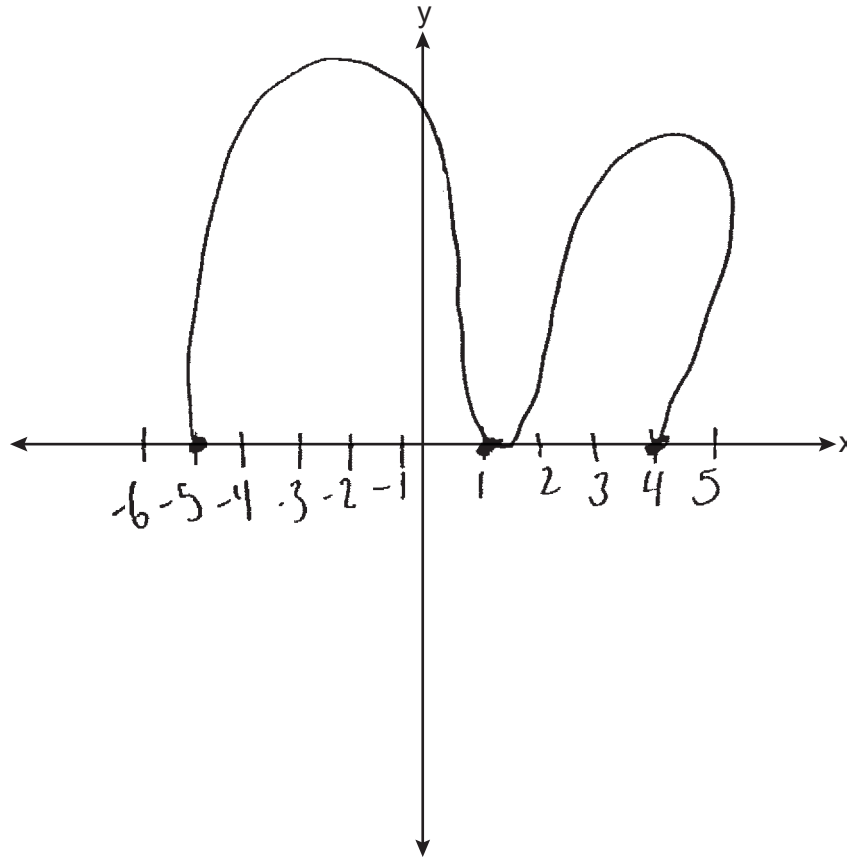


**Score 1:** The student did not graph a polynomial function.

**Question 28**

**28** Sketch a graph of polynomial  $P(x)$ , given the criteria below:

- $P(x)$  has zeros only at  $-5$ ,  $1$ , and  $4$
- As  $x \rightarrow \infty$ ,  $P(x) \rightarrow -\infty$
- As  $x \rightarrow -\infty$ ,  $P(x) \rightarrow -\infty$



**Score 0:** The student made multiple graphing errors.

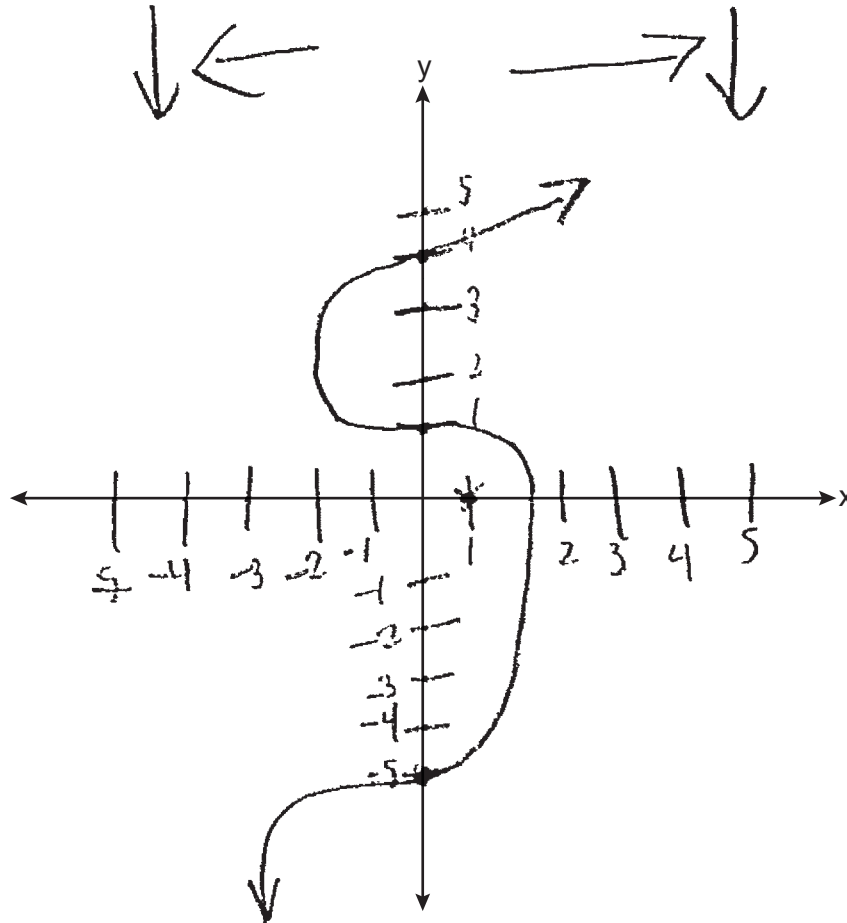


Question 28

28 Sketch a graph of polynomial  $P(x)$ , given the criteria below:

- $P(x)$  has zeros only at  $-5$ ,  $1$ , and  $4$
- As  $x \rightarrow \infty$ ,  $P(x) \rightarrow -\infty$
- As  $x \rightarrow -\infty$ ,  $P(x) \rightarrow -\infty$

$$P(x) = -5$$



**Score 0:** The student made multiple graphing errors.

**Question 29**

29 The height, above ground, of a Ferris wheel car can be modeled by the function

$$h(t) = -103.5\cos\left(\frac{2\pi t}{5}\right) + 108.5$$
 where  $h$  is measured in feet and  $t$  is measured in minutes.

State the period of the function and describe what the period represents in this context.

$$PB = 2\pi$$

$$\frac{2\pi}{5} P = 2\pi$$
$$\frac{2\pi}{5} \quad \frac{2\pi}{5}$$

$$P = 5$$

$$\frac{2\pi}{1} \cdot \frac{5}{2\pi} = \frac{10\pi}{2\pi} = 5$$

The period in this context is how long it takes for the ferris wheel to make one full rotation.

**Score 2:** The student gave a complete and correct response.

**Question 29**

**29** The height, above ground, of a Ferris wheel car can be modeled by the function

$$h(t) = -103.5\cos\left(\frac{2\pi t}{5}\right) + 108.5$$
 where  $h$  is measured in feet and  $t$  is measured in minutes.

State the period of the function and describe what the period represents in this context.

The period is 5 which means every 5 minutes the ferris wheel car will return to the bottom of the ride

**Score 2:** The student gave a complete and correct response.

**Question 29**

**29** The height, above ground, of a Ferris wheel car can be modeled by the function

$$h(t) = -103.5\cos\left(\frac{2\pi t}{5}\right) + 108.5$$
 where  $h$  is measured in feet and  $t$  is measured in minutes.

State the period of the function and describe what the period represents in this context.

The period represents how long it takes to go around the ferris wheel one time.

**Score 1:** The student did not state the period.

**Question 29**

**29** The height, above ground, of a Ferris wheel car can be modeled by the function

$$h(t) = -103.5\cos\left(\frac{2\pi t}{5}\right) + 108.5$$
 where  $h$  is measured in feet and  $t$  is measured in minutes.

State the period of the function and describe what the period represents in this context.

$$\frac{\frac{2\pi}{5}}{2}$$

The period represents how long it takes the ferris wheel to go around once

**Score 1:** The student stated the period incorrectly, but wrote a correct description.

**Question 29**

29 The height, above ground, of a Ferris wheel car can be modeled by the function

$$h(t) = -103.5\cos\left(\frac{2\pi t}{5}\right) + 108.5$$
 where  $h$  is measured in feet and  $t$  is measured in minutes.

State the period of the function and describe what the period represents in this context.

$$\frac{2\pi}{3} \left( \frac{2\pi}{5} \right)$$

$$\frac{2\pi}{3} \times \frac{5}{2\pi} = \frac{10\pi}{6\pi} = \frac{5}{3}$$

$$\text{Period} = \frac{5}{3}$$

The period represents how many full cycles the function will take

**Score 0:** The student determined the period incorrectly and did not state the description in context.

**Question 29**

29 The height, above ground, of a Ferris wheel car can be modeled by the function

$$h(t) = -103.5\cos\left(\frac{2\pi t}{5}\right) + 108.5$$
 where  $h$  is measured in feet and  $t$  is measured in minutes.

State the period of the function and describe what the period represents in this context.

The period is  $\frac{2\pi}{5}$  which represents the amount of time it takes for a certain part of the Ferris wheel to reach the top from the bottom  $\rightarrow$  one full cycle



**Score 0:** The student stated the period incorrectly and wrote an incorrect description.

**Question 30**

30 Solve algebraically for all values of  $x$ :

$$\frac{8}{x+5} - \frac{3}{x} = 5$$

$$8x - 3x - 15 = 5x^2 + 25x$$
$$- \underline{5x + 15} \quad - \underline{5x + 15}$$

$$0 = 5x^2 + 20x + 15$$

$$0 = 5(x^2 + 4x + 3)$$

$$0 = 5(x^2 + 3x | + x + 3)$$
$$5 | x(x+3) | (x+3) |$$

$$5 \neq 0 \quad x+1=0 \quad x+3=0$$
$$x = -1 \quad x = -3$$

**Score 2:** The student gave a complete and correct response.



**Question 30**

30 Solve algebraically for all values of  $x$ :

$$\frac{x}{x} \cdot \frac{8}{x+5} - \frac{3}{x} = 5$$

$$\frac{8}{2} - \frac{3}{-3} = 5$$

$$4 + 1 = 5$$
$$5 = 5 \checkmark$$

$$\frac{8x - 3x - 15}{x^2 + 5x} = \frac{5}{1}$$

$$5x^2 + 25x = 5x - 15$$

$$5x^2 + 20x + 15 = 0$$

$$5(x^2 + 4x + 3) = 0$$

$$5(x+3)(x+1) = 0$$

$$\boxed{x = -3, -1}$$

$$\frac{8}{4} - \frac{3}{-1} = 5$$

$$2 + 3 = 5$$
$$5 = 5 \checkmark$$

**Score 2:** The student gave a complete and correct response.

Question 30

30 Solve algebraically for all values of  $x$ :

$$\frac{(x) \cdot 8}{(x) \cdot (x+5)} - \frac{3(x+5)}{x(x+5)} = \frac{5}{(x+5)}$$

$$5x(x+5)$$

$$5(x^2+5x)$$

$$8x - 3x - 15 = 5x^2 + 25x$$

$$-8x + 3x + 15$$

$$5x^2 + 20x + 15 = 0$$

$$p \begin{matrix} 4 \\ 5 \end{matrix} \leftarrow +$$

$$8-3-5$$

**Score 1:** The student found a correct quadratic equation in standard form.

Question 30

30 Solve algebraically for all values of  $x$ :

$$\frac{8}{x+5} - \frac{3}{x} = 5$$

$$\frac{8}{x+5} - \frac{3}{x} = 5$$

$$x+1=0$$

$$x+3=0$$

$$x = -1$$

$$x = -3$$

**Score 1:** The student stated the correct values for  $x$ , but showed insufficient algebraic work.

Question 30

30 Solve algebraically for all values of  $x$ :

$$\frac{8}{x+5} - \frac{3}{x} = 5$$

$$\frac{8}{x+5} - \frac{3(x+5)}{x} = 5(x)(x+5)$$

~~$$\frac{(x)8}{(x)(x+5)} - \frac{(x+5)3}{(x+5)x} = 5$$~~

$$8x - 3x + 15$$

~~$$\frac{8x + 15 + 3x}{x(x+5)}$$~~

$$5x + 15 = 5(x)(x+5)$$

~~$$\frac{11x + 15}{(x^2 + 5x)} = 5$$~~

$$5x + 15 = 5x^2 + 25x$$

$$-25x \quad -25x$$

~~$$11x + 15 = 5x^2 + 25x$$~~
~~$$-5x^2 - 25x$$~~
~~$$-5x^2 - 14x + 15 = 0$$~~

$$-20x + 15 = 5x^2$$

$$-5x^2 \quad -5x^2$$

$$-5x^2 - 20x + 15 = 0$$

$$+5(x^2 - 4x + 3) = 0$$

$$-4 \quad 3$$

$$-3, -1$$

$$5(x-3)(x+1)$$

$$x = 3$$

$$x = -1$$

Score 0: The student made multiple errors.

Question 30

30 Solve algebraically for all values of  $x$ :

$$\frac{8}{x+5} - \frac{3}{x} = 5$$

$$\frac{8(x)}{x+5(x)} - \frac{3(x+5)}{x(x+5)} = \frac{5(x)(x+5)}{1(x)(x+5)}$$

$$\frac{8x - 3(x+5)}{(x+5)(x)} = \frac{5x(x+5)}{(x)(x+5)}$$

$$\frac{8x - 3(x+5)}{8x - 3x + 15} = 5x^2 + 25x$$

$$\begin{array}{r} 5x + 15 = 5x^2 + 25x \\ -25x \qquad \qquad -25x \\ \hline \end{array}$$

$$\begin{array}{r} -20x + 15 = 5x^2 \\ -5x^2 \qquad \qquad -5x^2 \\ \hline \end{array}$$

$$= 5x^2 - 20x + 15$$

$$-x^2 - 4x + 3$$

$$5(x+4)(x-1)$$

$$x = -4$$

$$x = 1$$

Score 0: The student made multiple errors.

**Question 31**

- 31 The transportation methods used by the upperclassmen at Calhoun High School are summarized in the table below.

**Upperclassmen Transportation Methods**

	Drive	Take the Bus	Walk
Junior	58	75	12
Senior	81	39	12

297

Are the events “being a junior” and “driving to school” independent? Using statistical evidence, justify your answer.

$$P(J) = 145/277 \quad P(D) = .523$$

$$P(J|D) = 58/139 \quad P(J|D) = .417$$

NO, they're not independent b/c

$$.523 \neq .417$$

**Score 2:** The student gave a complete and correct response.

Question 31

- 31 The transportation methods used by the upperclassmen at Calhoun High School are summarized in the table below.

Upperclassmen Transportation Methods

	Drive	Take the Bus	Walk	
Junior	58	75	12	145
Senior	81	39	12	132
	139	114	24	277

Are the events "being a junior" and "driving to school" independent? Using statistical evidence, justify your answer.

$$P(\text{Junior and drive}) \stackrel{?}{=} P(\text{Junior}) \times P(\text{drive})$$

$$\frac{58}{277} \stackrel{?}{=} \frac{145}{277} \times \frac{139}{277}$$

$$.209 \neq .263$$

No, the events "being a junior" & "driving to school" are not independent events, because  $P(\text{Junior} \& \text{drive})$  does not equal  $P(\text{Junior}) \times P(\text{drive})$ .

Score 2: The student gave a complete and correct response.

**Question 31**

31 The transportation methods used by the upperclassmen at Calhoun High School are summarized in the table below.

**Upperclassmen Transportation Methods**

	Drive	Take the Bus	Walk
Junior	58	75	12
Senior	81	39	12

Are the events “being a junior” and “driving to school” independent? Using statistical evidence, justify your answer.

(277) students

$$P(A \cap B) = P(A) \cdot P(B)$$

$$\frac{58}{277} = \frac{149}{277} \cdot \frac{114}{277}$$

$$.2093862816 \neq .2154335881$$

dependent

**Score 1:** The student found the probability of taking a bus rather than the probability of driving to school.



**Question 31**

31 The transportation methods used by the upperclassmen at Calhoun High School are summarized in the table below.

**Upperclassmen Transportation Methods**

	Drive	Take the Bus	Walk	T
Junior	58	75	12	145
Senior	81	39	12	132
T	139	114	24	277

Are the events “being a junior” and “driving to school” independent? Using statistical evidence, justify your answer.

$$P(J \cup D) = P(J) \cdot P(D)$$

$$P(J \cup D) = P(J) + P(D) - P(J \cap D)$$

$$P(J \cup D) = \frac{145}{277} + \frac{139}{277} - \frac{58}{277}$$

$$P(J \cup D) = \frac{226}{277}$$

$$\frac{226}{277} = \frac{145}{277} \cdot \frac{139}{277}$$

$$.8158844765 \neq .2626777359$$

No, they are not independent.

**Score 1:** The student used an incorrect method for proving independence.

**Question 31**

31 The transportation methods used by the upperclassmen at Calhoun High School are summarized in the table below.

**Upperclassmen Transportation Methods**

	Drive	Take the Bus	Walk	Total
Junior	58	75	12	145
Senior	81	39	12	132
Total	139	114	24	554

Are the events “being a junior” and “driving to school” independent? Using statistical evidence, justify your answer.

$$\frac{145}{554} = .261732852$$

$$\frac{139}{554} = .25090271$$

They are independent because they have different probabilities which means that they do not happen together.

**Score 0:** The student did not show enough relevant course-level work to receive any credit.

**Question 31**

- 31 The transportation methods used by the upperclassmen at Calhoun High School are summarized in the table below.

**Upperclassmen Transportation Methods**

	Drive	Take the Bus	Walk	
Junior	58	75	12	145
Senior	81	39	12	<del>132</del>
	139	114	24	277

Are the events “being a junior” and “driving to school” independent? Using statistical evidence, justify your answer.

$$A|B = 145 / 277 = 0.523$$

$$139 / 277 = 0.502$$

Not equal independent

**Score 0:** The student did not show enough relevant course-level work to receive any credit.

**Question 32**

32 Can  $f(x) = x^3 + 7$  be classified as an odd function? Justify your answer.

$$f(-x) = (-x)^3 + 7$$

$$-f(x) = -(x^3 + 7)$$

$$-f(x) = -x^3 - 7$$

$f(x) = x^3 + 7$  can not be classified  
as an odd function because  $f(-x) \neq -f(x)$

**Score 2:** The student gave a complete and correct response.

**Question 32**

32 Can  $f(x) = x^3 + 7$  be classified as an odd function? Justify your answer.

$$f(-x) = (-x)^3 + 7$$

$$f(-x) = -x^3 + 7$$

$f(x)$  cannot be classified as odd because when negative  $x$  is plugged in, the formula is not the same as opposite of the regular formula, so it is neither even nor odd.

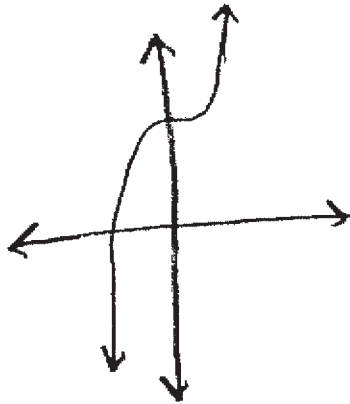
**Score 2:** The student gave a complete and correct response.

Question 32

32 Can  $f(x) = x^3 + 7$  be classified as an odd function? Justify your answer.

$$f(x) = x^3 + 7$$

No, it is not an odd function  
b/c it does not rotate  $180^\circ$



**Score 1:** The student wrote an incomplete justification.

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**Question 32**

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**32** Can  $f(x) = x^3 + 7$  be classified as an odd function? Justify your answer.

Yes. Because the exponent is odd (3)  
and 7 is odd.

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**Score 0:** The student did not satisfy the criteria for one or more credits.

Question 32

32 Can  $f(x) = x^3 + 7$  be classified as an odd function? Justify your answer.

NO, because the ~~y~~ intercept  
is at a positive 7, and it increases,  
it doesn't decrease at all, ~~it's not~~  
 $(-\infty, +\infty)$

**Score 0:** The student provided an incorrect justification.



**Question 33**

33 Solve the following system of equations algebraically for all values of  $x$ ,  $y$ , and  $z$ :

$$3x - 8y + 2z = -60$$

$$2x - 7y - 5z = -31$$

$$-6x + 2y - 4z = 36$$

$$\begin{array}{r} -6x + 16y - 4z = 120 \\ 6x - 21y - 5z = -93 \\ \hline -5y - 19z = 27 \end{array}$$

$$\begin{array}{r} 6x - 16y + 4z = -120 \\ -6x + 2y - 4z = 36 \\ \hline -14y = -84 \end{array}$$

$$y = 6$$

$$\begin{array}{r} -5(6) - 19z = 27 \\ -30 - 19z = 27 \\ -19z = 57 \\ z = -3 \end{array}$$

$$3x - 8(6) + 2(-3) = -60$$

$$3x - 48 - 6 = -60$$

$$3x - 54 = -60$$

$$3x = -6$$

$$x = -2$$

**Score 4:** The student gave a complete and correct response.

Question 33

33 Solve the following system of equations algebraically for all values of  $x$ ,  $y$ , and  $z$ :

$$\begin{array}{r} 2(3x - 8y + 2z = -60) \\ -3(2x - 7y - 5z = -31) \\ \hline -6x + 2y - 4z = 36 \\ 6x - 16y + 4z = -120 \\ \hline -6x + 2y - 4z = 36 \\ \hline -14y = -84 \\ \hline -14 \\ \hline y = 6 \end{array}$$

$$\begin{array}{r} 6x - 16y + 4z = -120 \\ -6x + 21y + 15z = 93 \\ \hline 5y + 19z = -27 \end{array}$$

$$\begin{array}{r} 5(6) + 19z = -27 \\ 30 + 19z = -27 \\ \hline -30 \quad -30 \\ \hline 19z = -57 \\ \hline 19 \\ \hline z = 3 \end{array}$$

$$\begin{array}{r} -6x + 2(6) - 4(3) = 36 \\ -6x + 12 - 12 = 36 \\ \hline -6x = 36 \\ \hline -6 \\ \hline x = -6 \end{array}$$

$x = -6$
$y = 6$
$z = 3$

**Score 3:** The student made an error solving for  $z$ .

Question 33

33 Solve the following system of equations algebraically for all values of  $x$ ,  $y$ , and  $z$ :

$$\begin{array}{r}
 5y - 19z = -27 \\
 3x - 8y + 2z = -60 \\
 2x - 7y - 5z = -31 \\
 -6x + 2y - 4z = 36
 \end{array}
 \begin{array}{l}
 36 \cdot 16 = -5.22 \\
 \left. \begin{array}{l} > \\ > \end{array} \right\} 5y - 19z = -27
 \end{array}$$

$$\begin{array}{l}
 y = 6 \\
 z = -3 \\
 x = -44
 \end{array}$$

$$\begin{array}{r}
 6x - 8y + 2z = -60 \\
 + \quad -6x + 2y - 4z = 36 \\
 \hline
 -14y = -84 \\
 y = 6
 \end{array}$$

$$\begin{array}{r}
 6x - 16y + 4z = -120 \\
 + \quad -6x + 21y + 15z = 93 \\
 \hline
 5y + 19z = -27
 \end{array}$$

$$\begin{array}{l}
 z = -2.61 \\
 y = 4.52
 \end{array}$$

$$\begin{array}{l}
 6(2x - 7y - 5z = -31) \\
 2(6x)
 \end{array}$$

$$\begin{array}{r}
 12x - 42y - 30z = -186 \\
 + \quad -12x + 4y - 8z = 72 \\
 \hline
 -38y - 22z = -114
 \end{array}$$

$$\begin{array}{r}
 38(5y + 19z = -27) \\
 5(38y - 22z = -144) \\
 + \quad 190y + 72z = -1026 \\
 + \quad -190y - 110z = -570 \\
 \hline
 612z = -1596 \\
 \frac{612z}{612} = \frac{-1596}{612}
 \end{array}$$

Score 2: The student correctly found one value.

Question 33

33 Solve the following system of equations algebraically for all values of  $x$ ,  $y$ , and  $z$ :

$$\begin{cases} 3x - 8y + 2z = -60 \\ 2x - 7y - 5z = -31 \\ -6x + 2y - 4z = 36 \end{cases}$$

$$\begin{array}{cccc} 3 & -8 & 2 & -60 \\ 2 & -7 & -5 & -31 \\ -6 & 2 & -4 & 36 \end{array}$$

$$\begin{array}{cccc} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & -3 \end{array}$$

$$\begin{array}{l} x = -2 \\ y = 6 \\ z = -3 \end{array}$$

**Score 2:** The student used a method other than algebraic.

**Question 33**

33 Solve the following system of equations algebraically for all values of  $x$ ,  $y$ , and  $z$ :

$$3x - 8y + 2z = -60$$

$$2x - 7y - 5z = -31$$

$$-6x + 2y - 4z = 36$$

$$x = -2$$

$$y = 6$$

$$z = -3$$

$$\begin{aligned} 3x - 8y + 2z &= -60 \\ -6x + 2y - 4z &= 36 \\ +24x - 8y + 16z &= 144 \end{aligned}$$

**Score 1:** The student found the correct answer with no supporting work.

**Question 33**

33 Solve the following system of equations algebraically for all values of  $x$ ,  $y$ , and  $z$ :

$$(3) \quad 2x - 7y - 5z = 31$$

$$\underline{-6x + 2y - 4z = 36}$$

$$6x - 21y - 15z = -93$$

$$\underline{-6x + 2y - 4z = 36}$$

$$-19y - 19z = -57$$

$$3x - 8y + 2z = -60$$

$$2x - 7y - 5z = -31$$

$$-6x + 2y - 4z = 36$$

$$(2) \quad 3x - 8y + 2z = -60$$

$$\underline{-6x + 2y - 4z = 36}$$

$$6x - 8y + 2z = -120$$

$$\underline{-6x + 2y - 4z = 36}$$

$$-6y - 2z = -84$$

**Score 0:** The student created an incorrect system of equations with two variables.

**Question 34**

34 In the town of Skaneateles, New York, house prices since 2008 have changed based on the function  $H(t) = 200,000(1.045)^t$ , where  $t$  is the number of years since 2008 and  $H(t)$  is the median house price. Determine the average rate of change for the median house price in Skaneateles from 2010 to 2018 to the nearest dollar per year.

$$2 \rightarrow 218405$$

$$10 \rightarrow 310593.88437$$

$$0 = 2008$$

$$2 = 2010$$

$$10 = 2018$$

$$\frac{310593.88437 - 218405}{10 - 2} = \frac{92188.88437}{8}$$

$$\boxed{\$11524/\text{yr}}$$

Explain what this rate of change means as it relates to median house prices.

On average, median house prices increase at a rate of 11524 per year for 2010-2018

**Score 4:** The student gave a complete and correct response.

**Question 34**

34 In the town of Skaneateles, New York, house prices since 2008 have changed based on the function  $H(t) = 200,000(1.045)^t$ , where  $t$  is the number of years since 2008 and  $H(t)$  is the median house price. Determine the average rate of change for the median house price in Skaneateles from 2010 to 2018 to the nearest dollar per year.

$$\frac{200000(1.045)^{10} - [200000(1.045)^2]}{2018 - 2010} = \frac{92188.9}{8}$$

$$\approx \$ 11524$$

Explain what this rate of change means as it relates to median house prices.

Between 2010 and 2018, house prices have risen by approximately 11524 dollars per year.

**Score 4:** The student gave a complete and correct response.



Question 34

34 In the town of Skaneateles, New York, house prices since 2008 have changed based on the function  $H(t) = 200,000(1.045)^t$ , where  $t$  is the number of years since 2008 and  $H(t)$  is the median house price. Determine the average rate of change for the median house price in Skaneateles from 2010 to 2018 to the nearest dollar per year.

$$\begin{aligned} H(2) &= 200,000(1.045)^2 \\ &= 209,000(1.092025) \\ &= 218,405 \end{aligned}$$

$$\begin{aligned} H(10) &= 200,000(1.045)^{10} \\ &= 200,000(1.552969422) \\ &= 310,593.8843 \end{aligned}$$

$$\begin{aligned} \text{ROC} &= \frac{\$ \text{final} - \text{initial}}{\# \text{final} - \text{initial}} \\ &= \frac{310,593.8843 - 218,405}{2018 - 2010} \\ &= \frac{92,188.8843}{8} \\ &= \$ 11,523.610 \\ &\approx \boxed{\$ 11,524 \text{ per year}} \end{aligned}$$

Explain what this rate of change means as it relates to median house prices.

This rate of change means that house prices have been increasing<sup>at</sup> an average of 11,524 dollars per year in Skaneateles.

**Score 3:** The student omitted the time frame 2010 to 2018.

**Question 34**

34 In the town of Skaneateles, New York, house prices since 2008 have changed based on the function  $H(t) = 200,000(1.045)^t$ , where  $t$  is the number of years since 2008 and  $H(t)$  is the median house price. Determine the average rate of change for the median house price in Skaneateles from 2010 to 2018 to the nearest dollar per year.

$$H(10) = 200,000(1.045)^{10} = 310,594$$

$$H(0) = 200,000(1.045)^0 = 200,000$$

$$AROC = \frac{H(10) - H(0)}{10 - 0}$$

$$AROC = \frac{310,594 - 200,000}{10}$$

$$AROC = 11,059$$

Explain what this rate of change means as it relates to median house prices.

This rate of change represents the average increase in price for the median house price in Skaneateles, NY from the year 2008-2018.

**Score 3:** The student calculated average rate of change using 2008.

**Question 34**

34 In the town of Skaneateles, New York, house prices since 2008 have changed based on the function  $H(t) = 200,000(1.045)^t$ , where  $t$  is the number of years since 2008 and  $H(t)$  is the median house price. Determine the average rate of change for the median house price in Skaneateles from 2010 to 2018 to the nearest dollar per year.

$\underbrace{\hspace{10em}}_{8 \text{ years}} \qquad 2008 - 2018 = 10 \text{ years}$

$$H(10) = 200000(1.045)^{10} = 310593.88$$

$$H(2) = 200000(1.045)^2 = 218405$$

$$\begin{array}{r} 310593.88 \\ - 218405 \\ \hline 91889 \end{array} \qquad \begin{array}{l} \lrcorner \$11486 \rceil \\ \lrcorner \end{array}$$

$$\frac{91889}{8} = 11486$$

Explain what this rate of change means as it relates to median house prices.

As each year passes, the house price in Skaneateles increases by \$11486

**Score 2:** The student incorrectly calculated the average rate of change and wrote an incomplete explanation.

**Question 34**

34 In the town of Skaneateles, New York, house prices since 2008 have changed based on the function  $H(t) = 200,000(1.045)^t$ , where  $t$  is the number of years since 2008 and  $H(t)$  is the median house price. Determine the average rate of change for the median house price in Skaneateles from 2010 to 2018 to the nearest dollar per year.

$$\begin{array}{l} H(2) = 218405 \\ H(10) = 310594 \end{array} \left. \vphantom{\begin{array}{l} H(2) \\ H(10) \end{array}} \right\} \text{rate}$$

$$\frac{310594 - 218405}{10 - 2} = \$11523.63$$

Explain what this rate of change means as it relates to median house prices.

On average, the houses value increases  
\$11523.63 every year

**Score 2:** The student made a rounding error and wrote an incomplete explanation.

**Question 34**

34 In the town of Skaneateles, New York, house prices since 2008 have changed based on the function  $H(t) = 200,000(1.045)^t$ , where  $t$  is the number of years since 2008 and  $H(t)$  is the median house price. Determine the average rate of change for the median house price in Skaneateles from 2010 to 2018 to the nearest dollar per year.

$$H(2) = 200,000(1.045)^2$$
$$= 218,405$$

$$H(8) = 200,000(1.045)^8$$
$$= 284,420.12$$

$$\frac{284,420.12 - 218,405}{8 - 2} = \frac{66,015.12}{6} = \boxed{\$11,002.52}$$

$$\text{avg} = \frac{f(b) - f(a)}{b - a}$$

Explain what this rate of change means as it relates to median house prices.

Every year, the price basically goes up about \$11,002.52.

**Score 1:** The student received one credit for an incomplete explanation.

**Question 34**

34 In the town of Skaneateles, New York, house prices since 2008 have changed based on the function  $H(t) = 200,000(1.045)^t$ , where  $t$  is the number of years since 2008 and  $H(t)$  is the median house price. Determine the average rate of change for the median house price in Skaneateles from 2010 to 2018 to the nearest dollar per year.

$$200,000(1.045)^8 = 284,420.1226$$

$$\$284,420$$

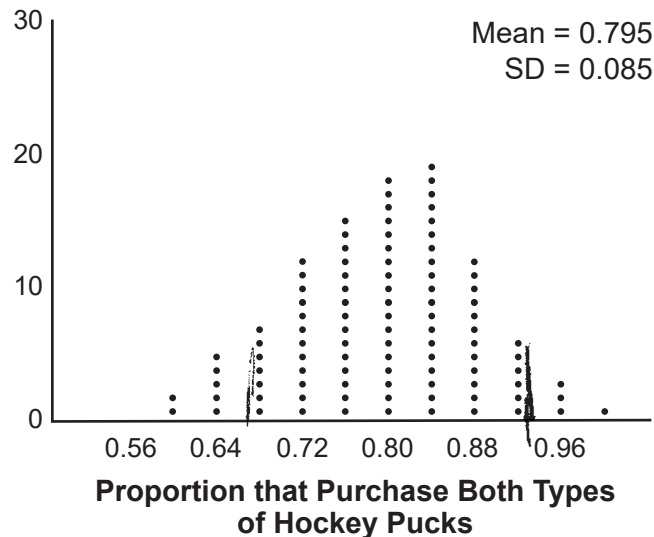
Explain what this rate of change means as it relates to median house prices.

This rate of change means that the price of a house in the town of Skaneateles, New York has increased.

**Score 0:** The student did not show enough relevant course-level work to receive any credit.

### Question 35

35 A sporting goods manufacturer is trying to determine if they should continue to produce multiple types of hockey pucks. The company surveyed 50 randomly chosen customers and asked them if they purchased both game regulation pucks and lighter training pucks. Of those surveyed, 40 of them said that they purchase both types of pucks. A simulation that was run 100 times based on the survey results produced the approximately normal results below.



- a) Determine an interval containing the middle 95% of plausible values that estimates the proportion of all customers who would purchase both types of pucks from the company.

$$\begin{aligned} 0.795 + 2(0.085) &= .965 \\ 0.795 - 2(0.085) &= .625 \\ .965 &\leftarrow .625 \quad (.625 - .965) \end{aligned}$$

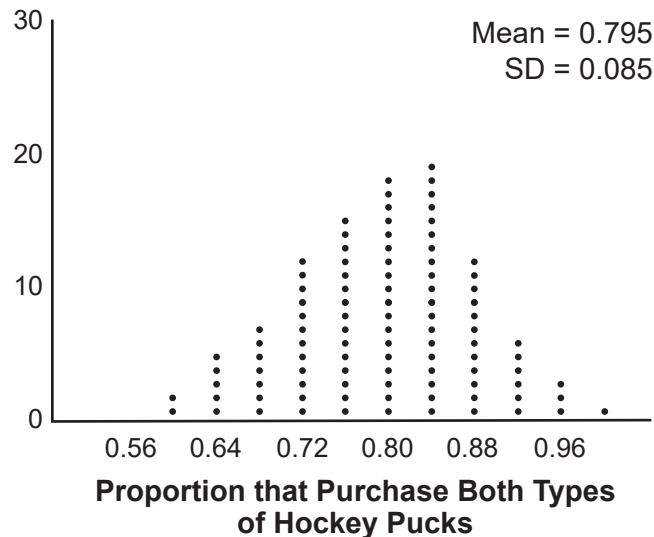
- b) The company will continue to manufacture both types of hockey pucks if it is reasonable to assume that the true proportion of customers who buy both types of hockey pucks is above 0.60. Using the interval from part a, explain whether or not the company should continue to produce both types of hockey pucks.

Yes. .625, the bottom number of the 95%, is above .60

**Score 4:** The student gave a complete and correct response.

### Question 35

35 A sporting goods manufacturer is trying to determine if they should continue to produce multiple types of hockey pucks. The company surveyed 50 randomly chosen customers and asked them if they purchased both game regulation pucks and lighter training pucks. Of those surveyed, 40 of them said that they purchase both types of pucks. A simulation that was run 100 times based on the survey results produced the approximately normal results below.



- a) Determine an interval containing the middle 95% of plausible values that estimates the proportion of all customers who would purchase both types of pucks from the company.

$(0.625, 0.965)$

- b) The company will continue to manufacture both types of hockey pucks if it is reasonable to assume that the true proportion of customers who buy both types of hockey pucks is above 0.60. Using the interval from part a, explain whether or not the company should continue to produce both types of hockey pucks.

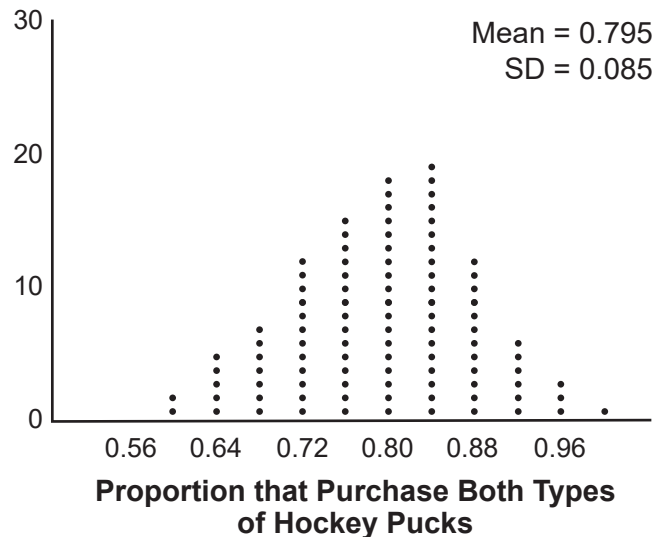
they should continue to produce them  
because the 95% confidence range is above  
0.6

**Score 3:** The student did not show any work to find the correct interval.



**Question 35**

**35** A sporting goods manufacturer is trying to determine if they should continue to produce multiple types of hockey pucks. The company surveyed 50 randomly chosen customers and asked them if they purchased both game regulation pucks and lighter training pucks. Of those surveyed, 40 of them said that they purchase both types of pucks. A simulation that was run 100 times based on the survey results produced the approximately normal results below.



- a) Determine an interval containing the <sup>2 SD's</sup> middle 95% of plausible values that estimates the proportion of all customers who would purchase both types of pucks from the company.

$$2 \cdot .085 = .17$$

$$.795 - .17 = .625$$

$$.795 + .17 = .965$$

between  
.625 and  
.965

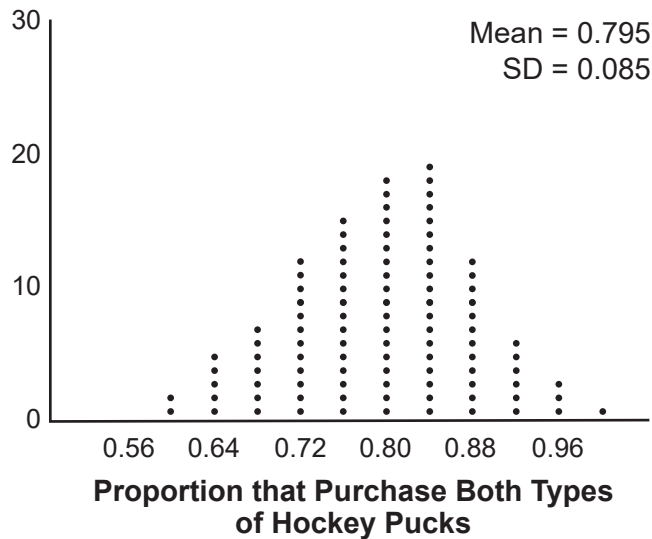
- b) The company will continue to manufacture both types of hockey pucks if it is reasonable to assume that the true proportion of customers who buy both types of hockey pucks is above 0.60. Using the interval from part a, explain whether or not the company should continue to produce both types of hockey pucks.

The true proportion of customers who buy both types of hockey pucks is above .60, so the company should continue to produce both types of hockey pucks.

**Score 3:** The student explanation did not reference the interval.

**Question 35**

**35** A sporting goods manufacturer is trying to determine if they should continue to produce multiple types of hockey pucks. The company surveyed 50 randomly chosen customers and asked them if they purchased both game regulation pucks and lighter training pucks. Of those surveyed, 40 of them said that they purchase both types of pucks. A simulation that was run 100 times based on the survey results produced the approximately normal results below.



- a) Determine an interval containing the middle 95% of plausible values that estimates the proportion of all customers who would purchase both types of pucks from the company.

$$.795 \pm 2(.085)$$

$$.965$$

$$.625$$

.625 to .965

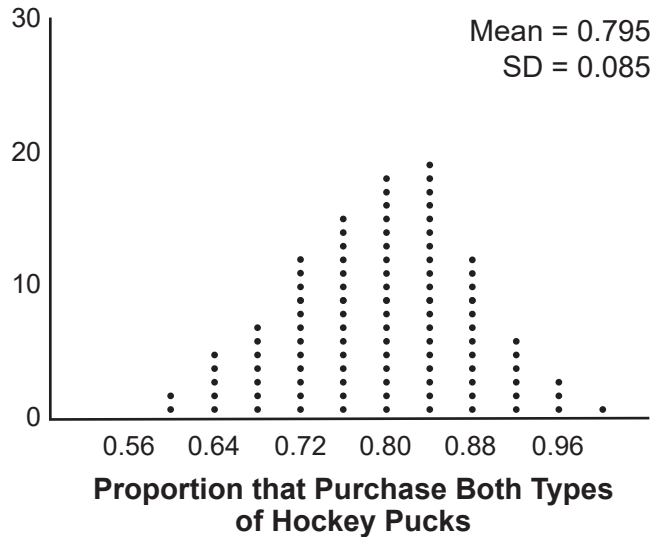
- b) The company will continue to manufacture both types of hockey pucks if it is reasonable to assume that the true proportion of customers who buy both types of hockey pucks is above 0.60. Using the interval from part a, explain whether or not the company should continue to produce both types of hockey pucks.

Yes they should produce more both types of hockey pucks because in the graph, only 3 proportions are outside the 95%, making them irrelevant.

**Score 2:** The student gave an incorrect explanation.

**Question 35**

**35** A sporting goods manufacturer is trying to determine if they should continue to produce multiple types of hockey pucks. The company surveyed 50 randomly chosen customers and asked them if they purchased both game regulation pucks and lighter training pucks. Of those surveyed, 40 of them said that they purchase both types of pucks. A simulation that was run 100 times based on the survey results produced the approximately normal results below.



- a) Determine an interval containing the middle 95% of plausible values that estimates the proportion of all customers who would purchase both types of pucks from the company.

$$\begin{aligned}
 .795 + 2(.085) &= .965 \\
 .795 - 2(.085) &= .625
 \end{aligned}
 \quad \left. \vphantom{\begin{aligned} .795 + 2(.085) &= .965 \\ .795 - 2(.085) &= .625 \end{aligned}} \right\} .625 - .965$$

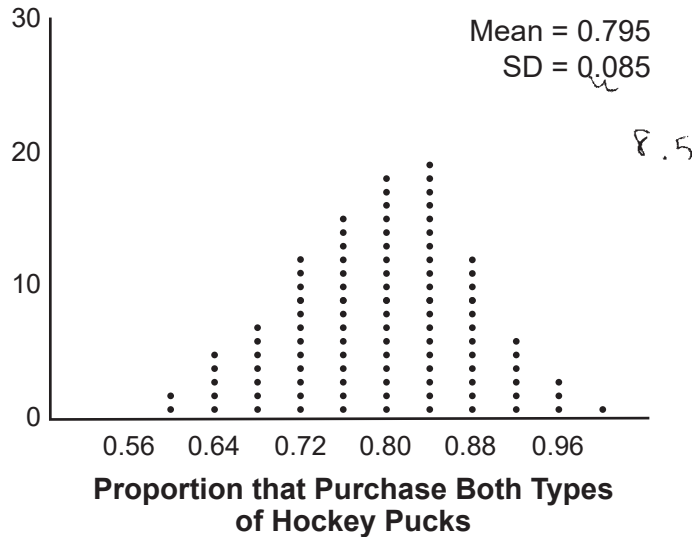
- b) The company will continue to manufacture both types of hockey pucks if it is reasonable to assume that the true proportion of customers who buy both types of hockey pucks is above 0.60. Using the interval from part a, explain whether or not the company should continue to produce both types of hockey pucks.

They should not since .60 is outside the middle 95%

**Score 2:** The student gave an incorrect explanation.

**Question 35**

35 A sporting goods manufacturer is trying to determine if they should continue to produce multiple types of hockey pucks. The company surveyed 50 randomly chosen customers and asked them if they purchased both game regulation pucks and lighter training pucks. Of those surveyed, 40 of them said that they purchase both types of pucks. A simulation that was run 100 times based on the survey results produced the approximately normal results below.



a) Determine an interval containing the middle 95% of plausible values that estimates the proportion of all customers who would purchase both types of pucks from the company.

$$0.84 \pm 0.085$$

$$[0.755, 0.925]$$

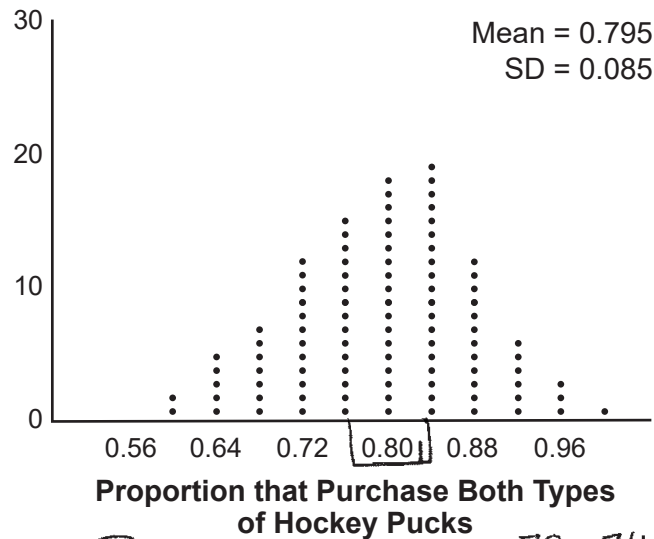
b) The company will continue to manufacture both types of hockey pucks if it is reasonable to assume that the true proportion of customers who buy both types of hockey pucks is above 0.60. Using the interval from part a, explain whether or not the company should continue to produce both types of hockey pucks.

The company should keep producing both types of hockey pucks because the proportion of customers who buy both types is above 0.60.

**Score 1:** The student determined an incorrect interval and gave an incomplete explanation.

**Question 35**

**35** A sporting goods manufacturer is trying to determine if they should continue to produce multiple types of hockey pucks. The company surveyed 50 randomly chosen customers and asked them if they purchased both game regulation pucks and lighter training pucks. Of those surveyed, 40 of them said that they purchase both types of pucks. A simulation that was run 100 times based on the survey results produced the approximately normal results below.



80 82 (84) 86 88 72 74 (76) 78 80

- a) Determine an interval containing the middle 95% of plausible values that estimates the proportion of all customers who would purchase both types of pucks from the company.

The interval that contains the middle 95% is from 0.76 to 0.84.

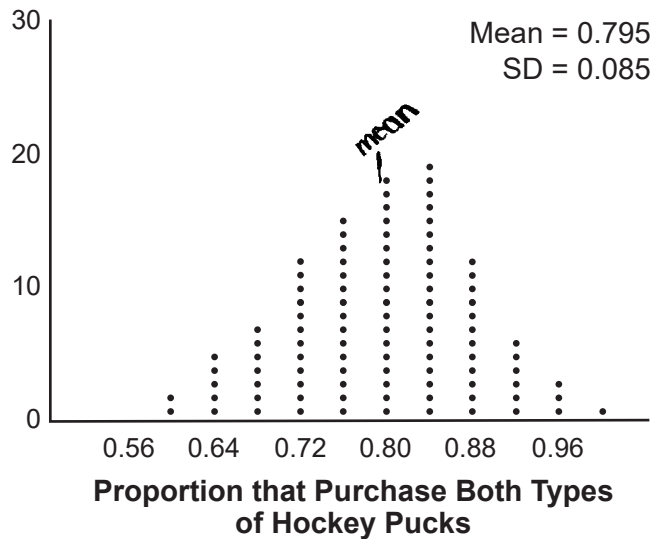
- b) The company will continue to manufacture both types of hockey pucks if it is reasonable to assume that the true proportion of customers who buy both types of hockey pucks is above 0.60. Using the interval from part a, explain whether or not the company should continue to produce both types of hockey pucks.

The company should continue to manufacture both types of hockey pucks because in the interval it ranges from 0.76 to 0.84. In the interval the people buy about 20 or more pucks which shows both types of hockey pucks are in high demand.

**Score 0:** The student's response did not contain enough course-level work to receive any credit.

**Question 35**

**35** A sporting goods manufacturer is trying to determine if they should continue to produce multiple types of hockey pucks. The company surveyed 50 randomly chosen customers and asked them if they purchased both game regulation pucks and lighter training pucks. Of those surveyed, 40 of them said that they purchase both types of pucks. A simulation that was run 100 times based on the survey results produced the approximately normal results below.



a) Determine an interval containing the middle 95% of plausible values that estimates the proportion of all customers who would purchase both types of pucks from the company.

*.76 - .82*

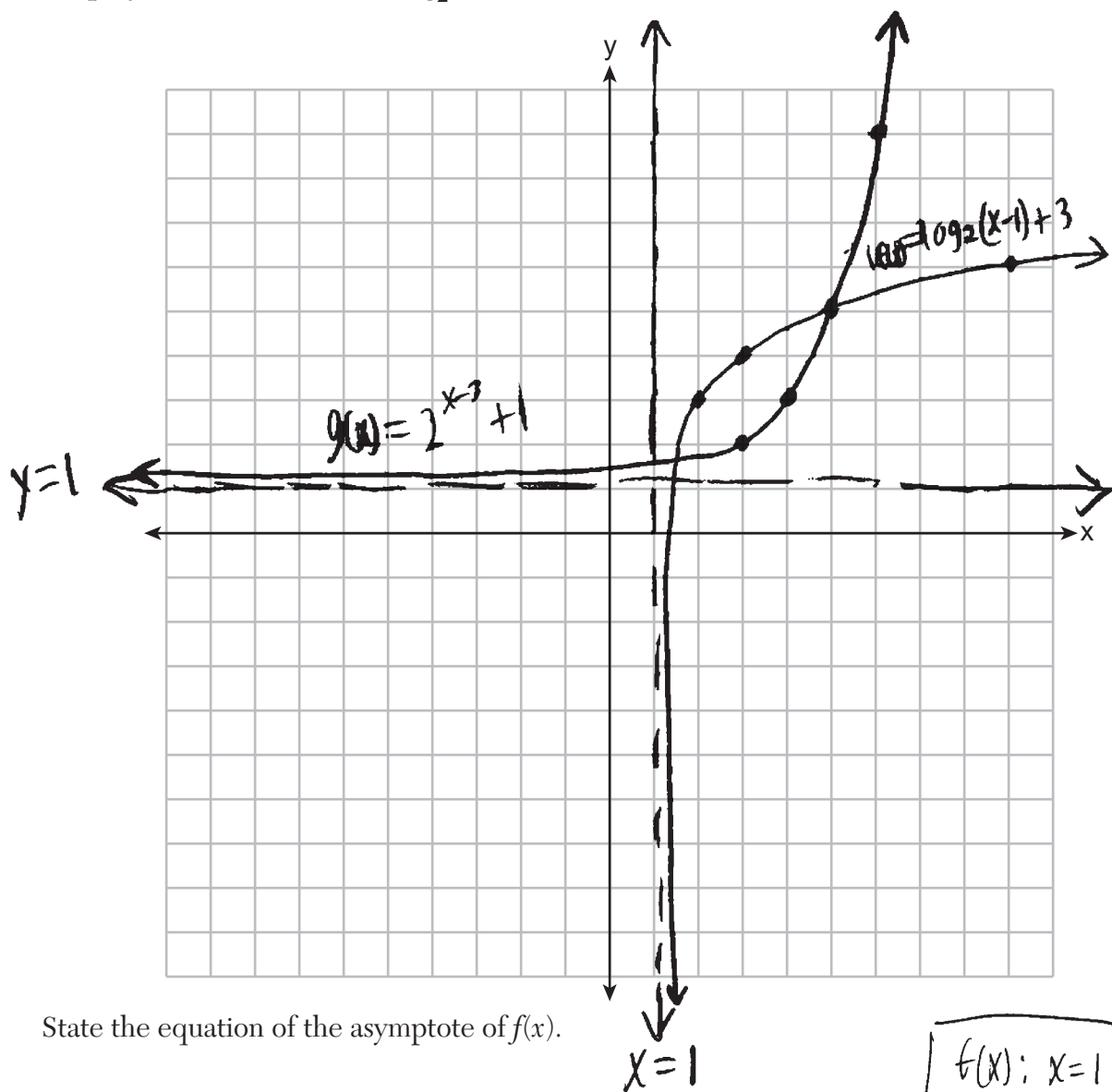
b) The company will continue to manufacture both types of hockey pucks if it is reasonable to assume that the true proportion of customers who buy both types of hockey pucks is above 0.60. Using the interval from part *a*, explain whether or not the company should continue to produce both types of hockey pucks.

*yes, the company should continue to produce both types of pucks because .80% of people are buying them & that's a large percentage.*

**Score 0:** The student's response did not contain enough course-level work to receive any credit.

Question 36

36 Graph  $y = f(x)$ , where  $f(x) = \log_2(x - 1) + 3$  on the set of axes below.



$f(x)$	
$x$	$y$
2	3
3	4
5	5
9	6

$g(x)$	
$x$	$y$
3	2
4	3
5	5
6	9

State the equation of the asymptote of  $f(x)$ .

$x=1$

$f(x): x=1$   
 $g(x): y=1$

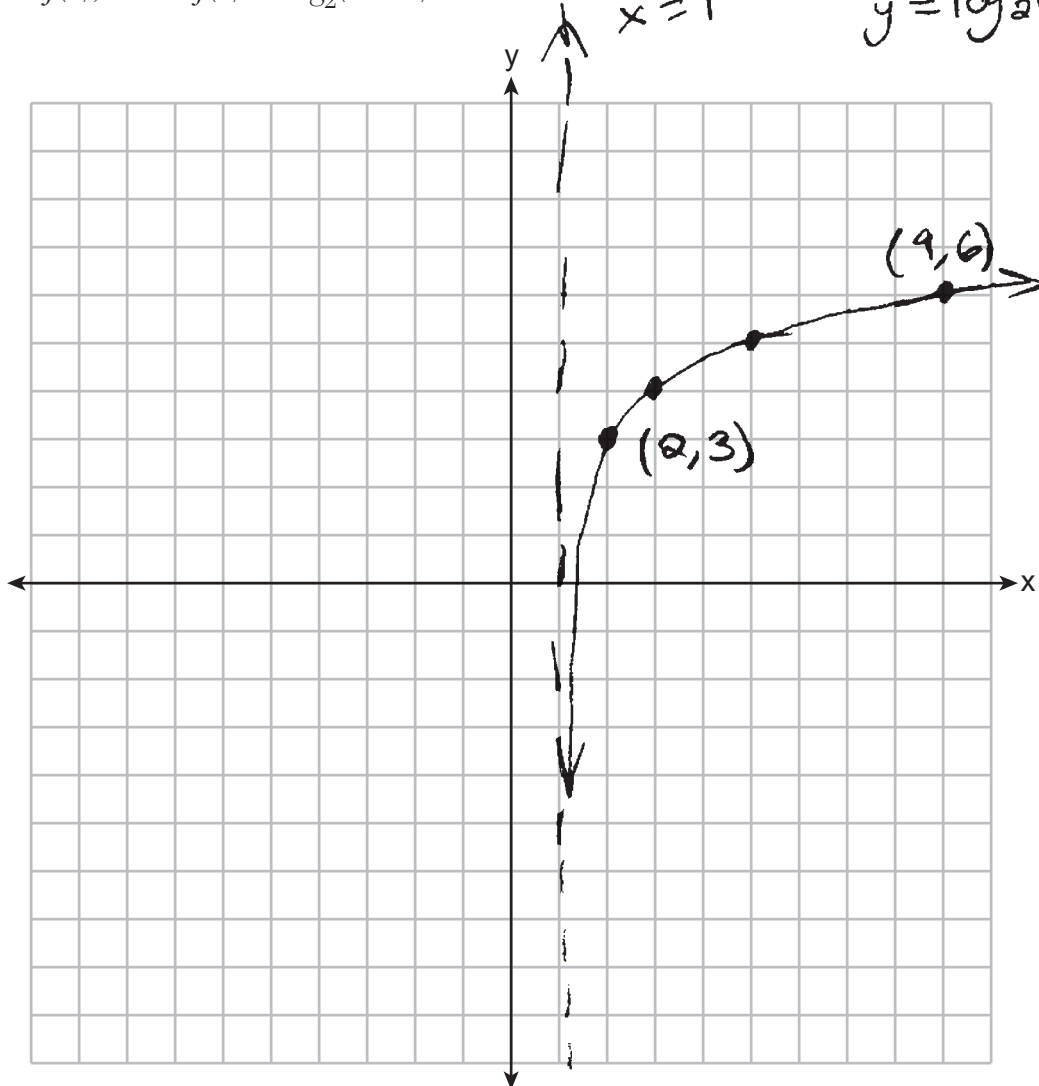
When  $f(x)$  is reflected over the line  $y = x$ , a new function is formed:  $g(x) = 2^{x-3} + 1$ .

State the equation of the asymptote of  $g(x)$ .

**Score 4:** The student gave a complete and correct response.

Question 36

36 Graph  $y = f(x)$ , where  $f(x) = \log_2(x - 1) + 3$  on the set of axes below.



State the equation of the asymptote of  $f(x)$ .

$f(x)$  asymptote =  $x = 1$

When  $f(x)$  is reflected over the line  $y = x$ , a new function is formed:  $g(x) = 2^{x - 3} + 1$ .

State the equation of the asymptote of  $g(x)$ .

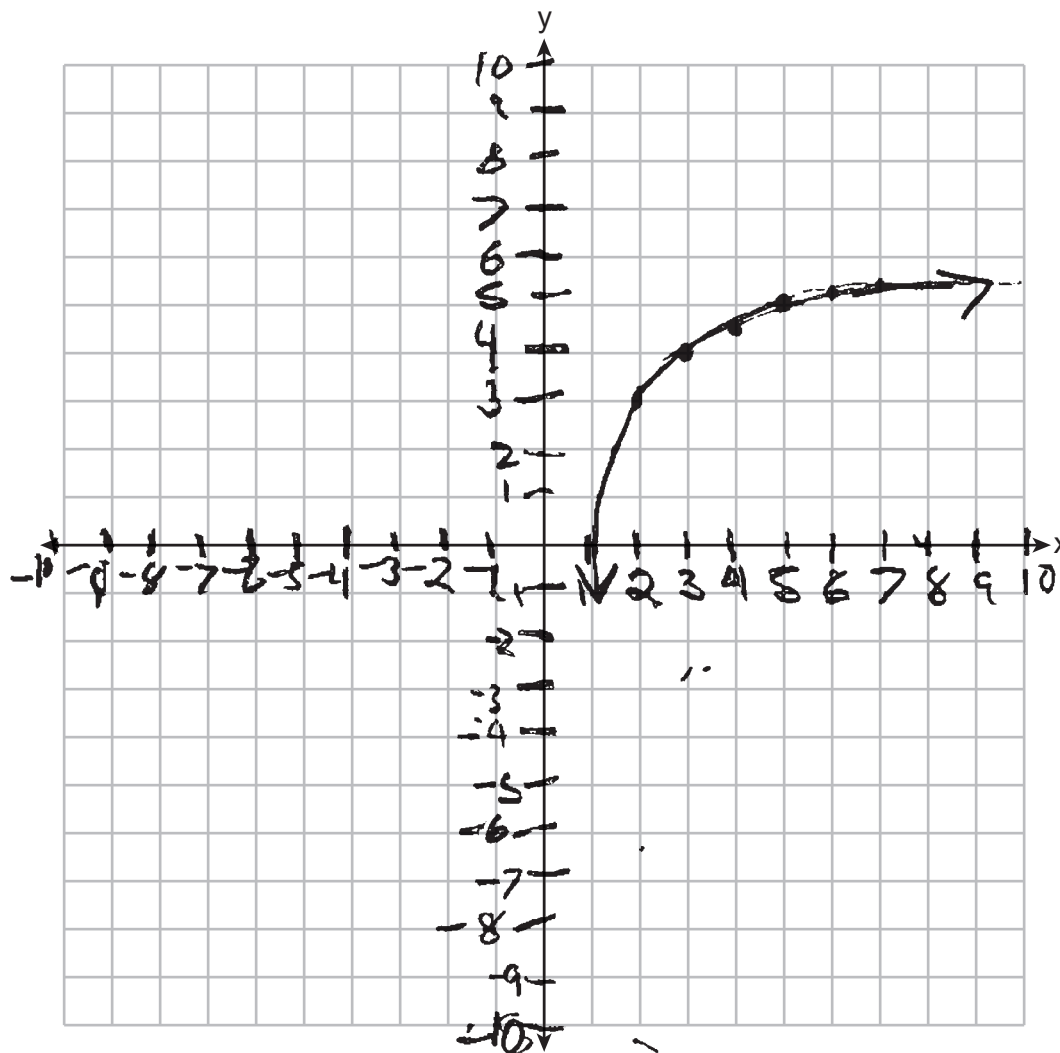
$g(x)$  asymptote =  $y = 1$

**Score 4:** The student gave a complete and correct response.



Question 36

36 Graph  $y = f(x)$ , where  $f(x) = \log_2(x - 1) + 3$  on the set of axes below.



State the equation of the asymptote of  $f(x)$ .

$$f(x) = x = 1$$

When  $f(x)$  is reflected over the line  $y = x$ , a new function is formed:  $g(x) = 2^{x-3} + 1$ .

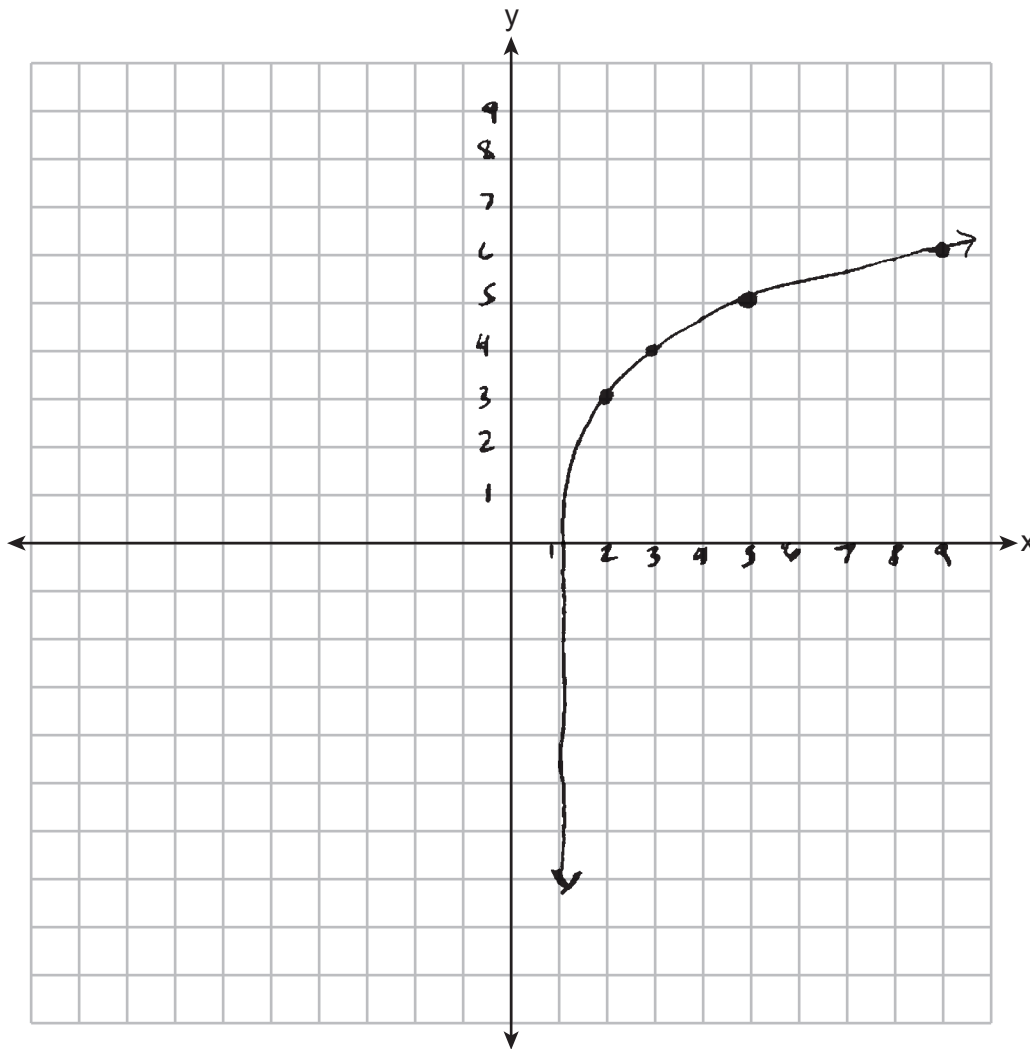
State the equation of the asymptote of  $g(x)$ .

$$g(x) = y = 1$$

**Score 3:** The student made a graphing error at  $x = 9$ .

Question 36

36 Graph  $y = f(x)$ , where  $f(x) = \log_2(x - 1) + 3$  on the set of axes below.



State the equation of the asymptote of  $f(x)$ .

asymptote of  $f(x) = x > 1$

When  $f(x)$  is reflected over the line  $y = x$ , a new function is formed:  $g(x) = 2^{x-3} + 1$ .

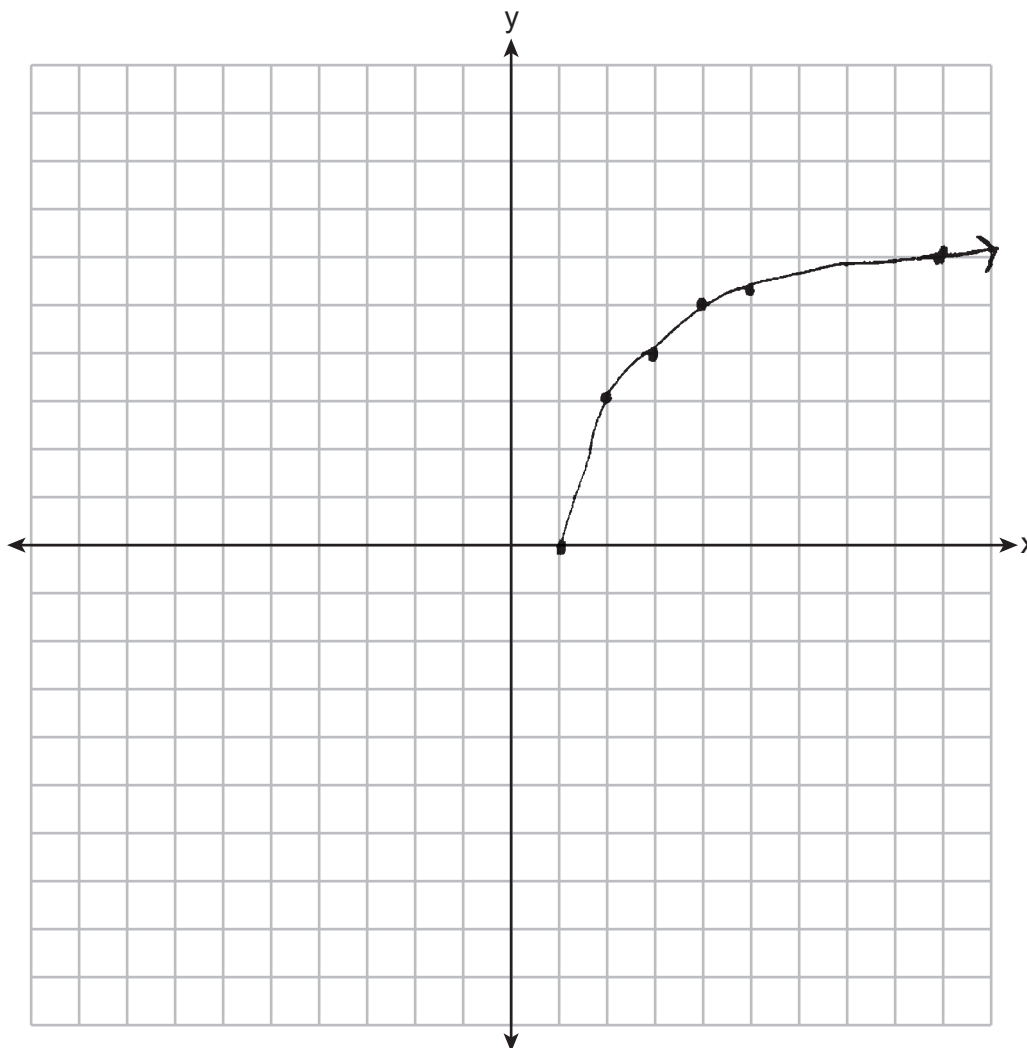
State the equation of the asymptote of  $g(x)$ .

asymptote of  $g(x) = y > 1$

**Score 2:** The student stated incorrect asymptotes.

**Question 36**

**36** Graph  $y = f(x)$ , where  $f(x) = \log_2(x - 1) + 3$  on the set of axes below.



State the equation of the asymptote of  $f(x)$ .

$$f(x): x=1$$

When  $f(x)$  is reflected over the line  $y = x$ , a new function is formed:  $g(x) = 2^{x-3} + 1$ .

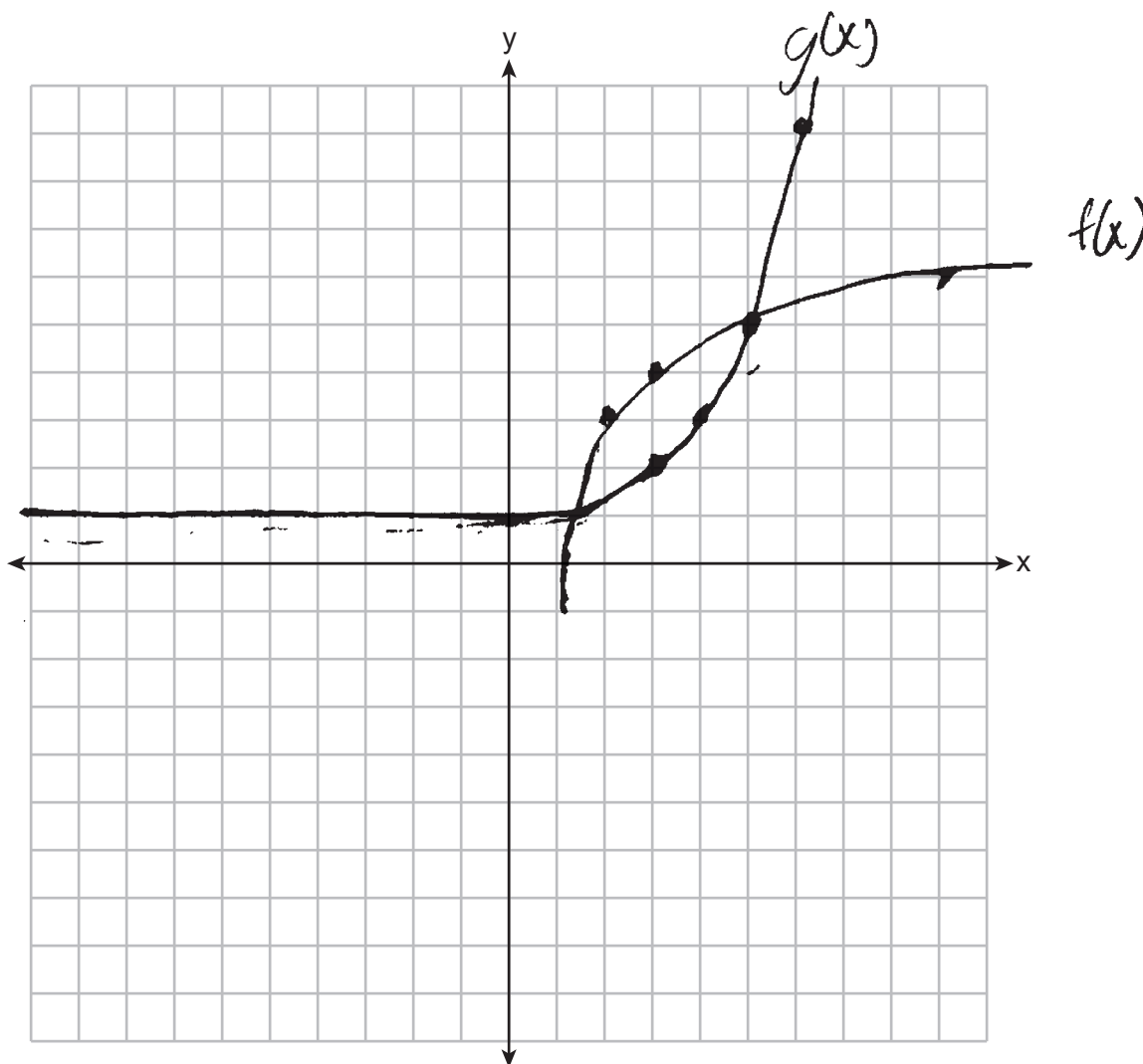
State the equation of the asymptote of  $g(x)$ .

$$g(x): y=1$$

**Score 2:** The student made two graphing errors.

Question 36

36 Graph  $y = f(x)$ , where  $f(x) = \log_2(x - 1) + 3$  on the set of axes below.



State the equation of the asymptote of  $f(x)$ .

$$f(x) \text{ asymptote} = (1, 0)$$

When  $f(x)$  is reflected over the line  $y = x$ , a new function is formed:  $g(x) = 2^{x-3} + 1$ .

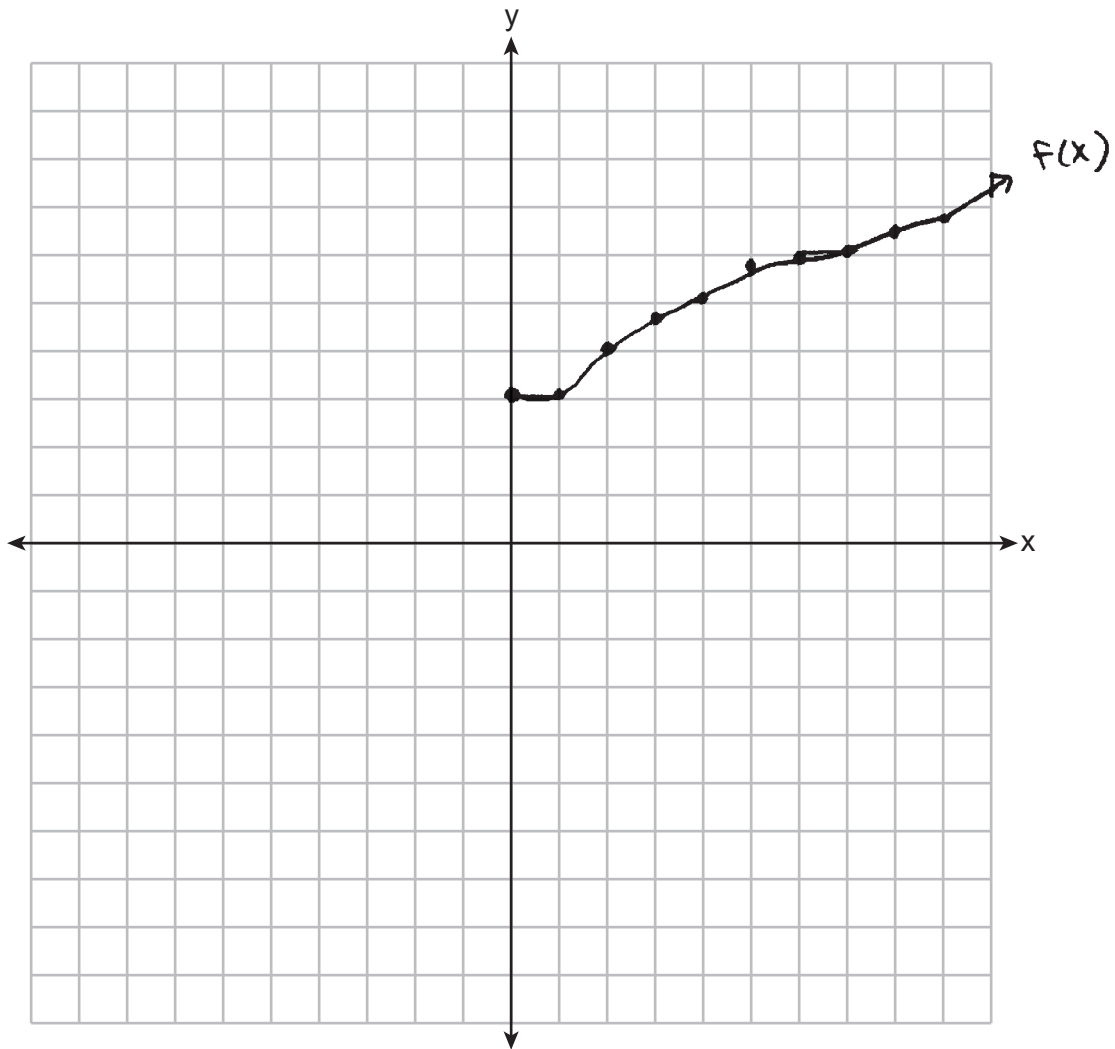
State the equation of the asymptote of  $g(x)$ .

$$g(x) \text{ asymptote} = (0, 0)$$

**Score 1:** The student made one graphing error and stated incorrect asymptotes.

Question 36

36 Graph  $y = f(x)$ , where  $f(x) = \log_2(x - 1) + 3$  on the set of axes below.



State the equation of the asymptote of  $f(x)$ .

$$f(x) \rightarrow \text{asymptote} \rightarrow x = -3$$

When  $f(x)$  is reflected over the line  $y = x$ , a new function is formed:  $g(x) = 2^{x-3} + 1$ .

State the equation of the asymptote of  $g(x)$ .

$$g(x) \rightarrow \text{asymptote} \rightarrow x = -1$$

**Score 0:** The student did not show enough course-level work to receive any credit.

**Question 37**

37 Megan is performing an experiment in a lab where the air temperature is a constant  $73^{\circ}\text{F}$  and the liquid is  $237^{\circ}\text{F}$ . One and a half hours later, the temperature of the liquid is  $112^{\circ}\text{F}$ . Newton's law of cooling states  $T(t) = T_a + (T_0 - T_a)e^{-kt}$  where:

$T(t)$ : temperature,  $^{\circ}\text{F}$ , of the liquid at  $t$  hours

$T_a$ : air temperature

$T_0$ : initial temperature of the liquid

$k$ : constant

Determine the value of  $k$ , to the nearest thousandth, for this liquid.

$$K = .958$$

$$112 = 73 + (237 - 73)e^{-k(1.5)}$$

$$\frac{39}{164} = \frac{164}{164}e^{-k(1.5)}$$

$$-\ln k = -1.426$$

$$\ln \frac{39}{164} = -k(1.5) \ln e$$

Question 37 is continued on the next page.

**Score 6:** The student gave a complete and correct response.

**Question 37**

Determine the temperature of the liquid using your value for  $k$ , to the nearest degree, after two and a half hours.

$$T(t) = 73 + (237 - 73)e^{-.958(2.5)}$$

$$T(t) = 88^\circ$$

Megan needs the temperature of the liquid to be  $80^\circ\text{F}$  to perform the next step in her experiment. Use your value for  $k$  to determine, to the nearest tenth of an hour, how much time she must wait since she first began the experiment.

$$80 = 73 + (237 - 73)e^{-.958(x)}$$

$$\frac{7}{164} = \frac{164}{164} e^{-.958(x)}$$

It would be 3.3 hours

$$\ln \frac{7}{164} = -.958(x) \ln e^1$$

$$-.958x = -3.15$$

**Question 37**

37 Megan is performing an experiment in a lab where the air temperature is a constant  $73^{\circ}\text{F}$  and the liquid is  $237^{\circ}\text{F}$ . One and a half hours later, the temperature of the liquid is  $112^{\circ}\text{F}$ . Newton's law of cooling states  $T(t) = T_a + (T_0 - T_a)e^{-kt}$  where:

$T(t)$ : temperature,  $^{\circ}\text{F}$ , of the liquid at  $t$  hours

$T_a$ : air temperature

$T_0$ : initial temperature of the liquid

$k$ : constant

Determine the value of  $k$ , to the *nearest thousandth*, for this liquid.

39 =  $k = .958$

$\frac{39}{164} = e^{-k(1.5)}$  ~~AAA~~  
 $-.415853 = -k \log e$   
 $112 = 73 + (237 - 73)e^{-k(1.5)}$   
 $-k = .957$   
 $\frac{\log 39 - \log 164}{1.5} = \frac{-k(1.5) / \log e}{1.5}$

Question 37 is continued on the next page.

**Score 6:** The student gave a complete and correct response.



Question 37

Determine the temperature of the liquid using your value for  $k$ , to the *nearest degree*, after two and a half hours.

$$T(t) = 73 + (237 - 73)e^{-.958(2.5)}$$

$$T(t) = 88^\circ\text{F}$$

Megan needs the temperature of the liquid to be  $80^\circ\text{F}$  to perform the next step in her experiment. Use your value for  $k$  to determine, to the *nearest tenth of an hour*, how much time she must wait since she first began the experiment.

$$x = 3.3 \text{ hours}$$

$$\frac{1.4298}{\ln e} = x \ln e$$

$$\frac{7}{164} = e^{-.958(x)}$$

$$80 = 73 + (237 - 73)e^{-.958(x)}$$

$$\frac{\ln 7 - \ln 164}{-.958} = \frac{-.958(x) \ln e}{-.958} \quad \frac{7}{(237-73)} = \frac{(237-73)e^{-.958(x)}}{(237-73)}$$

**Question 37**

37 Megan is performing an experiment in a lab where the air temperature is a constant  $73^\circ\text{F}$  and the liquid is  $237^\circ\text{F}$ . One and a half hours later, the temperature of the liquid is  $112^\circ\text{F}$ . Newton's law of cooling states  $T(t) = T_a + (T_0 - T_a)e^{-kt}$  where:

- $T(t)$ : temperature,  $^\circ\text{F}$ , of the liquid at  $t$  hours
- $T_a$ : air temperature
- $T_0$ : initial temperature of the liquid
- $k$ : constant

Determine the value of  $k$ , to the nearest thousandth, for this liquid.

$$\begin{aligned}
 112 &= 73 + (237 - 73)e^{-k \cdot 1.5} \\
 112 &= 73 + (164)e^{-k \cdot 1.5} \\
 \hline
 39 &= \frac{164e^{-k \cdot 1.5}}{1.5} \\
 \hline
 .2378 &= e^{-k \cdot 1.5} \\
 \ln(.2378) &= \ln(e^{-k \cdot 1.5}) \\
 \hline
 \frac{-1.44}{-1.5} &= k \\
 \boxed{.016} &= k
 \end{aligned}$$

Question 37 is continued on the next page.

**Score 5:** The student used incorrect time units when solving for  $k$ .

Question 37

Determine the temperature of the liquid using your value for  $k$ , to the *nearest degree*, after two and a half hours.

$$T(t) = 73 + (237 - 73)e^{(-.016)t/150} \quad \rightarrow 150 \text{ min}$$

$$T(t) = 73 + (164)e^{(-.016)(150)}$$

$$73 + (164)(.0907)$$

$$73 + (14.877)$$

$$T(t) = 88^\circ\text{F}$$

Megan needs the temperature of the liquid to be  $80^\circ\text{F}$  to perform the next step in her experiment. Use your value for  $k$  to determine, to the *nearest tenth of an hour*, how much time she must wait since she first began the experiment.

$$80 = 73 + (237 - 73)e^{(-.016)t}$$

$$7 = 164e^{-.016t}$$

$$\frac{7}{164} = \frac{164}{164}e^{-.016t}$$

$$.043 = e^{-.016t}$$

$$\ln(.043) = \ln(e^{-.016t})$$

$$\frac{\ln(.043)}{-.016} = \frac{-.016t}{-.016}$$

$$197.12 = t$$

$$\rightarrow t = 3.3 \text{ hrs}$$

**Question 37**

37 Megan is performing an experiment in a lab where the air temperature is a constant  $73^\circ\text{F}$  and the liquid is  $237^\circ\text{F}$ . One and a half hours later, the temperature of the liquid is  $112^\circ\text{F}$ . Newton's law of cooling states  $T(t) = T_a + (T_0 - T_a)e^{-kt}$  where:

$T(t)$ : temperature,  $^\circ\text{F}$ , of the liquid at  $t$  hours

$T_a$ : air temperature

$T_0$ : initial temperature of the liquid

$k$ : constant

Determine the value of  $k$ , to the *nearest thousandth*, for this liquid.

~~$112 = 73 + (237 - 73)e^{-k(1.5)}$~~   
 $112 = 73 + (237 - 73)e^{-k(1.5)}$   
 ~~$39 = 164e^{-k(1.5)}$~~   
 $39 = 164e^{-k(1.5)}$   
 ~~$\ln \frac{39}{164} = -k(1.5)$~~   
 $\ln \frac{39}{164} = -k(1.5)$   
 ~~$k = \frac{\ln \frac{39}{164}}{-1.5}$~~   
 $k = \frac{\ln \frac{39}{164}}{-1.5}$   
 $k = .958$

Question 37 is continued on the next page.

**Score 5:** The student made one computational error solving for  $t$ .

Question 37

Determine the temperature of the liquid using your value for  $k$ , to the *nearest degree*, after two and a half hours.

$$T(t) = 73 + (237 - 73)e^{-0.958(2.5)}$$

~~$T(t) = 84^\circ\text{F}$~~  88°F

Megan needs the temperature of the liquid to be  $80^\circ\text{F}$  to perform the next step in her experiment. Use your value for  $k$  to determine, to the *nearest tenth of an hour*, how much time she must wait since she first began the experiment.

$$80 = 73 + (237 - 73)e^{-0.958t}$$
$$7 = 164e^{-0.958t}$$
$$\ln \frac{7}{164} = t$$
$$\frac{-0.958}{3.02} = t$$
3.0 hours

**Question 37**

37 Megan is performing an experiment in a lab where the air temperature is a constant 73°F and the liquid is 237°F. One and a half hours later, the temperature of the liquid is 112°F. Newton's law of cooling states  $T(t) = T_a + (T_0 - T_a)e^{-kt}$  where:

$T(t)$ : temperature, °F, of the liquid at  $t$  hours

$T_a$ : air temperature

$T_0$ : initial temperature of the liquid

$k$ : constant

Determine the value of  $k$ , to the nearest thousandth, for this liquid.

$$\begin{aligned}
 & 112 = 73 + (237 - 73)e^{-k(1.5)} \\
 & \frac{112 - 73}{237 - 73} = e^{-k(1.5)} \\
 & \frac{39}{164} = e^{-k(1.5)} \\
 & \log e \frac{39}{164} = -1.5 \\
 & \frac{-1.436}{-1.436} = \frac{-1.5k}{-1.436} \\
 & \frac{39}{164} = e^{-k(1.5)}
 \end{aligned}$$

**$k = 1.049$**

Question 37 is continued on the next page.

**Score 4:** The student made an error solving for  $k$  and a rounding error determining the temperature.

Question 37

Determine the temperature of the liquid using your value for  $k$ , to the *nearest degree*, after two and a half hours.

$$T(t) = 85.03$$

$$T(t) = 73 + (237 - 73)e^{(-1.045)(2.5)}$$

$$T(t) = 73 + (164)e^{-2.6125}$$

$$T(t) = 73 + 12.03$$

Megan needs the temperature of the liquid to be  $80^\circ\text{F}$  to perform the next step in her experiment. Use your value for  $k$  to determine, to the *nearest tenth of an hour*, how much time she must wait since she first began the experiment.

$$t = 3.0$$

$$\frac{-3.15}{-1.045} = \frac{-1.045}{-1.045}$$

$$80 = 73 + (237 - 73)e^{-1.045t}$$

$$\ln\left(\frac{7}{164}\right) = \frac{-1.045t}{-1.045}$$

$$\frac{7}{164} = \frac{(164)}{164}e^{-1.045t}$$

$$\frac{7}{164} = e^{-1.045t}$$

**Question 37**

37 Megan is performing an experiment in a lab where the air temperature is a constant 73°F and the liquid is 237°F. One and a half hours later, the temperature of the liquid is 112°F. Newton's law of cooling states  $T(t) = T_a + (T_0 - T_a)e^{-kt}$  where:

- $T(t)$ : temperature, °F, of the liquid at  $t$  hours
- $T_a$ : air temperature
- $T_0$ : initial temperature of the liquid
- $k$ : constant

Determine the value of  $k$ , to the nearest thousandth, for this liquid.

Handwritten work showing calculations for  $k$ :

$$\ln \frac{39}{164} = -k(1.5)$$

$$\frac{\ln \frac{39}{164}}{1.5} = -k$$

$$-0.95753652 = -k$$

$$k = 0.957$$

Alternative calculation path:

$$T(t) = 73 + (237 - 73)e^{-kt}$$

$$112 = 73 + (237 - 73)e^{-k(1.5)}$$

$$39 = (237 - 73)e^{-k(1.5)}$$

$$\frac{39}{164} = e^{-k(1.5)}$$

Final answer circled:  $k = 0.96$

Question 37 is continued on the next page.

**Score 3:** The student made a rounding error solving for  $k$  and received no credit for the third part.



Question 37

Determine the temperature of the liquid using your value for  $k$ , to the nearest degree, after two and a half hours.

$$T(t) = 73 + (237 - 73)e^{-.96(t)}$$

$$T(2.5) = 87.87$$

$$T(2.5) = 88^\circ\text{F}$$

Megan needs the temperature of the liquid to be  $80^\circ\text{F}$  to perform the next step in her experiment. Use your value for  $k$  to determine, to the nearest tenth of an hour, how much time she must wait since she first began the experiment.

$$80^\circ\text{F} = 73 + (237 - 73)e^{-.96(x)}$$

$$77 = (237 - 73)e^{-.96(x)}$$

**Question 37**

37 Megan is performing an experiment in a lab where the air temperature is a constant  $73^{\circ}\text{F}$  and the liquid is  $237^{\circ}\text{F}$ . One and a half hours later, the temperature of the liquid is  $112^{\circ}\text{F}$ . Newton's law of cooling states  $T(t) = T_a + (T_0 - T_a)e^{-kt}$  where:

$T(t)$ : temperature,  $^{\circ}\text{F}$ , of the liquid at  $t$  hours

$T_a$ : air temperature

$T_0$ : initial temperature of the liquid

$k$ : constant

Determine the value of  $k$ , to the nearest thousandth, for this liquid.

$$T(t) = T_a + (T_0 - T_a)e^{-kt}$$
$$237 = 73 + (112 - 73)e^{-k(1.5)}$$

$$\frac{164}{139} = \frac{39e^{-k(1.5)}}{39}$$

$$\ln \frac{164}{39} = \ln e^{-1.5k}$$
$$\frac{-1.5}{-1.5} = \frac{-1.5k}{-1.5}$$

$$k = .9575$$

Question 37 is continued on the next page.

**Score 2:** The student determined an appropriate time.

Question 37

Determine the temperature of the liquid using your value for  $k$ , to the nearest degree, after two and a half hours.

$$237 = 73 + (112 - 73)e^{-k(2.5)}$$

$$\frac{164}{39} = \frac{\cancel{39}e^{-k(2.5)}}{\cancel{39}}$$

$$-.5$$

$-0.1^\circ$

$$\frac{\ln \frac{164}{39}}{-2.5} = \frac{\cancel{\ln} e^{-2.5k}}{-2.5}$$

Megan needs the temperature of the liquid to be  $80^\circ\text{F}$  to perform the next step in her experiment. Use your value for  $k$  to determine, to the nearest tenth of an hour, how much time she must wait since she first began the experiment.

$$80^\circ = 73 + (112 - 73)e^{-.9575t}$$

$$\frac{7}{39} = \frac{\cancel{39}e^{-.9575t}}{\cancel{39}}$$

$$t = 1.79$$

$$\ln \frac{7}{39} = \cancel{\ln} e^{-.9575t}$$

$$\frac{\ln \frac{7}{39}}{-.9575} = \frac{\cancel{\ln} e^{-.9575t}}{-.9575}$$

$t = 1.8 \text{ hrs}$

**Question 37**

37 Megan is performing an experiment in a lab where the air temperature is a constant  $73^{\circ}\text{F}$  and the liquid is  $237^{\circ}\text{F}$ . One and a half hours later, the temperature of the liquid is  $112^{\circ}\text{F}$ . Newton's law of cooling states  $T(t) = T_a + (T_0 - T_a)e^{-kt}$  where:

$T(t)$ : temperature,  $^{\circ}\text{F}$ , of the liquid at  $t$  hours

$T_a$ : air temperature

$T_0$ : initial temperature of the liquid

$k$ : constant

Determine the value of  $k$ , to the nearest thousandth, for this liquid.

$$\begin{aligned} 112 &= 73 + (237 - 73)e^{-k(1\frac{1}{2})} \\ 39 &= 164e^{-k(1\frac{1}{2})} \\ \ln\left(\frac{39}{164}\right) &= -k(1\frac{1}{2}) \\ \frac{-2\ln\frac{39}{164}}{3} &= k \qquad k \approx 2.496 \end{aligned}$$

Question 37 is continued on the next page.

**Score 1:** The student made one computational error in finding  $k$ .

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**Question 37**

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Determine the temperature of the liquid using your value for  $k$ , to the *nearest degree*, after two and a half hours.

$$1 \text{ degree} = \frac{\pi}{180} \text{ radians}$$

$$2\frac{1}{2} = 112^\circ\text{F} (237^\circ\text{F} - 112^\circ\text{F})e^{-}$$

Megan needs the temperature of the liquid to be  $80^\circ\text{F}$  to perform the next step in her experiment. Use your value for  $k$  to determine, to the *nearest tenth of an hour*, how much time she must wait since she first began the experiment.

**Question 37**

37 Megan is performing an experiment in a lab where the air temperature is a constant  $73^{\circ}\text{F}$  and the liquid is  $237^{\circ}\text{F}$ . One and a half hours later, the temperature of the liquid is  $112^{\circ}\text{F}$ . Newton's law of cooling states  $T(t) = T_a + (T_0 - T_a)e^{-kt}$  where:

$73$  )  $164$   $T(t)$ : temperature,  $^{\circ}\text{F}$ , of the liquid at  $t$  hours  
 $237$  )  $125$   $T_a$ : air temperature  
 $112$  )  $125$   $T_0$ : initial temperature of the liquid  
 $k$ : constant

Determine the value of  $k$ , to the *nearest thousandth*, for this liquid.

$$\begin{aligned}112 &= 73 + (237 - 73)e^{-k/2} \\112 &= 73 + (164)e^{-k/2} \\112 &= 237e^{-k/2} \\ \log 112 &= -k/2 \log 237 - \\ \frac{\log 112}{\log 237} &= -k/2 \quad k = -1.726\end{aligned}$$

**Question 37 is continued on the next page.**

**Score 0:** The student did not show enough relevant course-level work to receive any credit.

### Question 37

Determine the temperature of the liquid using your value for  $k$ , to the *nearest degree*, after two and a half hours.

$$T(t) = 73 + (237 - 73)e^{-.863t}$$

$$T(2.5) = 73 + 164e^{-.863 \cdot 2.5}$$

$$T(2.5) = 237e^{-.863}$$

$$T(2.5) = -.863 \log(237)$$

$$T(2.5) = -2$$

Megan needs the temperature of the liquid to be  $80^\circ\text{F}$  to perform the next step in her experiment. Use your value for  $k$  to determine, to the *nearest tenth of an hour*, how much time she must wait since she first began the experiment.

$$80 = 237e^{-1.726T}$$

$$\log 80 = -1.726T \log(237)$$

$$1.903 = -1.726T(2.37)$$

$$1.903 = T(-4.09)$$

$$T = 2 \text{ hours and } 19 \text{ minutes}$$

**Question 37**

37 Megan is performing an experiment in a lab where the air temperature is a constant  $73^{\circ}\text{F}$  and the liquid is  $237^{\circ}\text{F}$ . One and a half hours later, the temperature of the liquid is  $112^{\circ}\text{F}$ . Newton's law of cooling states  $T(t) = T_a + (T_0 - T_a)e^{-kt}$  where:

$T(t)$ : temperature,  $^{\circ}\text{F}$ , of the liquid at  $t$  hours

$T_a$ : air temperature

$T_0$ : initial temperature of the liquid

$k$ : constant

Determine the value of  $k$ , to the *nearest thousandth*, for this liquid.

$$T(t) = T_a + (T_0 - T_a)e^{-kt}$$

$$112(1.5) = 73 + (237 - 73)e^{-k(1.5)}$$

$$108 = 73 + (164)e^{-k(1.5)}$$

$$(164)$$

**Question 37 is continued on the next page.**

**Score 0:** The student did not show enough relevant course-level work to receive any credit.



Question 37

Determine the temperature of the liquid using your value for  $k$ , to the *nearest degree*, after two and a half hours.

$$237 - 112 = 125$$

$$1\frac{1}{2} \text{ hrs} = -125 / 112$$

$$125 \div 3 \approx 42$$

$$42 \times 2 = 84$$

$$84 = 1 \text{ hr}$$

$$112 - 84 = 28$$

$$\boxed{28^\circ\text{F}}$$

Megan needs the temperature of the liquid to be  $80^\circ\text{F}$  to perform the next step in her experiment. Use your value for  $k$  to determine, to the *nearest tenth of an hour*, how much time she must wait since she first began the experiment.

$$\boxed{2 \text{ hours}}$$

$$1\frac{1}{2} \text{ hrs} = -125 / 112$$

$$125 / 3 \approx 42$$

$$42 \times 2 = 84$$

$$84 = 1 \text{ hr}$$

$$42 = 30 \text{ min}$$

$$112 - 42 = 70$$