

The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

COURSE III

Thursday, August 13, 1998 — 8:30 to 11:30 a.m., only

Notice . . .

Scientific calculators must be available to all students taking this examination.

The formulas which you may need to answer some questions in this examination are found on page 2. The last page of the booklet is the answer sheet. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

When you have completed the examination, you must sign the statement printed at the end of the answer paper, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer paper cannot be accepted if you fail to sign this declaration.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Formulas

Pythagorean and Quotient Identities

$$\begin{array}{l} \sin^2 A + \cos^2 A = 1 \\ \tan^2 A + 1 = \sec^2 A \\ \cot^2 A + 1 = \csc^2 A \end{array} \quad \begin{array}{l} \tan A = \frac{\sin A}{\cos A} \\ \cot A = \frac{\cos A}{\sin A} \end{array}$$

Functions of the Sum of Two Angles

$$\begin{array}{l} \sin(A + B) = \sin A \cos B + \cos A \sin B \\ \cos(A + B) = \cos A \cos B - \sin A \sin B \\ \tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \end{array}$$

Functions of the Difference of Two Angles

$$\begin{array}{l} \sin(A - B) = \sin A \cos B - \cos A \sin B \\ \cos(A - B) = \cos A \cos B + \sin A \sin B \\ \tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B} \end{array}$$

Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Functions of the Double Angle

$$\begin{array}{l} \sin 2A = 2 \sin A \cos A \\ \cos 2A = \cos^2 A - \sin^2 A \\ \cos 2A = 2 \cos^2 A - 1 \\ \cos 2A = 1 - 2 \sin^2 A \\ \tan 2A = \frac{2 \tan A}{1 - \tan^2 A} \end{array}$$

Functions of the Half Angle

$$\begin{array}{l} \sin \frac{1}{2}A = \pm \sqrt{\frac{1 - \cos A}{2}} \\ \cos \frac{1}{2}A = \pm \sqrt{\frac{1 + \cos A}{2}} \\ \tan \frac{1}{2}A = \pm \sqrt{\frac{1 - \cos A}{1 + \cos A}} \end{array}$$

Area of Triangle

$$K = \frac{1}{2}ab \sin C$$

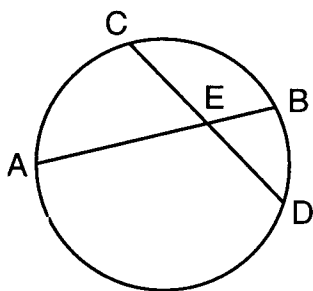
Standard Deviation

$$\text{S.D.} = \sqrt{\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2}$$

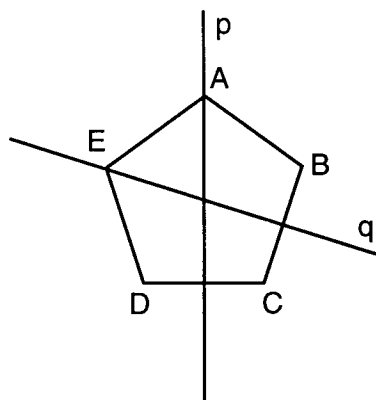
Part I

Answer 30 questions from this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet. Where applicable, answers may be left in terms of π or in radical form. [60]

- Express $\frac{7\pi}{5}$ radians in degrees.
- If $f(x) = x - 2$ and $g(x) = x^2$, find $f(g(3))$.
- If $\sin A = -1$ and $0^\circ \leq A < 360^\circ$, find $m\angle A$.
- In $\triangle ABC$, $a = 15$, $c = 10$, and $\sin A = 0.45$. Find $\sin C$.
- Find the coordinates of P' , the image of $P(3, -1)$ under the transformation $(x, y) \rightarrow (-y, -x)$.
- Find the value of $\sin 135^\circ$ in radical form.
- Evaluate: $\sum_{x=1}^4 (x^2 - 3)$
- When the graphs of $2 + 4i$ and $3 - 7i$ are drawn on the same set of axes, in which quadrant will the sum of these expressions lie?
- Solve for the positive value of x : $\frac{x}{3} - \frac{4}{x} = \frac{4}{3}$
- If $a = 4$, evaluate $a^{\frac{1}{2}} + a^0 + a^{-2}$.
- In the accompanying diagram, chords \overline{AB} and \overline{CD} intersect at E . If $m\widehat{AC} = 75$ and $m\widehat{DB} = 45$, find $m\angle AED$.



- What is the image of $(6, 5)$ under a counterclockwise rotation of 180° ?
- If $\log_n 8 = 3$, find the value of n .
- In the accompanying diagram, p and q are lines of symmetry in regular pentagon $ABCDE$. Find $r_p \circ r_q(B)$.



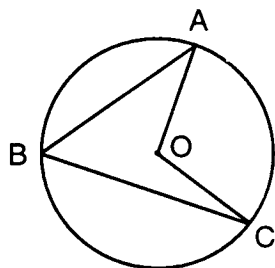
- If x varies inversely as y and $x = 12$ when $y = 4$, what is the value of y when $x = 16$?
- For which value of θ is $\frac{\cos \theta}{\sin \theta}$ undefined in the interval $-\pi < \theta < \pi$?

Directions (17–35): For each question chosen, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question.

- What is the value of y if $y = \sin\left(\text{Arc tan } \frac{5}{12}\right)$?

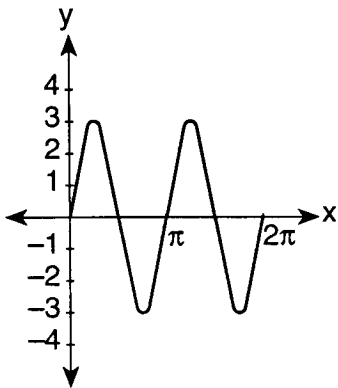
(1) $\frac{5}{13}$	(3) $\frac{13}{12}$
(2) $\frac{12}{13}$	(4) $\frac{13}{5}$

- 18 In the accompanying diagram of circle O , $m\angle AOC = 108$.



What is $m\angle ABC$?

- (1) 27 (3) 108
 (2) 54 (4) 216
- 19 What is the value of x in the equation $3^{x-3} = 1$?
- (1) 1 (3) 3
 (2) $\frac{1}{3}$ (4) 0
- 20 Which equation is represented by the graph in the accompanying diagram?



- (1) $y = 3 \sin 2x$ (3) $y = 3 \sin x$
 (2) $y = 2 \sin 3x$ (4) $y = 2 \sin 4x$
- 21 The expression $\cos^2 40 - \sin^2 40$ has the same value as
- (1) $\sin 20$ (3) $\cos 80$
 (2) $\sin 80$ (4) $\cos 20$
- 22 If a fair die is tossed five times, what is the probability of getting exactly three 6's?
- (1) $\frac{125}{7776}$ (3) ${}_5C_3 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^3$
 (2) ${}_5C_3 \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^2$ (4) $\frac{25}{7776}$

- 23 What is the solution set for the inequality $x^2 - 2x < 8$?
- (1) $-2 < x < 4$ (3) $x < -2$ or $x > 4$
 (2) $-4 < x < 2$ (4) $x < -4$ or $x > 2$

- 24 What is the solution set of the equation $|2x - 5| = 3$?
- (1) $\{ \}$ (3) $\{4\}$
 (2) $\{-4, 4\}$ (4) $\{1, 4\}$

- 25 The expression $\frac{\sin 2x}{\sin(-x)}$ is equivalent to
- (1) $-2 \sin x$ (3) $-2 \cos x$
 (2) $2 \sin x$ (4) $2 \cos x$

- 26 Which equation has both 3 and 6 as roots?
- (1) $\sqrt{x-2} = x-4$ (3) $\sqrt{x-2} = \frac{3}{x}$
 (2) $\sqrt{x-2} = 4-x$ (4) $\sqrt{x-2} = \frac{x}{3}$

- 27 In $\triangle ABC$, $a = 4$, $b = 3$, and $\cos C = -\frac{1}{2}$. What is the length of c ?
- (1) 7 (3) $\sqrt{37}$
 (2) $\sqrt{13}$ (4) $\sqrt{19}$

- 28 Which equation has rational roots?
- (1) $x^2 + 8x - 8 = 0$ (3) $2x^2 + 4x + 5 = 0$
 (2) $x^2 + 8x + 9 = 0$ (4) $3x^2 + 8x + 4 = 0$

- 29 When drawn on a set of axes, which equation is an ellipse?
- (1) $2x^2 + y = 12$ (3) $2x^2 + 2y^2 = 12$
 (2) $2x^2 + y^2 = 12$ (4) $2x^2 - y^2 = 12$

- 30 When simplified, i^{99} is equivalent to
- (1) 1 (3) i
 (2) -1 (4) $-i$

- 31 On a standardized test with a normal distribution of scores, the mean score is 82 and the standard deviation is 6. Which interval contains 95% of the scores?
- (1) 70-82 (3) 76-88
 (2) 70-94 (4) 76-94

32 In the interval $0 \leq \theta < 2\pi$, the number of solutions of the equation $\sin \theta = \cos \theta$ is

- (1) 1 (3) 3
(2) 2 (4) 4

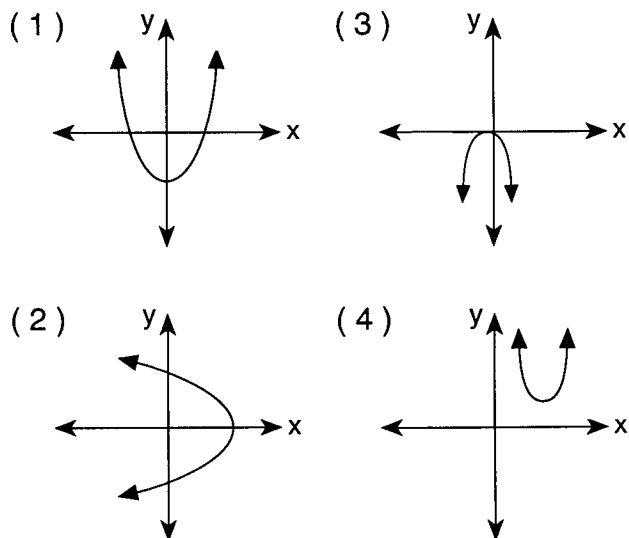
33 In isosceles triangle ABC , $\overline{AB} \cong \overline{BC}$, $m\angle B = 45^\circ$, and $AB = 3\sqrt{2}$. The area of the triangle is

- (1) $\frac{9}{2}$ (3) $\frac{9\sqrt{2}}{2}$
(2) $9\sqrt{2}$ (4) $\frac{3\sqrt{2}}{2}$

34 What is the fourth term in the expansion $(a + b)^5$?

- (1) $10a^2b^3$ (3) $5a^2b^3$
(2) $10a^3b^2$ (4) $5a^3b^2$

35 Which diagram could represent the graph of an equation with imaginary roots?



Answers to the following questions are to be written on paper provided by the school.

Part II

Answer four questions from this part. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Calculations that may be obtained by mental arithmetic or the calculator do not need to be shown. [40]

36 a On the same set of axes, sketch and label the graphs of the equations $y = 3 \cos \frac{1}{2}x$ and $y = -2 \sin x$ in the interval $0 \leq x \leq 2\pi$. [8]

b In the interval $0 \leq x \leq 2\pi$, which value of x satisfies the equation $3 \cos \frac{1}{2}x = -2 \sin x$? [2]

37 Find all values of x in the interval $0^\circ \leq x < 360^\circ$ that satisfy the equation $3 \cos 2x + 2 \sin x = -1$. Express your answer to the nearest ten minutes or nearest tenth of a degree. [10]

38 a Simplify: $\frac{\frac{x}{x-3} + \frac{4}{x}}{1 - \frac{1}{3-x}}$ [6]

b Solve the equation for x and express the roots in simplest $a + bi$ form:

$$4x^2 - 12x + 25 = 0 \quad [4]$$

39 a (1) On graph paper, sketch and label the graph of the equation $y = 2^x$ in the interval $-2 \leq x \leq 2$. [2]

(2) On the same set of axes, reflect the graph drawn in part a (1) in the y -axis and label it r . [2]

(3) Write an equation of the graph drawn in part a (2). [2]

b Solve for x to the nearest hundredth:

$$2^x = \frac{3}{16} \quad [4]$$

40 In parallelogram $ABCD$, $AB = 14$, $BC = 20$, and $m\angle B = 54^\circ$.

a Find, to the nearest tenth, the length of diagonal \overline{BD} . [6]

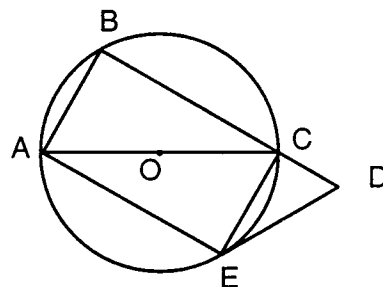
b Find $m\angle DBC$ to the nearest degree. [4]

- 41 The table below shows the heights of a group of 20 students.

Height (inches)	Frequency
72	3
71	2
70	1
69	2
68	4
67	2
66	4
65	2

- a Find the mean and the standard deviation to the nearest tenth. [4]
- b If one student's height is chosen at random, what is the probability that the height falls within one standard deviation of the mean? [2]
- c If three students' heights are chosen at random, what is the probability that *at most* one of them falls within one standard deviation of the mean? [4]

- 42 In the accompanying diagram of circle O , $m\widehat{AB}:m\widehat{BC} = 1:2$; diameter \overline{CA} and chord \overline{AE} are drawn; chord \overline{EC} is parallel to chord \overline{AB} ; chord \overline{BC} is extended through C to D ; and tangent \overline{DE} is drawn.



Find:

a $m\widehat{BC}$ [2]

b $m\widehat{CE}$ [2]

c $m\angle AEC$ [2]

d $m\angle CED$ [2]

e $m\angle BDE$ [2]

The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

SEQUENTIAL MATH – COURSE III

Thursday, August 13, 1998 — 8:30 to 11:30 a.m., only

Part I Score
Part II Score	<u>.....</u>
Total Score
Rater's Initials:

ANSWER SHEET

Pupil Sex: Male Female Grade

Teacher School

Your answers to Part I should be recorded on this answer sheet.

Part I

Answer 30 questions from this part.

- | | | | |
|----------|----------|----------|----------|
| 1 | 11 | 21 | 31 |
| 2 | 12 | 22 | 32 |
| 3 | 13 | 23 | 33 |
| 4 | 14 | 24 | 34 |
| 5 | 15 | 25 | 35 |
| 6 | 16 | 26 | |
| 7 | 17 | 27 | |
| 8 | 18 | 28 | |
| 9 | 19 | 29 | |
| 10 | 20 | 30 | |

Your answers for Part II should be placed on paper provided by the school.

The declaration below should be signed when you have completed the examination.

I do hereby affirm, at the close of this examination, that I had no unlawful knowledge of the questions or answers prior to the examination, and that I have neither given nor received assistance in answering any of the questions during the examination.

Signature

Tear Here

Tear Here