Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, the result is written in simplest form, when the expression \(-x^2 + 2x - 3\) is written in simplest form, the result is

1. The point \((3, w)\) is on the graph of \(y = 2x + 7\). What is the value of \(w\)?

   - (1) \(3\)
   - (2) \(6\)
   - (3) \(9\)
   - (4) \(12\)

2. When the expression \(2x(x - 3)\) is written in simplest form, which of the following is correct?

   - (1) \(2x^2 - 6x\)
   - (2) \(2x^2 - 5x + 7\)
   - (3) \(2x^2 - 6x + 7\)
   - (4) \(2x^2 - 7x + 12\)

Use this space for computations.
Students were asked to write $\frac{2x^3}{10} + \frac{3x^2}{1} + \frac{4x}{1}$ in standard form.

Four student responses are shown below.

- **Alexa:** $\frac{4}{1}x^3 + \frac{2}{1}x^2 + \frac{3}{1}x + \frac{4}{1}
- **Carol:** $\frac{2}{1}x^3 + \frac{3}{1}x^2 + \frac{4}{1}x + \frac{3}{1}
- **Ryan:** $\frac{2}{1}x^3 + \frac{3}{1}x^2 + \frac{4}{1}x + \frac{3}{1}
- **Eric:** $\frac{1}{1}x^3 + \frac{2}{1}x^2 + \frac{3}{1}x + \frac{4}{1}$

Which student’s response is correct?

(1) Alexa (3) Ryan (2) Carol (4) Eric

Given $\frac{f(x)}{10} = -3x^2 + 10$, what is the value of $f(2)$?

(1) $\frac{26}{10}$ (3) $\frac{22}{10}$
(2) $\frac{2}{10}$ (4) $\frac{46}{10}$

Use this space for computations.
Which relation is a function?

(1) \{ (1,3), (2,1), (3,1), (4,7) \}

(2) \[
\begin{array}{c|c}
\text{Input} & \text{Output} \\
\hline
5 & 7 \\
3 & 7 \\
2 & -4 \\
-2 & -6 \\
\end{array}
\]

(3) \{(1,3), (2,1), (3,1), (4,7)\}

(4) \[
\begin{array}{c|c}
\text{Input} & \text{Output} \\
\hline
1 & 7 \\
8 & 5 \\
6 & 5 \\
3 & 4 \\
\end{array}
\]
6. What is the value of the third quartile in the box plot shown below?

7. What is the solution to \( \frac{3}{x} - \frac{7}{3} - \frac{3}{x} = \frac{3}{x} \)?

(1) 18 (3) 36
(2) 22 (4) 46

Use this space for computations.
One Saturday afternoon, three friends decided to keep track of the number of text messages they received each hour from 8 a.m. to noon. The results are shown below.

Emily said that the number of messages she received increased by 8 each hour. 

Jessica said that the number of messages she received doubled every hour, none the third hour, and 15 the last hour.

Chris said that he received 3 messages the first hour, 10 the second hour, none the third hour, and 15 the last hour.

Which of the friends’ responses best classifies the number of messages they received each hour as a linear function?

(1) Emily, only (3) Emily and Chris  
(2) Jessica, only (4) Jessica and Chris

Which expression is equivalent to $(\frac{x}{4})^2$? 

(1) $(\frac{x}{16})^6$  
(2) $(\frac{x}{4})^5$  
(3) $(\frac{x}{16})^6$  
(4) $(\frac{x}{16})^5$
Caitlin graphs the function \( f(x) = ax^2 \), where \( a \) is a positive integer.

If Caitlin multiplies \( a \) by \(-1\), when compared to \( f(x) \), the new graph

10. Caitlin graphs the function \( f(x) = ax^2 \), where \( a \) is a positive integer.

If Caitlin multiplies \( a \) by \(-1\), when compared to \( f(x) \), the new graph

1. Which expression can be used to determine the value of the car after \( t \) years?

- \( 29,873(0.20)^t \)
- \( 29,873(1 - 0.20)^t \)
- \( 29,873(1 + 0.20)^t \)
- \( 29,873(0.20)^t \)

Which of the following is true for which values of \( x \) is \( 2x^2 + x = f(x) \)?

- \( x = -1 \) and \( x = 2 \)
- \( x = 3 \) and \( x = -2 \)
- \( x = 0 \) and \( x = 2 \)
- \( x = 0 \) and \( x = -2 \)

Use this space for computations.
Skyler mows lawns in the summer. The function $f(x)$ is used to model the amount of money earned, where $x$ is the number of lawns completely mowed. A reasonable domain for this function would be

1. real numbers
2. rational numbers
3. irrational numbers
4. natural numbers

Which expression is equivalent to $2x^2/100 + 8x$?

1. $2(x + 1)(x - 5)$
2. $2(x - 1)(x + 5)$
3. $2(x + 1)(x - 5)$
4. $2(x + 1)(x - 5)$

Ian throws a ball up in the air and lets it fall to the ground. The height of the ball, $h(t)$, is modeled by the equation $h(t) = -16t^2 + 6t + 3$, with $h(t)$ measured in feet, time, $t$, measured in seconds.

The number 3 in $h(t)$ represents

1. the maximum height of the ball
2. the height from which the ball is thrown
3. the number of seconds it takes for the ball to reach the ground
4. the number of seconds it takes for the ball to reach its maximum height

Which expression is equivalent to $2x^2 + 8x - 100$?

1. $(x + 1)(x - 5)$
2. $(x - 1)(x + 5)$
3. $(x + 1)(x - 5)$
4. $(x + 1)(x - 5)$

Use this space for computations.
Thirty-two teams are participating in a basketball tournament. Only the winning teams in each round advance to the next round, as shown in the table below.

<table>
<thead>
<tr>
<th>Number of Rounds Completed, x</th>
<th>Number of Teams Remaining, f(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>32</td>
</tr>
<tr>
<td>1</td>
<td>16</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
</tr>
</tbody>
</table>

Which function type best models the relationship between the number of rounds completed and the number of teams remaining?

1. (1) absolute value
2. (2) exponential
3. (3) linear
4. (4) quadratic

In a geometric sequence, the first term is 4 and the common ratio is \( \frac{3}{2} \). The fifth term of this sequence is

1. (1) 324
2. (2) 108
3. (3) 108
4. (4) 324
The amount of energy, $Q$, in joules, needed to raise the temperature of $m$ grams of a substance is given by the formula $Q = \frac{mC(T_f - T_i)}{1005}$, where $C$ is the specific heat capacity of the substance. If its initial temperature is $T_i$, an equation to find its final temperature, $T_f$, is

$$T_f = \frac{Q}{mC} + T_i$$

When using the method of completing the square, which equation is equivalent to $x^2 - \frac{12}{5}x - 10 = 0$?

- $$(x - 6)^2 = \frac{46}{5}$$
- $$(x + 6)^2 = \frac{46}{5}$$
- $$(x + 6)^2 = \frac{26}{5}$$
- $$(x - 6)^2 = \frac{26}{5}$$
Which quadratic function has the smallest minimum value?

1. \( f(x) = x^2 - 6x + 9 \)
2. \( g(x) = 5(x-3)^2 + 1 \)
3. \( h(x) = -\frac{1}{2}(x-2)^2 \)
4. \( j(x) = 4(x-1)^2 - 3 \)

Use this space for computations.
Which representation yields the same outcome as the sequence defined recursively below?

$\begin{align*}
(a_1) & : 3, 7, 11, 15, 19, \ldots \\
(a_n) & : 3, 1, 5, 9, 13, \ldots
\end{align*}$

(1) 3, 7, 11, 15, 19, \ldots (3) 3, 1, 5, 9, 13, \ldots

If the zeros of the function $g(x)$ are $\{3, 0, 4\}$, which function could represent $g(x)$?

(1) $g(x) = (x-3)(x+4)$ (3) $g(x) = x(x-3)(x+4)$

(2) $g(x) = (x+3)(x+4)$ (4) $g(x) = x(x+3)(x+4)$

Morgan read that a snail moves about $72$ feet per day. He performs the calculation to convert this rate to different units. What are the units for the converted rate?

Use this space for computations.

\[
\begin{align*}
\text{1 day} & \rightarrow \frac{24 \text{ hours}}{1 \text{ day}} \cdot \frac{60 \text{ minutes}}{1 \text{ hour}} \cdot \frac{12 \text{ inches}}{1 \text{ foot}} \cdot 72 \text{ feet} \\
& = 4320 \text{ inches/hour}
\end{align*}
\]

\[
\begin{align*}
\text{1 day} & \rightarrow \frac{24 \text{ hours}}{1 \text{ day}} \cdot \frac{1 \text{ hour}}{60 \text{ minutes}} \cdot \frac{1 \text{ foot}}{12 \text{ inches}} \cdot 72 \text{ feet} \\
& = 120 \text{ hours/inch}
\end{align*}
\]

\[
\begin{align*}
\text{1 day} & \rightarrow \frac{24 \text{ hours}}{1 \text{ day}} \cdot \frac{1 \text{ hour}}{24 \text{ hours}} \cdot \frac{1 \text{ foot}}{1 \text{ inch}} \cdot 72 \text{ feet} \\
& = 72 \text{ minutes/inch}
\end{align*}
\]

\[
\begin{align*}
\text{1 day} & \rightarrow \frac{24 \text{ hours}}{1 \text{ day}} \cdot \frac{1 \text{ hour}}{72 \text{ feet}} \cdot \frac{1 \text{ inch}}{12 \text{ inches}} \cdot 72 \text{ feet} \\
& = 2 \text{ inches/hour}
\end{align*}
\]
During summer vacation, Ben decides to sell hot dogs and pretzels on a food cart in Manhattan. It costs Ben $0.50 for each hot dog and $0.40 for each pretzel. He has only $100 to spend each day on hot dogs and pretzels. He wants to sell at least 200 items each day. If \( h \) is the number of hot dogs and \( p \) is the number of pretzels, which inequality would be part of a system of inequalities used to determine the total number of hot dogs and pretzels Ben can sell?

1. \( 0.50h + 0.40p \leq 100 \)  
2. \( h + p \geq 200 \)
3. \( 0.50h + 0.40p \leq 200 \)  
4. \( h + p \geq 200 \)

The total number of hot dogs and pretzels Ben can sell would be part of a system of inequalities used to determine the total number of hot dogs and pretzels, which Ben wants to sell at least 200 items each day.
Part II

Answer all 8 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs.

25 Graph the function \( g(x) = \sqrt{x} + 3 \) on the set of axes on the next page.

The set of axes for question 25 is on the next page.
Question 25 continued

\[ g(x) \]
The sixth-grade classes at West Road Elementary School were asked to vote on the location of their class trip. The results are shown in the table below.

<table>
<thead>
<tr>
<th></th>
<th>Playland</th>
<th>Fun Central</th>
<th>Splashdown</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Boys</strong></td>
<td>37</td>
<td>25</td>
<td>46</td>
</tr>
<tr>
<td><strong>Girls</strong></td>
<td>39</td>
<td>53</td>
<td>38</td>
</tr>
</tbody>
</table>

Determine, to the nearest percent, the percentage of girls who voted for Splashdown.
Solve the inequality \( \frac{x}{3} - \frac{3}{2} + 6 < 12 \) algebraically for \( x \).
Determine the common difference of the arithmetic sequence in which $a_1 = 3$ and $a_4 = 17$. 

Work space for question 28 is continued on the next page.
Given: $A = \sqrt{363}$ and $B = \sqrt{27}$

Explain why $A + B$ is irrational.

**Civen:** $A = \sqrt{363}$ and $B = \sqrt{27}$
Question 29 continued

Explain why $A \cdot B$ is rational.
Work space for question 30 is continued on the next page.

30 Use the quadratic formula to solve \( x^2 + 4x - 1 = 0 \) for \( x \).

Round the solutions to the nearest hundredth.
Factor completely:

$4x^3 - 49x$
The function $g$ is defined as

$$g(x) = \begin{cases} x^2, & \text{if } x \geq 2 \\ \lfloor \frac{x}{2} \rfloor + \lfloor \frac{x}{3} \rfloor, & \text{if } -2 < x < 2 \\ -1 + \frac{x}{3}, & \text{if } -\infty < x \leq -2 \end{cases}$$

On the set of axes on the next page, graph $g(x)$. The set of axes for question 32 is on the next page.
33. Anessa is studying the changes in population in a town. The graph below shows the population over 50 years.

Question 33 is continued on the next page.
Question 33 continued

State the entire interval during which the population remained constant.

State the maximum population of the town over the 50-year period.

Determine the average rate of change from Year 30 to Year 40.

Explain what your average rate of change means from Year 30 to Year 40 in the context of the problem.
The table below shows the number of math classes missed during a school year for nine students, and their final exam scores.

<table>
<thead>
<tr>
<th>Number of Classes Missed (x)</th>
<th>Final Exam Score (y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>99</td>
</tr>
<tr>
<td>3</td>
<td>72</td>
</tr>
<tr>
<td>15</td>
<td>60</td>
</tr>
<tr>
<td>18</td>
<td>90</td>
</tr>
<tr>
<td>20</td>
<td>35</td>
</tr>
<tr>
<td>22</td>
<td>40</td>
</tr>
<tr>
<td>2</td>
<td>43</td>
</tr>
<tr>
<td>10</td>
<td>90</td>
</tr>
</tbody>
</table>

Write the linear regression equation for this data set. Round all values to the nearest hundredth.

\[ y = mx + b \]

where:
- \( m \) is the slope of the line.
- \( b \) is the y-intercept.

To find the slope (\( m \)) and y-intercept (\( b \)) of the line, you can use the following formulas:

\[ m = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2} \]

\[ b = \bar{y} - m \bar{x} \]

where:
- \( \bar{x} \) is the mean of the x-values.
- \( \bar{y} \) is the mean of the y-values.

The coefficients of determination (\( R^2 \)) can also be calculated to determine how well the data fits the regression line.

\[ R^2 = \frac{\text{variance explained by the model}}{\text{total variance}} \]

The value of \( R^2 \) ranges from 0 to 1, with 1 indicating a perfect fit and 0 indicating no relation.

In the context of this problem, you would need to calculate the slope and y-intercept using the given data and the formulas provided.
State what the correlation coefficient indicates about the linear fit of the data.
A fence was installed around the edge of a rectangular garden. The length, \( l \), of the fence was 5 feet less than 3 times its width, \( w \). The amount of fencing used was 90 feet.

Write a system of equations or write an equation using one variable that models this situation.

A fence was installed around the edge of a rectangular garden. The length of the fence was 5 feet less than 3 times its width, \( w \). The amount of fencing used was 90 feet.
Determine algebraically the dimensions, in feet, of the garden.
Given: \[
\frac{y}{110} < \frac{9}{134}, \quad \frac{2x}{110} > \frac{2}{4}, \quad \frac{3y}{12} \leq 6 - 12
\]

Graph the system of inequalities on the set of axes on the next page.
Question 36 is continued on the next page.
Question 36 continued

State the coordinates of a point that satisfies both inequalities. Justify your answer.
Aidan and his sister Ella are having a race. Aidan runs at a rate of 10 feet per second. Ella runs at a rate of 6 feet per second. Since Ella is younger, Aidan is letting her begin 30 feet ahead of the starting line.

Let $y$ represent the distance from the starting line and $x$ represent the time elapsed, in seconds.

Write an equation to model the distance Aidan traveled.

Let $y$ represent the distance from the starting line.

The starting line is 30 feet ahead of the race. Aidan runs at a rate of 10 feet per second. Ella runs at a rate of 6 feet per second. Since Ella is younger, Aidan is letting her begin 30 feet ahead of the starting line.

Question 37 is continued on the next page.

[6] Aidan and his sister Ella are having a race. Aidan runs at a rate of 10 feet per second. Ella runs at a rate of 6 feet per second. Since Ella is younger, Aidan is letting her begin 30 feet ahead of the starting line.

Let $y$ represent the distance from the starting line and $x$ represent the time elapsed, in seconds.

Write an equation to model the distance Aidan traveled.

Let $y$ represent the distance from the starting line.

The starting line is 30 feet ahead of the race. Aidan runs at a rate of 10 feet per second. Ella runs at a rate of 6 feet per second. Since Ella is younger, Aidan is letting her begin 30 feet ahead of the starting line.

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Let $y$ represent the distance from the starting line and $x$ represent the time elapsed, in seconds.

Write an equation to model the distance Aidan traveled.

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The starting line is 30 feet ahead of the race. Aidan runs at a rate of 10 feet per second. Ella runs at a rate of 6 feet per second. Since Ella is younger, Aidan is letting her begin 30 feet ahead of the starting line.

Question 37 is continued on the next page.
Write an equation to model the distance Ella traveled.
On the set of axes below, graph your equations.
Exactly how many seconds does it take Aidan to catch up to Ella? Justify your answer.
Scrap Graph Paper — this sheet will not be scored.
### Algebra I – Jan. ’23

#### High School Math Reference Sheet

<table>
<thead>
<tr>
<th>1 gallon</th>
<th>16 quarts</th>
<th>4 quarts</th>
<th>2 pints</th>
<th>2 cups</th>
<th>8 fluid ounces</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 liter</td>
<td>3.785 liters</td>
<td>0.95 quarts</td>
<td>2 pints</td>
<td>2 cups</td>
<td>8 fluid ounces</td>
</tr>
<tr>
<td>1 liter</td>
<td>2.2048 liters</td>
<td>1.1052 quarts</td>
<td>2 pints</td>
<td>2 cups</td>
<td>8 fluid ounces</td>
</tr>
</tbody>
</table>

#### Conversion Table

<table>
<thead>
<tr>
<th>Length</th>
<th>Equivalents</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 mile</td>
<td>1.609 kilometers</td>
</tr>
<tr>
<td>1 kilometer</td>
<td>0.62137 miles</td>
</tr>
<tr>
<td>1 meter</td>
<td>39.37 inches</td>
</tr>
<tr>
<td>1 inch</td>
<td>2.54 centimeters</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Weight</th>
<th>Equivalents</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 pound</td>
<td>0.45359 kilograms</td>
</tr>
<tr>
<td>1 kilogram</td>
<td>2.20462 pounds</td>
</tr>
<tr>
<td>1 ounce</td>
<td>28.3495 grams</td>
</tr>
<tr>
<td>1 gram</td>
<td>0.0352739 kilograms</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Volume</th>
<th>Equivalents</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 gallon</td>
<td>3.785 liters</td>
</tr>
<tr>
<td>1 liter</td>
<td>0.26417 gallons</td>
</tr>
<tr>
<td>1 cubic foot</td>
<td>7.5 cubic decimeters</td>
</tr>
</tbody>
</table>

#### Geometry Formulas

- **Triangle**: $A = \frac{1}{2}bh$
- **Parallelogram**: $A = bh$
- **Circle (Area)**: $A = \pi r^2$
- **Circle (Circumference)**: $C = \pi d$ or $C = 2\pi r$
- **Pythagorean Theorem**: $a^2 + b^2 = c^2$
- **Quadratic Formula**: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

#### Series and Sequences

- **Arithmetic Sequence**: $a_n = a_1 + (n-1)d$
- **Geometric Sequence**: $a_n = a_1 \cdot r^{n-1}$
- **Geometric Series**: $S_n = \frac{a_1(1-r^n)}{1-r}$ for $r \neq 1$
<table>
<thead>
<tr>
<th></th>
<th>Cylinder</th>
<th>Sphere</th>
<th>Cone</th>
<th>Pyramid</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exponential Growth/Decay</td>
<td>$V = \pi r^2 h$</td>
<td>$V = \frac{4}{3} \pi r^3$</td>
<td>$V = \frac{1}{3} \pi r^2 h$</td>
<td>$V = \frac{1}{3} Bh$</td>
</tr>
<tr>
<td>$A_0 + e^{(t-t_0)}$</td>
<td>$180 \degree$</td>
<td>$180 \degree$</td>
<td>$180 \degree$</td>
<td>$180 \degree$</td>
</tr>
<tr>
<td>$1$ radian</td>
<td>$90 \degree$</td>
<td>$90 \degree$</td>
<td>$90 \degree$</td>
<td>$90 \degree$</td>
</tr>
<tr>
<td>$1$ degree</td>
<td>$\frac{\pi}{180}$ radian</td>
<td>$\frac{\pi}{180}$ radian</td>
<td>$\frac{\pi}{180}$ radian</td>
<td>$\frac{\pi}{180}$ radian</td>
</tr>
<tr>
<td>Reference Sheet — concluded</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>