

The University of the State of New York  
REGENTS HIGH SCHOOL EXAMINATION

# GEOMETRY

Wednesday, January 25, 2023 — 9:15 a.m. to 12:15 p.m., only

Student Name: \_\_\_\_\_

School Name: \_\_\_\_\_

**The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.**

Print your name and the name of your school on the lines above.

A separate answer sheet for **Part I** has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 35 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in **Parts II, III, and IV** directly in this booklet. All work should be written in pen, except for graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

**Notice...**

**A graphing calculator, a straightedge (ruler), and a compass must be available for you to use while taking this examination.**

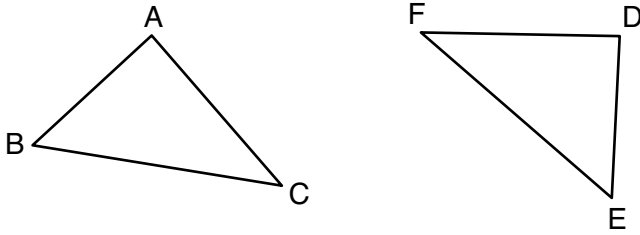
**DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.**

## Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

Use this space for  
computations.

- 1 In the diagram below, a line reflection followed by a rotation maps  $\triangle ABC$  onto  $\triangle DEF$ .

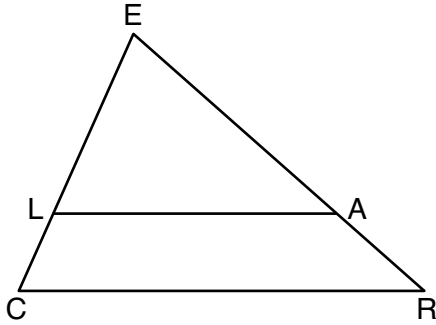


Which statement is always true?

- (1)  $\overline{BC} \cong \overline{EF}$                       (3)  $\angle A \cong \angle F$   
(2)  $\overline{AC} \cong \overline{DE}$                       (4)  $\angle B \cong \angle D$
- 2 A circle is continuously rotated about its diameter. Which three-dimensional object will be formed?
- (1) cone                                      (3) sphere  
(2) prism                                      (4) cylinder

Use this space for  
computations.

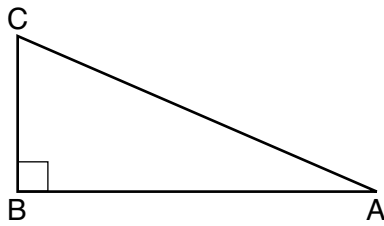
3 In the diagram below of  $\triangle CER$ ,  $\overline{LA} \parallel \overline{CR}$ .



If  $CL = 3.5$ ,  $LE = 7.5$ , and  $EA = 9.5$ , what is the length of  $\overline{AR}$ , to the nearest tenth?

- (1) 5.5                                      (3) 3.0  
(2) 4.4                                      (4) 2.8

4 Right triangle  $ABC$  is shown below.

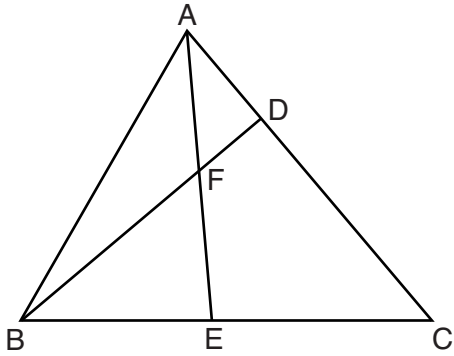


Which trigonometric equation is always true for triangle  $ABC$ ?

- (1)  $\sin A = \cos C$                       (3)  $\cos A = \cos C$   
(2)  $\cos A = \sin A$                       (4)  $\tan A = \tan C$

**Use this space for computations.**

- 5 In the diagram of  $\triangle ABC$  below,  $\overline{AE}$  bisects angle  $BAC$ , and altitude  $\overline{BD}$  is drawn.

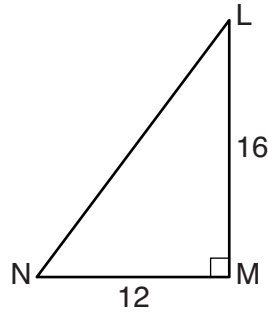


If  $m\angle C = 50^\circ$  and  $m\angle ABC = 60^\circ$ ,  $m\angle FEB$  is

- |                |                |
|----------------|----------------|
| (1) $35^\circ$ | (3) $55^\circ$ |
| (2) $40^\circ$ | (4) $85^\circ$ |
- 6 A jewelry company makes copper heart pendants. Each heart uses  $0.75 \text{ in}^3$  of copper and there is 0.323 pound of copper per cubic inch. If copper costs \$3.68 per pound, what is the total cost for 24 copper hearts?
- |             |              |
|-------------|--------------|
| (1) \$5.81  | (3) \$66.24  |
| (2) \$21.40 | (4) \$205.08 |

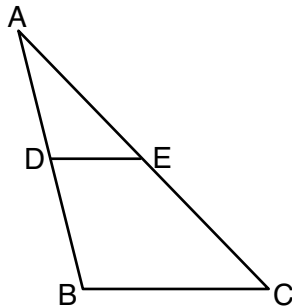
Use this space for computations.

- 7 In right triangle  $LMN$  shown below,  $m\angle M = 90^\circ$ ,  $MN = 12$ , and  $LM = 16$ .



The ratio of  $\cos N$  is

- (1)  $\frac{12}{20}$  (2)  $\frac{16}{20}$  (3)  $\frac{12}{16}$  (4)  $\frac{16}{12}$
- 8 In  $\triangle ABC$  below,  $\overline{DE}$  is drawn such that  $D$  and  $E$  are on  $\overline{AB}$  and  $\overline{AC}$ , respectively.



If  $\overline{DE} \parallel \overline{BC}$ , which equation will always be true?

- (1)  $\frac{AD}{DE} = \frac{DB}{BC}$  (2)  $\frac{AD}{DE} = \frac{AB}{BC}$  (3)  $\frac{AD}{BC} = \frac{DE}{DB}$  (4)  $\frac{AD}{BC} = \frac{DE}{AB}$
- 9 Which polygon does *not* always have congruent diagonals?
- (1) square (2) rectangle (3) rhombus (4) isosceles trapezoid

**Use this space for computations.**

**10** If the circumference of a standard lacrosse ball is 19.9 cm, what is the volume of this ball, to the *nearest cubic centimeter*?

- (1) 42
- (2) 133
- (3) 415
- (4) 1065

**11** Which polygon always has a minimum rotation of  $180^\circ$  about its center to carry it onto itself?



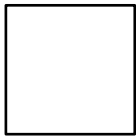
Rectangle

(1)



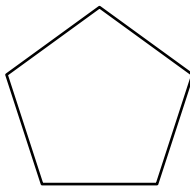
Isosceles trapezoid

(3)



Square

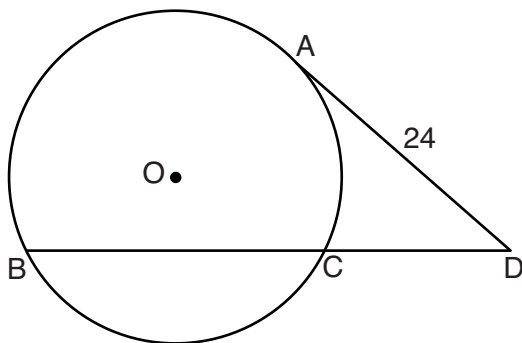
(2)



Regular pentagon

(4)

**12** Circle  $O$  is drawn below with secant  $\overline{BCD}$ . The length of tangent  $\overline{AD}$  is 24.



If the ratio of  $DC:CB$  is 4:5, what is the length of  $\overline{CB}$ ?

- (1) 36
- (2) 20
- (3) 16
- (4) 4

Use this space for computations.

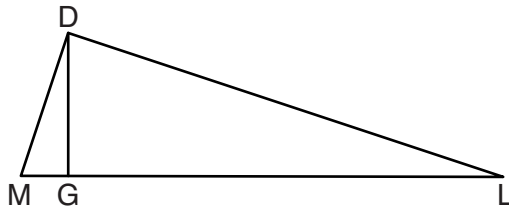
13 The equation of a line is  $3x - 5y = 8$ . All lines perpendicular to this line must have a slope of

- (1)  $\frac{3}{5}$                                       (3)  $-\frac{3}{5}$   
(2)  $\frac{5}{3}$                                       (4)  $-\frac{5}{3}$

14 What are the coordinates of the center and length of the radius of the circle whose equation is  $x^2 + y^2 + 2x - 16y + 49 = 0$ ?

- (1) center  $(1, -8)$  and radius 4  
(2) center  $(-1, 8)$  and radius 4  
(3) center  $(1, -8)$  and radius 16  
(4) center  $(-1, 8)$  and radius 16

15 In the diagram below of right triangle  $MDL$ , altitude  $\overline{DG}$  is drawn to hypotenuse  $\overline{ML}$ .



If  $MG = 3$  and  $GL = 24$ , what is the length of  $\overline{DG}$ ?

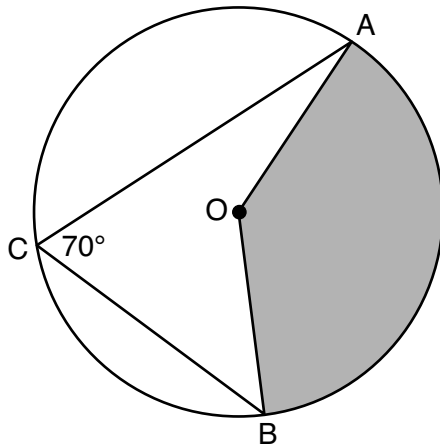
- (1) 8    (3)  $\sqrt{63}$   
(2) 9    (4)  $\sqrt{72}$

**Use this space for computations.**

**16** Segment  $AB$  is the perpendicular bisector of  $\overline{CD}$  at point  $M$ . Which statement is always true?

- (1)  $\overline{CB} \cong \overline{DB}$                       (3)  $\triangle ACD \sim \triangle BCD$   
(2)  $\overline{CD} \cong \overline{AB}$                       (4)  $\triangle ACM \sim \triangle BCM$

**17** In the diagram below of circle  $O$ ,  $\overline{AC}$  and  $\overline{BC}$  are chords, and  $m\angle ACB = 70^\circ$ .



If  $OA = 9$ , the area of the shaded sector  $AOB$  is

- (1)  $3.5\pi$                                       (3)  $15.75\pi$   
(2)  $7\pi$                                         (4)  $31.5\pi$



**Use this space for  
computations.**

**18** Quadrilateral  $BEST$  has diagonals that intersect at point  $D$ . Which statement would *not* be sufficient to prove quadrilateral  $BEST$  is a parallelogram?

(1)  $\overline{BD} \cong \overline{SD}$  and  $\overline{ED} \cong \overline{TD}$

(2)  $\overline{BE} \cong \overline{ST}$  and  $\overline{ES} \cong \overline{TB}$

(3)  $\overline{ES} \cong \overline{TB}$  and  $\overline{BE} \parallel \overline{TS}$

(4)  $\overline{ES} \parallel \overline{BT}$  and  $\overline{BE} \parallel \overline{TS}$

**19** The equation of line  $t$  is  $3x - y = 6$ . Line  $m$  is the image of line  $t$  after a dilation with a scale factor of  $\frac{1}{2}$  centered at the origin. What is an equation of line  $m$ ?

(1)  $y = \frac{3}{2}x - 3$

(3)  $y = 3x + 3$

(2)  $y = \frac{3}{2}x - 6$

(4)  $y = 3x - 3$

**20** A cylindrical pool has a diameter of 16 feet and height of 4 feet. The pool is filled to  $\frac{1}{2}$  foot below the top. How much water does the pool contain, to the *nearest gallon*? [ $1 \text{ ft}^3 = 7.48$  gallons]

(1) 704

(3) 5264

(2) 804

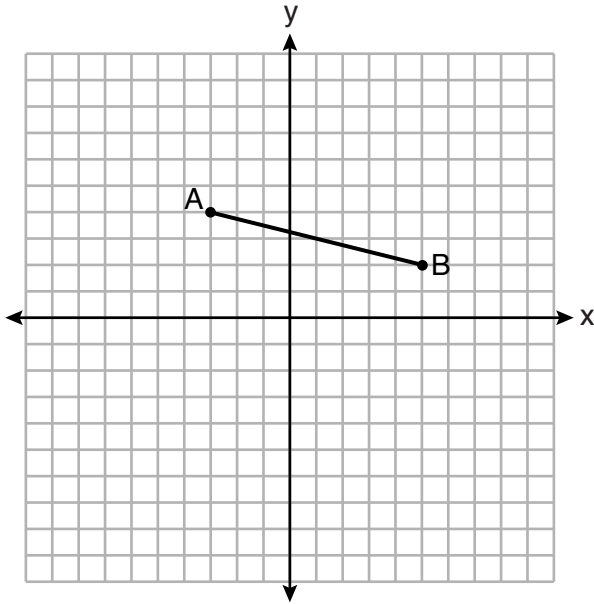
(4) 6016

**Use this space for computations.**

**21** The area of  $\triangle TAP$  is  $36 \text{ cm}^2$ . A second triangle,  $\triangle JOE$ , is formed by connecting the midpoints of each side of  $\triangle TAP$ . What is the area of  $\triangle JOE$ , in square centimeters?

- (1) 9
- (2) 12
- (3) 18
- (4) 27

**22** On the set of axes below, the endpoints of  $\overline{AB}$  have coordinates  $A(-3,4)$  and  $B(5,2)$ .

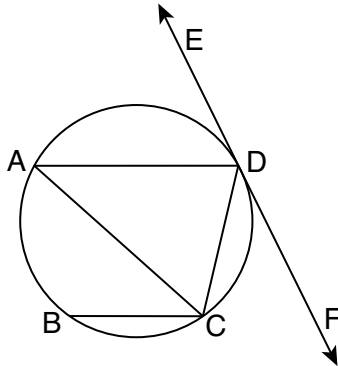


If  $\overline{AB}$  is dilated by a scale factor of 2 centered at  $(3,5)$ , what are the coordinates of the endpoints of its image,  $\overline{A'B'}$ ?

- (1)  $A'(-7,5)$  and  $B'(9,1)$
- (2)  $A'(-1,6)$  and  $B'(7,4)$
- (3)  $A'(-6,8)$  and  $B'(10,4)$
- (4)  $A'(-9,3)$  and  $B'(7,-1)$

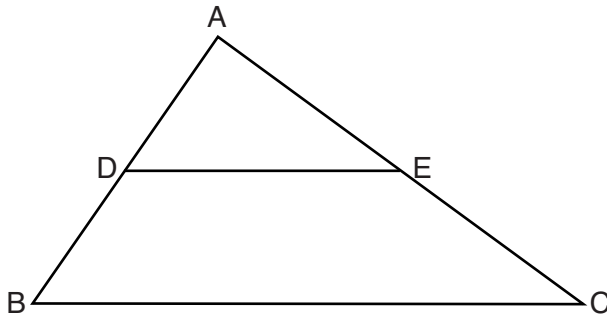
Use this space for computations.

- 23 In the circle below,  $\overline{AD}$ ,  $\overline{AC}$ ,  $\overline{BC}$ , and  $\overline{DC}$  are chords,  $\overline{EDF}$  is tangent at point  $D$ , and  $\overline{AD} \parallel \overline{BC}$ .



Which statement is always true?

- (1)  $\angle ADE \cong \angle CAD$                       (3)  $\angle BCA \cong \angle DCA$   
 (2)  $\angle CDF \cong \angle ACB$                       (4)  $\angle ADC \cong \angle ADE$
- 24 In the diagram below of  $\triangle ABC$ ,  $D$  and  $E$  are the midpoints of  $\overline{AB}$  and  $\overline{AC}$ , respectively, and  $\overline{DE}$  is drawn.



- I. AA similarity  
 II. SSS similarity  
 III. SAS similarity

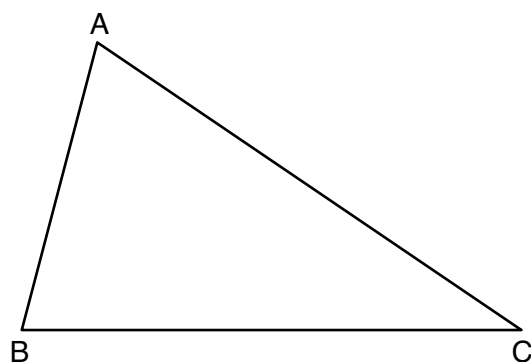
Which methods could be used to prove  $\triangle ABC \sim \triangle ADE$ ?

- (1) I and II, only                      (3) I and III, only  
 (2) II and III, only                      (4) I, II, and III

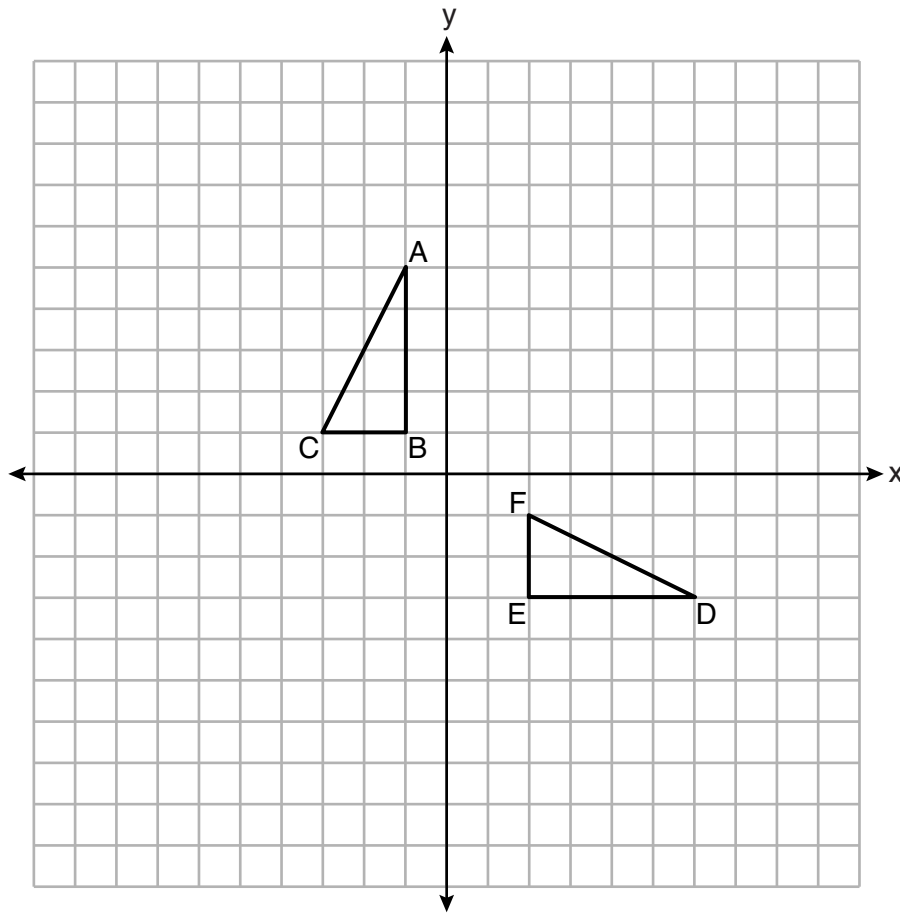
## Part II

Answer all 7 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [14]

- 25 Using a compass and straightedge, construct the angle bisector of  $\angle ABC$ .  
[Leave all construction marks.]

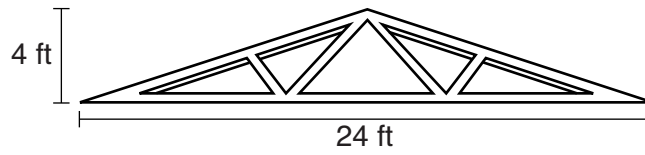


26 On the set of axes below,  $\triangle ABC$  and  $\triangle DEF$  are graphed.



Describe a sequence of rigid motions that would map  $\triangle ABC$  onto  $\triangle DEF$ .

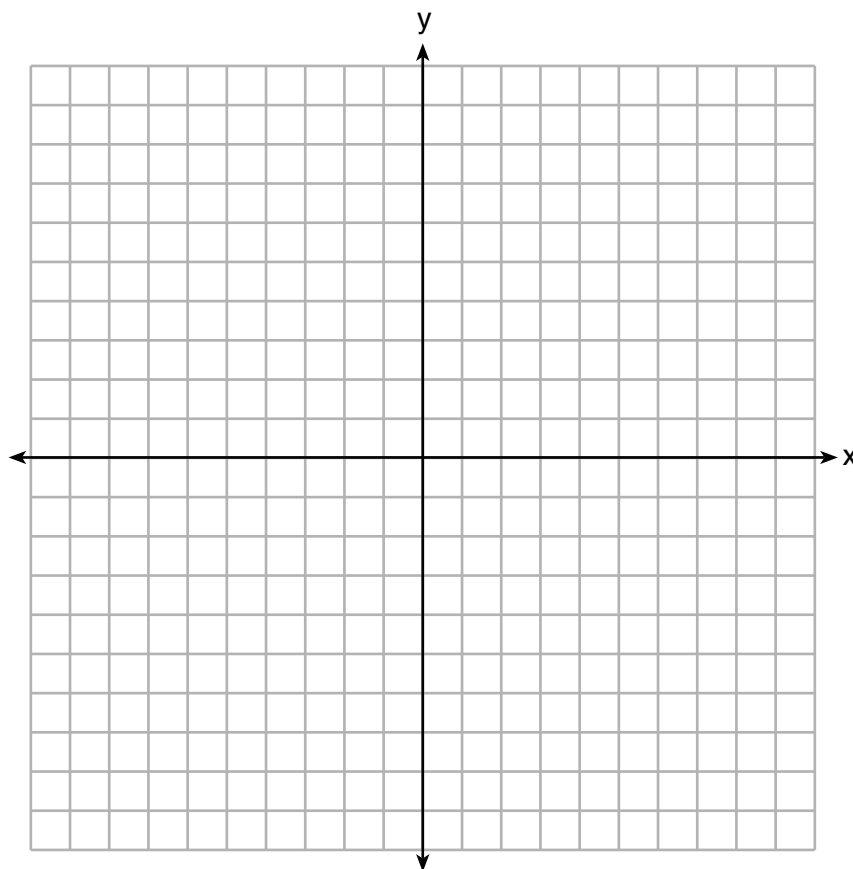
27 As shown in the diagram below, a symmetrical roof frame rises 4 feet above a house and has a width of 24 feet.



Determine and state, to the *nearest degree*, the angle of elevation of the roof frame.

**28** Directed line segment  $AB$  has endpoints whose coordinates are  $A(-2,5)$  and  $B(8,-1)$ . Determine and state the coordinates of  $P$ , the point which divides the segment in the ratio 3:2.

[The use of the set of axes below is optional.]

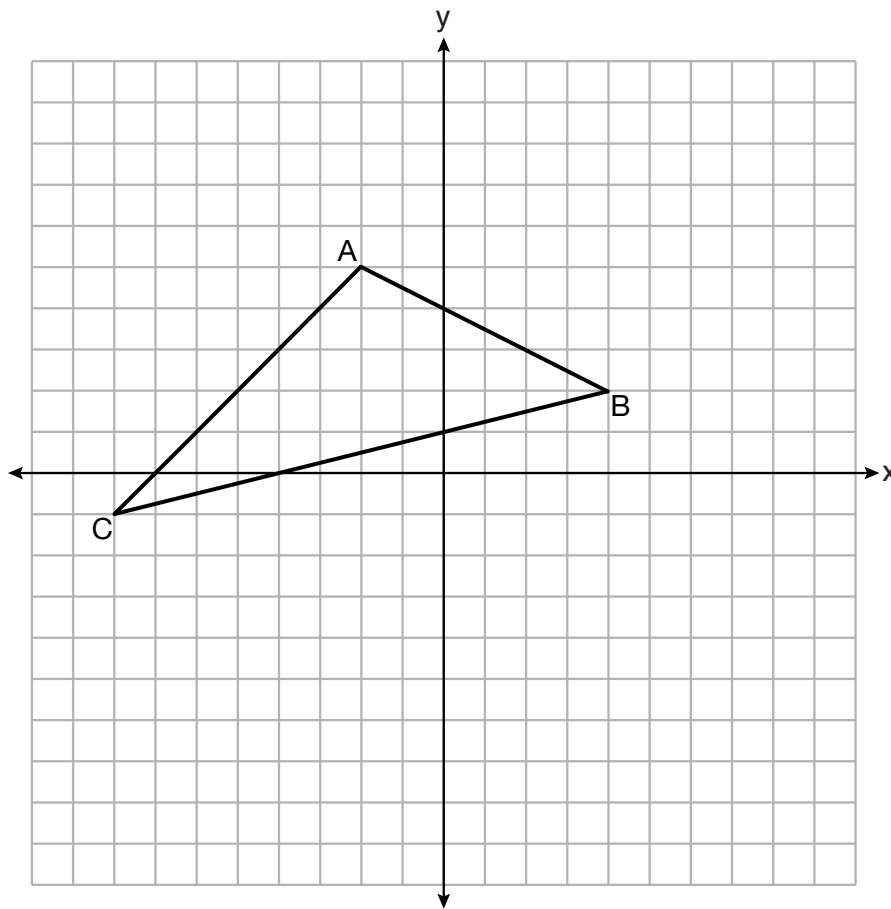


**29** In  $\triangle ABC$ ,  $AB = 5$ ,  $AC = 12$ , and  $m\angle A = 90^\circ$ . In  $\triangle DEF$ ,  $m\angle D = 90^\circ$ ,  $DF = 12$ , and  $EF = 13$ . Brett claims  $\triangle ABC \cong \triangle DEF$  and  $\triangle ABC \sim \triangle DEF$ .  
Is Brett correct? Explain why.



**30** The volume of a triangular prism is  $70 \text{ in}^3$ . The base of the prism is a right triangle with one leg whose measure is 5 inches. If the height of the prism is 4 inches, determine and state the length, in inches, of the other leg of the triangle.

31 Triangle  $ABC$  with coordinates  $A(-2,5)$ ,  $B(4,2)$ , and  $C(-8,-1)$  is graphed on the set of axes below.

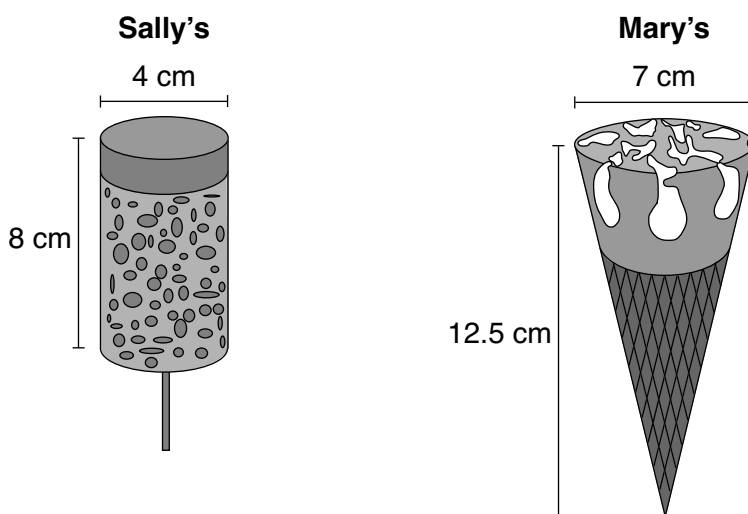


Determine and state the area of  $\triangle ABC$ .

### Part III

Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

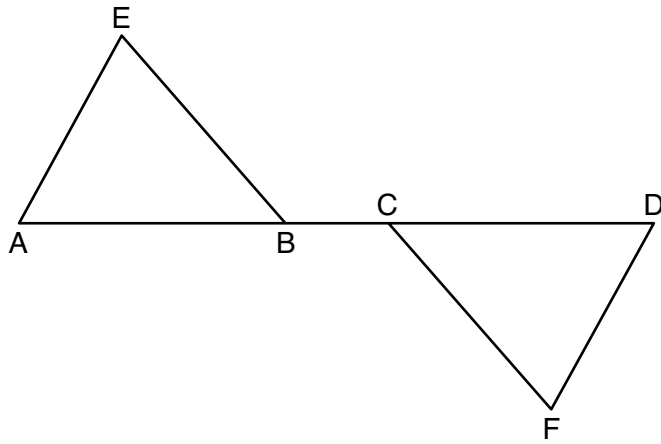
- 32 Sally and Mary both get ice cream from an ice cream truck. Sally's ice cream is served as a cylinder with a diameter of 4 cm and a total height of 8 cm. Mary's ice cream is served as a cone with a diameter of 7 cm and a total height of 12.5 cm. Assume that ice cream fills Sally's cylinder and Mary's cone.



Who was served more ice cream, Sally or Mary? Justify your answer.

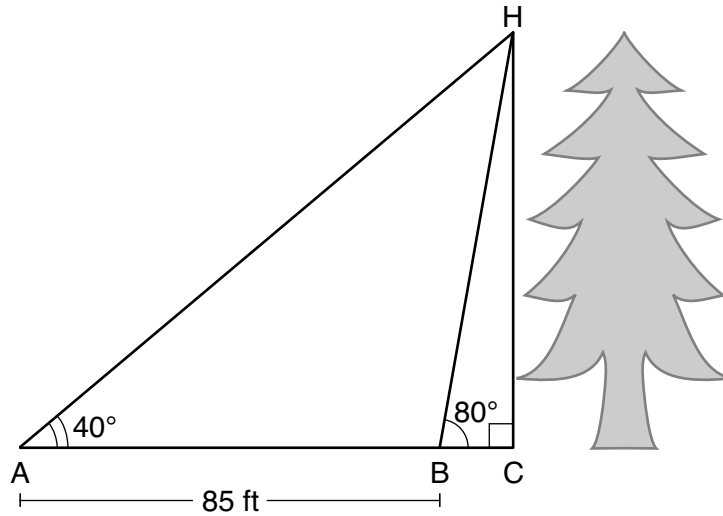
Determine and state how much more is served in the larger ice cream than the smaller ice cream, to the nearest cubic centimeter.

33 Given:  $\triangle AEB$  and  $\triangle DFC$ ,  $\overline{ABCD}$ ,  $\overline{AE} \parallel \overline{DF}$ ,  $\overline{EB} \parallel \overline{FC}$ ,  $\overline{AC} \cong \overline{DB}$



Prove:  $\triangle EAB \cong \triangle FDC$

34 Barry wants to find the height of a tree that is modeled in the diagram below, where  $\angle C$  is a right angle. The angle of elevation from point  $A$  on the ground to the top of the tree,  $H$ , is  $40^\circ$ . The angle of elevation from point  $B$  on the ground to the top of the tree,  $H$ , is  $80^\circ$ . The distance between points  $A$  and  $B$  is 85 feet.



Barry claims that  $\triangle ABH$  is isosceles. Explain why Barry is correct.

Determine and state, to the *nearest foot*, the height of the tree.

**Part IV**

**Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided to determine your answer. Note that diagrams are not necessarily drawn to scale. A correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]**

**35** Given: Triangle  $DUC$  with coordinates  $D(-3,-1)$ ,  $U(-1,8)$ , and  $C(8,6)$

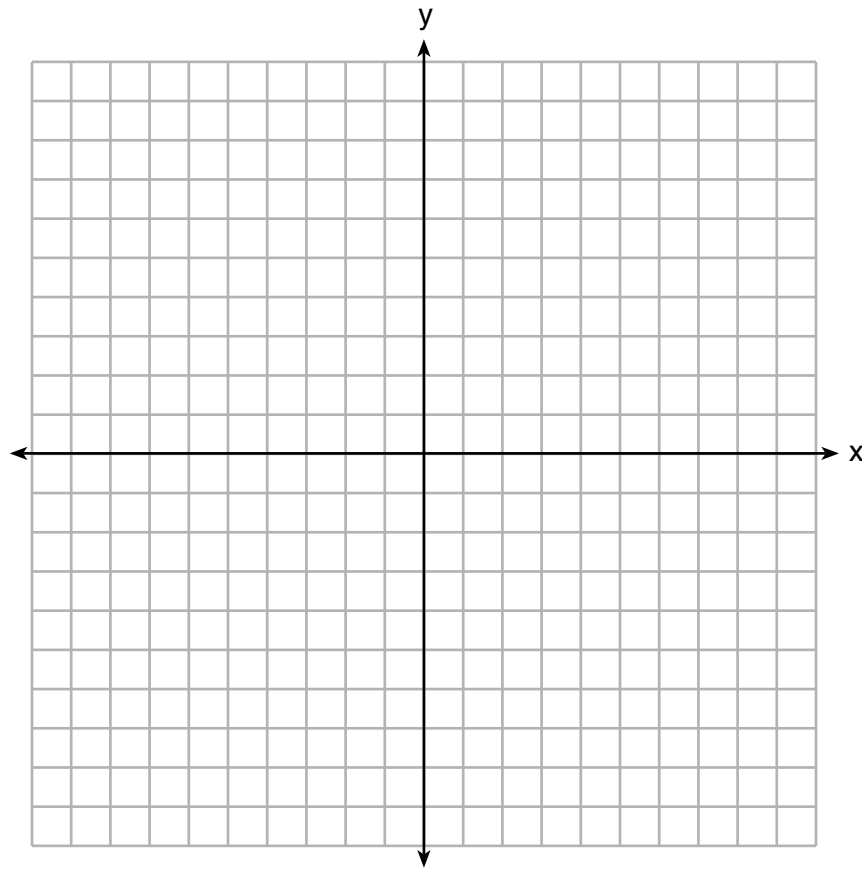
Prove:  $\triangle DUC$  is a right triangle

[The use of the set of axes on the next page is optional.]

**Question 35 is continued on the next page.**

**Question 35 continued**

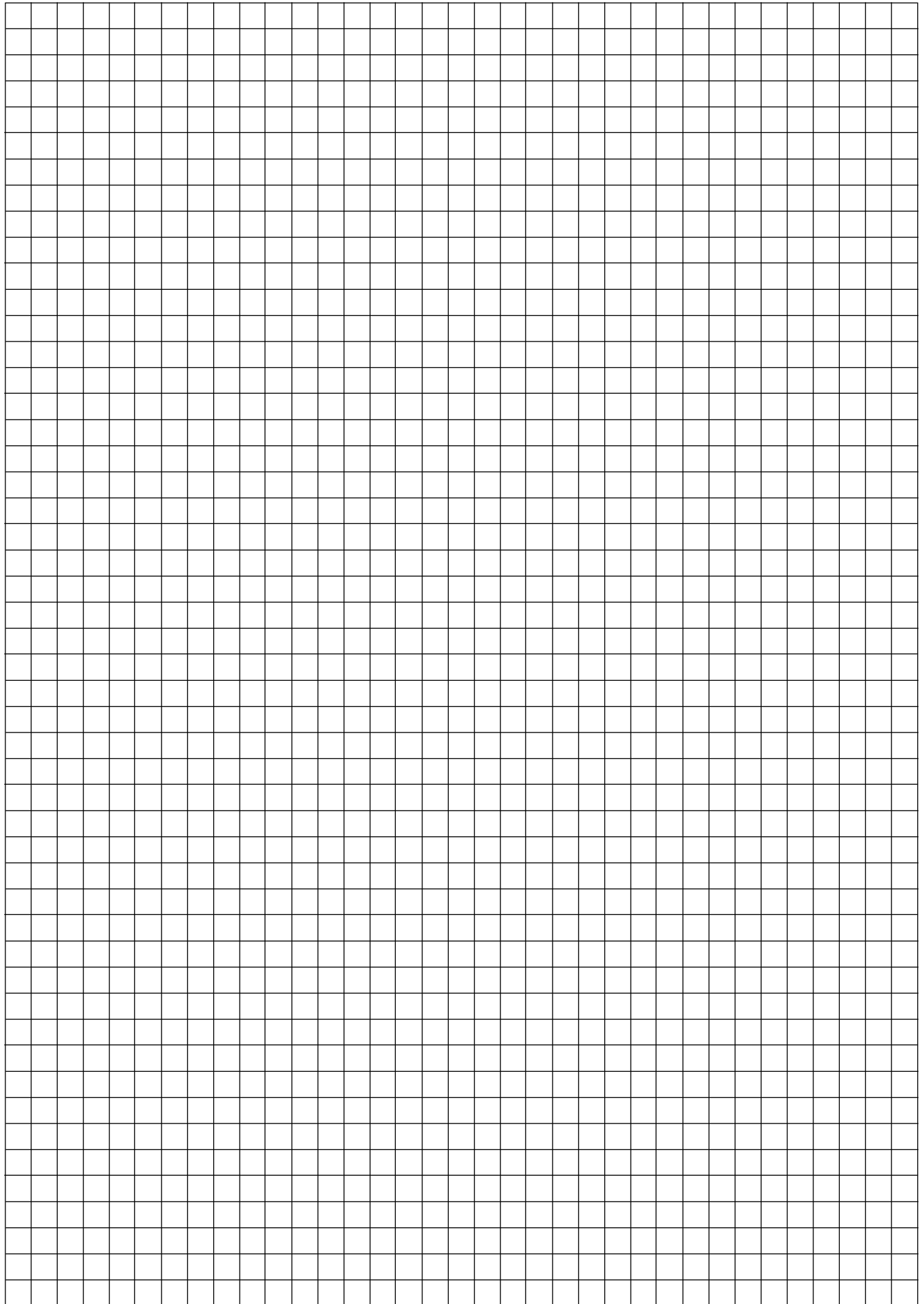
Point  $U$  is reflected over  $\overline{DC}$  to locate its image point,  $U'$ , forming quadrilateral  $DUCU'$ .  
Prove quadrilateral  $DUCU'$  is a square.







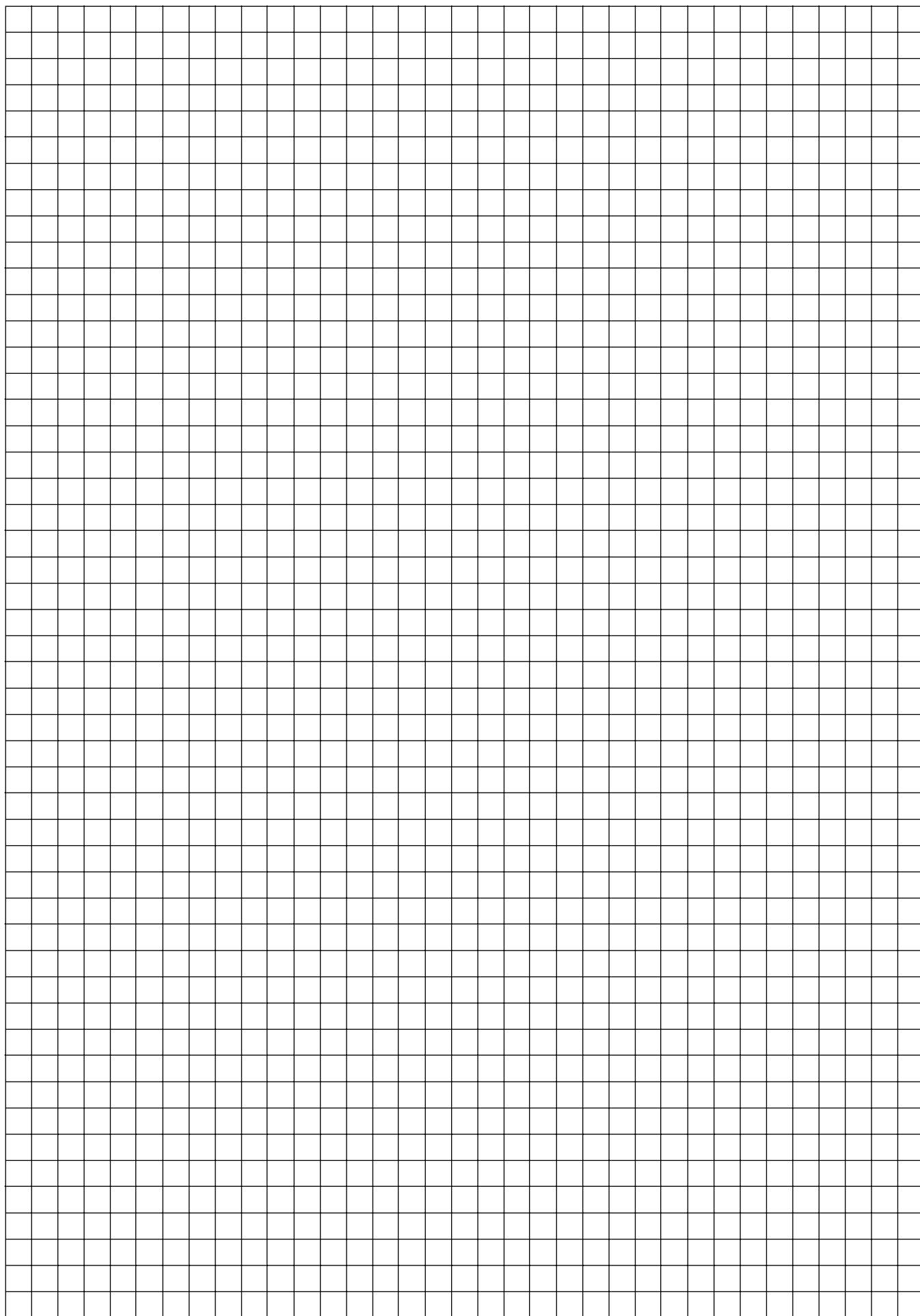
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**Scrap Graph Paper – This sheet will *not* be scored.**



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## High School Math Reference Sheet

|                           |                          |                                  |
|---------------------------|--------------------------|----------------------------------|
| 1 inch = 2.54 centimeters | 1 kilometer = 0.62 mile  | 1 cup = 8 fluid ounces           |
| 1 meter = 39.37 inches    | 1 pound = 16 ounces      | 1 pint = 2 cups                  |
| 1 mile = 5280 feet        | 1 pound = 0.454 kilogram | 1 quart = 2 pints                |
| 1 mile = 1760 yards       | 1 kilogram = 2.2 pounds  | 1 gallon = 4 quarts              |
| 1 mile = 1.609 kilometers | 1 ton = 2000 pounds      | 1 gallon = 3.785 liters          |
|                           |                          | 1 liter = 0.264 gallon           |
|                           |                          | 1 liter = 1000 cubic centimeters |

|                |                             |
|----------------|-----------------------------|
| Triangle       | $A = \frac{1}{2}bh$         |
| Parallelogram  | $A = bh$                    |
| Circle         | $A = \pi r^2$               |
| Circle         | $C = \pi d$ or $C = 2\pi r$ |
| General Prisms | $V = Bh$                    |
| Cylinder       | $V = \pi r^2 h$             |
| Sphere         | $V = \frac{4}{3}\pi r^3$    |
| Cone           | $V = \frac{1}{3}\pi r^2 h$  |
| Pyramid        | $V = \frac{1}{3}Bh$         |

|                          |  |
|--------------------------|--|
| Pythagorean Theorem      | $a^2 + b^2 = c^2$                                    |
| Quadratic Formula        | $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$             |
| Arithmetic Sequence      | $a_n = a_1 + (n - 1)d$                               |
| Geometric Sequence       | $a_n = a_1 r^{n-1}$                                  |
| Geometric Series         | $S_n = \frac{a_1 - a_1 r^n}{1 - r}$ where $r \neq 1$ |
| Radians                  | 1 radian = $\frac{180}{\pi}$ degrees                 |
| Degrees                  | 1 degree = $\frac{\pi}{180}$ radians                 |
| Exponential Growth/Decay | $A = A_0 e^{k(t - t_0)} + B_0$                       |

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# GEOMETRY

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