## The University of the State of New York REGENTS HIGH SCHOOL EXAMINATION GEOMETRY

Tuesday, June 20, 2023 - 9:15 a.m. to 12:15 p.m., only

## MODEL RESPONSE SET

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## Question 25

25 In $\triangle A B C$ below, use a compass and straightedge to construct the altitude from $C$ to $\overline{A B}$. [Leave all construction marks.]


Score 2: The student gave a complete and correct response.

## Question 25

25 In $\triangle A B C$ below, use a compass and straightedge to construct the altitude from $C$ to $\overline{A B}$. [Leave all construction marks.]


Score 2: The student gave a complete and correct response.

## Question 25

25 In $\triangle A B C$ below, use a compass and straightedge to construct the altitude from $C$ to $\overline{A B}$. [Leave all construction marks.]


Score 1: The student constructed all appropriate arcs, but the altitude was not drawn.

## Question 25

25 In $\triangle A B C$ below, use a compass and straightedge to construct the altitude from $C$ to $\overline{A B}$. [Leave all construction marks.]


Score 0: The student made a drawing that was not an appropriate construction.

## Question 25

25 In $\triangle A B C$ below, use a compass and straightedge to construct the altitude from $C$ to $\overline{A B}$. [Leave all construction marks.]


Score 0: The student gave a completely incorrect response.

## Question 25

25 In $\triangle A B C$ below, use a compass and straightedge to construct the altitude from $C$ to $\overline{A B}$. [Leave all construction marks.]


Score 0: The student gave a completely incorrect response.

## Question 26

26 Triangles $A B C$ and $D E F$ are graphed on the set of axes below.


Describe a sequence of transformations that maps $\triangle A B C$ onto $\triangle D E F$.
translate $\triangle A B C$ down 8 units, then rotate
$\triangle A B C$ 90 clock ie e around point f

Score 2: The student gave a complete and correct response.

## Question 26

26 Triangles $A B C$ and $D E F$ are graphed on the set of axes below.


Describe a sequence of transformations that maps $\triangle A B C$ onto $\triangle D E F$.

$$
\text { rotation } 90^{\circ} \text { clockwise around the origin }
$$

Score 2: The student gave a complete and correct response.

## Question 26

26 Triangles $A B C$ and $D E F$ are graphed on the set of axes below.


Describe a sequence of transformations that maps $\triangle A B C$ onto $\triangle D E F$.

$$
T_{4,-4} \circ R_{A,-90^{\circ}}
$$

Score 2: The student gave a complete and correct response.

## Question 26

26 Triangles $A B C$ and $D E F$ are graphed on the set of axes below.


Describe a sequence of transformations that maps $\triangle A B C$ onto $\triangle D E F$.
Rotation about point $C, 90^{\circ}$ clock wise, followed by a translation down by 8 .

Score 2: The student gave a complete and correct response.

## Question 26

26 Triangles $A B C$ and $D E F$ are graphed on the set of axes below.


Describe a sequence of transformations that maps $\triangle A B C$ onto $\triangle D E F$.
$90^{\circ}$ counterclockwise rotation about
the origin

Score 1: The student wrote an incorrect direction for the rotation.

## Question 26

26 Triangles $A B C$ and $D E F$ are graphed on the set of axes below.


Describe a sequence of transformations that maps $\triangle A B C$ onto $\triangle D E F$.


Score 1: The student did not state the center of rotation.

## Question 26

26 Triangles $A B C$ and $D E F$ are graphed on the set of axes below.


Describe a sequence of transformations that maps $\triangle A B C$ onto $\triangle D E F$.

$$
\begin{aligned}
& \text { rotation } 90^{\circ} \text { clock wisc } 2 b \text { out } C \\
& \text { translate } 4 \text { mats down } \\
& \text { reflect over the } x-2 x \text { is }^{2}
\end{aligned}
$$

Score 1: The student gave a correct description of the rotation and translation, but no further correct work was shown.

## Question 26

26 Triangles $A B C$ and $D E F$ are graphed on the set of axes below.


Describe a sequence of transformations that maps $\triangle A B C$ onto $\triangle D E F$.

$$
\text { Rotate } 90^{\circ} \text { clochwise }
$$

Score 1: The student did not state the center of rotation.

## Question 26

26 Triangles $A B C$ and $D E F$ are graphed on the set of axes below.


Describe a sequence of transformations that maps $\triangle A B C$ onto $\triangle D E F$.

$$
\text { Translation }\langle 4,4\rangle \text { rotation ao cociswise }
$$

Score 1: The student correctly stated the translation as a vector, but did not state the center of rotation.

## Question 26

26 Triangles $A B C$ and $D E F$ are graphed on the set of axes below.


Describe a sequence of transformations that maps $\triangle A B C$ onto $\triangle D E F$.

$$
\begin{aligned}
& \text { Mac took a counter clockinise rotation } \\
& 90^{\circ} \mathrm{L} \text { times }
\end{aligned}
$$

Score 0: The student did not show enough correct relevant work to receive any credit.

## Question 26

26 Triangles $A B C$ and $D E F$ are graphed on the set of axes below.


Describe a sequence of transformations that maps $\triangle A B C$ onto $\triangle D E F$.

Rotation and Reflection

Score 0: The student gave an incomplete rotation and an incorrect reflection.

## Question 26

26 Triangles $A B C$ and $D E F$ are graphed on the set of axes below.


Describe a sequence of transformations that maps $\triangle A B C$ onto $\triangle D E F$.

$$
\begin{aligned}
& \text { A to } d \text { is } 4 \text { down and } 4 \text { to the right } \\
& \text { B to } E \text { is } 10 \text { down and } 6 \text { to the fight } \\
& C \text { to } F \text { is } 8 \text { down and stops }
\end{aligned}
$$

Score 0: The student gave a completely incorrect description.

## Question 26

26 Triangles $A B C$ and $D E F$ are graphed on the set of axes below.


Describe a sequence of transformations that maps $\triangle A B C$ onto $\triangle D E F$.

1. rotate $A B C \quad 180^{\circ}$
2. translate $A^{\prime} B^{\prime} C(-8,0)$ 3. Done.

Score 0: The student gave a completely incorrect description.

## Question 27

27 Line segment $P Q$ has endpoints $P(-5,1)$ and $Q(5,6)$, and point $R$ is on $\overline{P Q}$. Determine and state the coordinates of $R$, such that $P R: R Q=2: 3$.
[The use of the set of axes below is optional.]

$$
\begin{aligned}
& \left(x+\frac{2}{5}(\Delta x) ; y+\frac{2}{5}(\Delta y)\right) \\
& -5+\frac{2}{5}(10) 1+\frac{2}{5}(5) \\
& (-1,3)
\end{aligned}
$$



Score 2: The student gave a complete and correct response.

Question 27

27 Line segment $P Q$ has endpoints $P(-5,1)$ and $Q(5,6)$, and point $R$ is on $\overline{P Q}$. Determine and state the coordinates of $R$, such that $P R: R Q=2: 3$.
[The use of the set of axes below is optional.]


$$
\begin{array}{ll}
5--5=10 & 6-1=5 \\
10 \cdot \frac{2}{5}=4 & 5 \cdot \frac{2}{5}=2 \\
-5+4=-1 & 1+2=3
\end{array}
$$



Score 2: The student gave a complete and correct response.

## Question 27

27 Line segment $P Q$ has endpoints $P(-5,1)$ and $Q(5,6)$, and point $R$ is on $\overline{P Q}$.
Determine and state the coordinates of $R$, such that $P R: R Q=2: 3$.
[The use of the set of axes below is optional.]

$$
\begin{array}{ll}
-5+\frac{2}{5}(5+5) \\
(-1,3) & \\
& R(-1,3)
\end{array}
$$

$$
1+\frac{2}{5}(6-1)
$$



Score 2: The student gave a complete and correct response.

## Question 27

27 Line segment $P Q$ has endpoints $P(-5,1)$ and $Q(5,6)$, and point $R$ is on $\overline{P Q}$.
Determine and state the coordinates of $R$, such that $P R: R Q=2: 3$.
[The use of the set of axes below is optional.]

$$
R(-1,3) \text {, for that I the }
$$ proper location to place it it fie line seymin $P Q$ is evenly dialed into 5 segments to give the $P R$ had two segments within it wad that RQ has three.



Score 2: The student gave a complete and correct response.

Question 27


27 Line segment $P Q$ has endpoints $P(-5,1)$ and $Q(5,6)$, and point $R$ is on $\overline{P Q}$.
Determine and state the coordinates of $R$, such that $P R: R Q=2: 3$.
[The use of the set of axes below is optional.]

$$
\begin{array}{cl}
x & y \\
\frac{2}{5}(-10)+5=1 & \frac{2}{5}(-5)+6=
\end{array}
$$

$$
R(1,4)
$$



Score 1: The student determined the coordinates of $R$ such that $P R: R Q$ was in a 3:2 ratio.

## Question 27

27 Line segment $P Q$ has endpoints $P(-5,1)$ and $Q(5,6)$, and point $R$ is on $\overline{P Q}$.
Determine and state the coordinates of $R$, such that $P R: R Q=2: 3$.
[The use of the set of axes below is optional.]



Score 1: The student determined the coordinates of R, but did not show work.

## Question 27

27 Line segment $P Q$ has endpoints $P(-5,1)$ and $Q(5,6)$, and point $R$ is on $\overline{P Q}$.
Determine and state the coordinates of $R$, such that $P R: R Q=2: 3$.
[The use of the set of axes below is optional.]
The distance
of $\overline{P R}$ to
the distance $: R(-1,3)$
of $\overline{R Q}$ when in the ratio $2: 3$
simplified results in


Score 1: The student determined the coordinates of R, but did not show work.

## Question 27

27 Line segment $P Q$ has endpoints $P(-5,1)$ and $Q(5,6)$, and point $R$ is on $\overline{P Q}$.
Determine and state the coordinates of $R$, such that $P R: R Q=2: 3$.
[The use of the set of axes below is optional.]

$$
\left.\begin{array}{ll}
2=\sqrt{(-5-x)^{2}+(1-x)^{2}} & d=\sqrt{\left(x_{1}-x_{2}\right)^{2}+\left(y_{1}-y^{2}\right)^{2}} \\
2=\sqrt{\left(-25+x^{2}\right)+1+x^{2}} & d \\
2^{2}=\sqrt{(-5-5)^{2}+(1-6)^{2}} \\
4=26+2 x^{2} &
\end{array}=\sqrt{(0)^{2}+(-5)^{2}}\right)
$$



Score 0: The student did not show enough correct relevant work to receive any credit.

## Question 27

27 Line segment $P Q$ has endpoints $P(-5,1)$ and $Q(5,6)$, and point $R$ is on $\overline{P Q}$.
Determine and state the coordinates of $R$, such that $P R: R Q=2: 3$.
[The use of the set of axes below is optional.]

$$
\begin{gathered}
2 x+3 x=10 \\
\frac{5}{5}=\frac{10}{5}
\end{gathered}
$$

$$
x=2
$$




Score 0: The student did not show enough correct relevant work to receive any credit.

Question 28

28 A circle has a radius of 6.4 inches. Determine and state, to the nearest square inch, the area of a sector whose arc measures $80^{\circ}$.


$$
\frac{360 x}{360}=\frac{10294,3708}{360}
$$



Score 2: The student gave a complete and correct response.

Question 28

28 A circle has a radius of 6.4 inches. Determine and state, to the nearest square inch, the area of a sector whose arc measures $80^{\circ}$.


$$
\begin{array}{ll}
A=\pi r^{2} & \frac{80}{360}(40.96 \pi) \\
A=x(6.4)^{2} & =28.59547 \\
A=40.96 \pi & \\
&
\end{array}
$$

Score 2: The student gave a complete and correct response.

## Question 28

28 A circle has a radius of 6.4 inches. Determine and state, to the nearest square inch, the area of a sector whose arc measures $80^{\circ}$.

$A=\pi(6.4)^{2}$
$A=128.6796351$
$\frac{80}{360}=\frac{x}{128.6796331}$
$\frac{360 x=10294.37081}{360}$
$\quad \begin{aligned} & x=28.59547447 \\ & \approx 29 \text { in }^{2} \quad\end{aligned} \quad \approx 29$

Score 2: The student gave a complete and correct response.

Question 28

28 A circle has a radius of 6.4 inches. Determine and state, to the nearest square inch, the area of a sector whose arc measures $80^{\circ}$.


$$
\begin{aligned}
& \pi 6.4^{2}= 128.6796357 \\
& \frac{80}{360}= x \\
& 128.6796357 \\
& 360 x=10294.37081 \\
& x=29 \mathrm{nn}
\end{aligned}
$$

Score 1: The student determined the area of the sector in inches, not square inches.

## Question 28

28 A circle has a radius of 6.4 inches. Determine and state, to the nearest square inch, the area of a sector whose arc measures $80^{\circ}$.
$\downarrow$
$\frac{\theta}{30}$
$\frac{80}{300} \cdot 2+t(6.4)=8.936 \ldots$
$\approx 9 \mathrm{in} .^{2}$

Score 1: The student made an error in using a formula for arc length, but found an appropriate answer.

## Question 28

28 A circle has a radius of 6.4 inches. Determine and state, to the nearest square inch, the area of a sector whose arc measures $80^{\circ}$.

$$
\begin{aligned}
& -\frac{x}{360} \cdot \dot{\pi} \cdot r^{2}=A \\
& \frac{80}{360} \cdot \pi \cdot 6 \cdot 4^{2}=\frac{64 \pi}{45} \text { or } 4.468
\end{aligned}
$$

Score 1: The student made an error in not squaring the radius, but found an appropriate answer.

## Question 28

28 A circle has a radius of 6.4 inches. Determine and state, to the nearest square inch, the area of a sector whose arc measures $80^{\circ}$.


Score 0: The student used an incorrect formula and made one rounding error.

## Question 28

28 A circle has a radius of 6.4 inches. Determine and state, to the nearest square inch, the area of a sector whose arc measures $80^{\circ}$.


Score 0: The student did not show enough correct relevant course-level work to receive any credit.

## Question 29

29 A large snowman is made of three spherical snowballs with radii of 1 foot, 2 feet, and 3 feet, respectively. Determine and state the amount of snow, in cubic feet, that is used to make the snowman.
[Leave your answer in terms of $\pi$.]

$$
\begin{array}{lc}
V=\frac{4}{3} \pi 1^{3} & 1 . \overline{3} \pi \\
V=\frac{4}{3} \pi r^{3} \\
V=\frac{4}{3} \pi 3^{3} & \frac{10.6 \pi}{48 \pi+t^{3}-} \\
& \frac{36 \pi}{48 \pi f^{3}}
\end{array}
$$

Score 2: The student gave a complete and correct response.

## Question 29

29 A large snowman is made of three spherical snowballs with radii of 1 foot, 2 feet, and 3 feet, respectively. Determine and state the amount of snow, in cubic feet, that is used to make the snowman.
[Leave your answer in terms of $\pi$.]


Score 2: The student gave a complete and correct response.

## Question 29

29 A large snowman is made of three spherical snowballs with radii of 1 foot, 2 feet, and 3 feet, respectively. Determine and state the amount of snow, in cubic feet, that is used to make the snowman.
[Leave your answer in terms of $\pi$.]

$$
V=\frac{4}{3} \pi r^{3} \quad 1.3 \pi
$$



Score 1: The student made a rounding error.

## Question 29

29 A large snowman is made of three spherical snowballs with radii of 1 foot, 2 feet, and 3 feet, respectively. Determine and state the amount of snow, in cubic feet, that is used to make the snowman.
[Leave your answer in terms of $\pi$.]


Score 1: The student made an error by squaring the radius when using the volume formula, but found an appropriate answer.

## Question 29

29 A large snowman is made of three spherical snowballs with radii of 1 foot, 2 feet, and 3 feet, respectively. Determine and state the amount of snow, in cubic feet, that is used to make the snowman.
[Leave your answer in terms of $\pi$.]


$$
V=150.8
$$

Score 1: The student determined the volume of the snowman, but not in terms of $\pi$.

## Question 29

29 A large snowman is made of three spherical snowballs with radii of 1 foot, 2 feet, and 3 feet, respectively. Determine and state the amount of snow, in cubic feet, that is used to make the snowman.
[Leave your answer in terms of $\pi$.]


$$
c=\pi d \quad 12 \pi
$$

$$
c=12 \pi
$$

Score 0: The student did not show enough correct relevant course-level work to receive any credit.

## Question 29

29 A large snowman is made of three spherical snowballs with radii of 1 foot, 2 feet, and 3 feet, respectively. Determine and state the amount of snow, in cubic feet, that is used to make the snowman.
[Leave your answer in terms of $\pi$.]


Score 0: The student did not show enough correct relevant course-level work to receive any credit.

## Question 30

30 In the diagram below of right triangle $A C B$, altitude $\overline{C D}$ is drawn to hypotenuse $\overline{A B}$, $A D=2$ and $A C=6$.


Determine and state the length of $\overline{A B}$.

$$
\begin{aligned}
& \frac{2}{6}=\frac{6}{x} \\
& 2 x=36 \\
& x=18
\end{aligned}
$$

$$
A B=18
$$

Score 2: The student gave a complete and correct response.

## Question 30

30 In the diagram below of right triangle $A C B$, altitude $\overline{C D}$ is drawn to hypotenuse $\overline{A B}$, $A D=2$ and $A C=6$.


Determine and state the length of $\overline{A B}$.


Score 2: The student gave a complete and correct response.

Question 30

30 In the diagram below of right triangle $A C B$, altitude $\overline{C D}$ is drawn to hypotenuse $\overline{A B}$, $A D=2$ and $A C=6$.


Determine and state the length of $\overline{A B}$.

$$
\begin{gathered}
\text { Find } C D \\
2^{2}+(C D)^{2}=6^{2} \\
(C D)^{2}=36-4 \\
C D=\sqrt{32}
\end{gathered} \left\lvert\, \begin{gathered}
\triangle A D C \sim \triangle C D B \\
\frac{C D}{A D}=\frac{B D}{C D} \\
\frac{\sqrt{32}}{2}=\frac{x}{\sqrt{32}} \\
2 x=32 \\
B D=x=16 \\
A B=16+2 \\
A B=18
\end{gathered}\right.
$$

Score 2: The student gave a complete and correct response.

## Question 30

30 In the diagram below of right triangle $A C B$, altitude $\overline{C D}$ is drawn to hypotenuse $\overline{A B}$, $A D=2$ and $A C=6$.


Determine and state the length of $\overline{A B}$.


$$
12=2 x
$$

$$
x=6
$$

Score 1: The student made a computational error.

## Question 30

30 In the diagram below of right triangle $A C B$, altitude $\overline{C D}$ is drawn to hypotenuse $\overline{A B}$, $A D=2$ and $A C=6$.


Determine and state the length of $\overline{A B}$.

$$
\begin{gathered}
\frac{2}{6}=\frac{6}{2+x} \\
4+2 x=36 \\
2 x=32 \\
y=16
\end{gathered}
$$

Score 1: The student correctly determined the length of $\overline{D B}$, but did not find the length of $\overline{A B}$.

## Question 30

30 In the diagram below of right triangle $A C B$, altitude $\overline{C D}$ is drawn to hypotenuse $\overline{A B}$, $A D=2$ and $A C=6$.


Determine and state the length of $\overline{A B}$.

$$
\begin{array}{ll}
a^{2}+b^{2}=c^{2} & a^{2}+b^{2}=c^{2} \\
2^{2}+x^{2}=6^{2} & x^{2}+x^{2}=\sqrt{32} \\
\frac{4+x^{2}=36}{-4}-\frac{4}{x^{2}-\sqrt{32}} & \frac{2 x^{2}}{2}=\sqrt{32} \\
x=\sqrt{32} & x^{2}=\sqrt{2} \\
x=4
\end{array}
$$

Score 0: The student made a conceptual error and a computational error in determining the length of $\overline{D B}$.

## Question 30

30 In the diagram below of right triangle $A C B$, altitude $\overline{C D}$ is drawn to hypotenuse $\overline{A B}$, $A D=2$ and $A C=6$.


Determine and state the length of $\overline{A B}$.

$$
\begin{array}{lr}
a^{2}+b^{2}=c^{2} & \\
2^{2}+x^{2}=6^{2} & \sqrt{38+x^{2}}=\sqrt{y^{2}} \\
4+x^{n}=36 & 6^{2} 4 x^{2}=y^{2}-2 \sqrt{38+x}=y \\
-4 & 364 x^{2}=y^{2}-9 \\
\sqrt{x^{2}}=\sqrt{32} & 6^{2}+8^{2}=10^{2} \\
x=5.7 & 3.6464=100
\end{array}
$$

$$
\overline{A B}=10
$$

Score 0: The student did not show enough correct relevant work to receive any credit.

## Question 31

31 Triangle $R S T$ has vertices with coordinates $R(-3,-2), S(3,2)$ and $T(4,-4)$. Determine and state an equation of the line parallel to $\overline{R T}$ that passes through point $S$.
[The use of the set of axes below is optional.]

$$
\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \quad \frac{-4+2}{4+3}=-2 / 7
$$

$y-2=-2 / 7(x-3)$ For a line segment to be
parallel, it must have the same slope, and pass through the point $(3,2)$.
$\begin{aligned} y-2 & =-2 / 7(x-3) \\ y+z & =(-2 / 7) x+6 / 7\end{aligned}$


Score 2: The student gave a complete and correct response.

## Question 31

31 Triangle $R S T$ has vertices with coordinates $R(-3,-2), S(3,2)$ and $T(4,-4)$. Determine and state an equation of the line parallel to $\overline{R T}$ that passes through point $S$.
[The use of the set of axes below is optional.]

$$
\frac{\text { Slope of }}{R T}=\frac{-4+2}{4+3}=\frac{-2}{7}
$$

$$
\begin{aligned}
& 7 \\
& y-2=\frac{-2}{7}(x-3) \\
& y=\frac{-2}{7} x+\frac{6}{7}
\end{aligned}
$$




Score 2: The student gave a complete and correct response.

## Question 31

31 Triangle $R S T$ has vertices with coordinates $(-3,-2), S(3,2)$ and $T(4,-4)$. Determine and state an equation of the line parallel to $\overline{R T}$ that passes through point $S$.
[The use of the set of axes below is optional.]

$$
\begin{aligned}
& y=-0,3 x+b \\
& z=-0,3(3)+b \\
& z=-0.9+b \\
& +0,9+10.9 \\
& 2,9=b
\end{aligned}
$$



Score 1: The student made an error when determining the slope of $\overline{R T}$.

## Question 31

31 Triangle $R S T$ has vertices with coordinates $R(-3,-2), S(3,2)$ and $T(4,-4)$. Determine and state an equation of the line parallel to $\overline{R T}$ that passes through point $S$.
[The use of the set of axes below is optional.]

$$
\begin{aligned}
\mu_{R T}=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}=\frac{-4+2}{4+3}=\frac{-2}{7}= & \frac{-2}{7} \\
& \frac{1}{2} \text { neg anile recirnocal }
\end{aligned}
$$

$$
y=\frac{7}{2} x+b \quad y=\frac{7}{2}(3)+b \quad y=\frac{7}{2} x-8 \cdot 5 \text { is the }
$$

$$
\begin{aligned}
& 2=10.5+b \\
& -8.5=b
\end{aligned}
$$

equation belle the globe is $n$ apatite reciprocal ald

Score 1: The student wrote an equation of the line perpendicular to $\overline{R T}$ through point $S$.

## Question 31

31 Triangle $R S T$ has vertices with coordinates $R(-3,-2), S(3,2)$ and $T(4,-4)$. Determine and state an equation of the line parallel to $\overline{R T}$ that passes through point $S$.
[The use of the set of axes below is optional.]

$$
m=-\frac{2}{7}
$$

$$
y=-\frac{2}{7} x+3
$$



Score 1: The student correctly determined the slope of the line parallel to $\overline{R T}$, but no further correct work was shown.

## Question 31

31 Triangle $R S T$ has vertices with coordinates $R(-3,-2), S(3,2)$ and $T(4,-4)$. Determine and state an equation of the line parallel to $\overline{R T}$ that passes through point $S$.
[The use of the set of axes below is optional.]
(3, 2 )



Score 0: The student did not show enough correct relevant work to receive any credit.

## Question 32

32 Cape Canaveral, Florida is where NASA launches rockets into space. As modeled in the diagram below, a person views the launch of a rocket from observation area $A, 3280$ feet away from launch $\operatorname{pad} B$. After launch, the rocket was sighted at $C$ with an angle of elevation of $15^{\circ}$. The rocket was later sighted at $D$ with an angle of elevation of $31^{\circ}$.


Determine and state, to the nearest foot, the distance the rocket traveled between the two sightings, $C$ and $D$.

$$
\begin{gathered}
\frac{\operatorname{Tan} 15}{1}=\frac{x}{3280} \quad \frac{\operatorname{Tan} 31}{1}=\frac{y}{3280} \\
x=\operatorname{Tan} 15(3280 \quad y=\operatorname{Tan} 31(3280) \\
B-C \quad x=878.873 \quad B-D y=1970.822 \\
\frac{1970.822}{1091.849 \mathrm{ft}}
\end{gathered}
$$

## Distance traveled between: 1092 ft

Score 4: The student gave a complete and correct response.

## Question 32

32 Cape Canaveral, Florida is where NASA launches rockets into space. As modeled in the diagram below, a person views the launch of a rocket from observation area $A, 3280$ feet away from launch $\operatorname{pad} B$. After launch, the rocket was sighted at $C$ with an angle of elevation of $15^{\circ}$. The rocket was later sighted at $D$ with an angle of elevation of $31^{\circ}$.


Determine and state, to the nearest foot, the distance the rocket traveled between the two sightings, $C$ and $D$.

$$
\begin{aligned}
\tan 15^{\circ}=\frac{x}{3280} & \tan 31^{\circ}=\frac{y}{3280} \\
x=3280 \tan 15^{\circ} & Y=3280 \tan 31^{\circ} \\
Z & =Y-x \\
Z= & 3280 \tan 31^{\circ}-3280 \tan 15^{\circ} \\
& =3280\left(\tan 31^{\circ}-\tan 15^{\circ}\right) \\
& =3280(.3329114266) \\
& =1091.949471 \\
& =1092
\end{aligned}
$$

Score 4: The student gave a complete and correct response.

Question 32

32 Cape Canaveral, Florida is where NASA launches rockets into space. As modeled in the diagram below, a person views the launch of a rocket from observation area $A, 3280$ feet away from launch $\operatorname{pad} B$. After launch, the rocket was sighted at $C$ with an angle of elevation of $15^{\circ}$. The rocket was later sighted at $D$ with an angle of elevation of $31^{\circ}$.


Fine Al


$$
\begin{aligned}
&\cos 3)=\frac{3280}{(A D)} \\
& 3280 \div(\cos 31)=A D \\
& A D=3826.56
\end{aligned}
$$

Determine and state, to the nearest foot, the distance the rocket traveled between the two sightings, $C$ and $D$.


Score 4: The student gave a complete and correct response.

## Question 32

32 Cape Canaveral, Florida is where NASA launches rockets into space. As modeled in the diagram below, a person views the launch of a rocket from observation area $A, 3280$ feet away from launch $\operatorname{pad} B$. After launch, the rocket was sighted at $C$ with an angle of elevation of $15^{\circ}$. The rocket was later sighted at $D$ with an angle of elevation of $31^{\circ}$.


Determine and state, to the nearest foot, the distance the rocket traveled between the two sightings, $C$ and $D$.

$$
\begin{aligned}
\tan \theta & =\frac{\text { opp }}{\text { adj }} \\
\tan \left(31^{\circ}\right) & =\frac{x}{3280} \\
x & =(3280)\left(\tan \left(31^{\circ}\right)\right) \\
x & =1970.82283 \\
\tan \left(15^{\circ}\right) & =\frac{y}{3280} \\
y & =(3280)(\tan 18) \\
y & =878.8733512 \quad 1970.82283-878.8733512= \\
x-y & =z
\end{aligned}
$$

Score 3: The student made a rounding error when determining the length of $\overline{D C}$.

## Question 32

32 Cape Canaveral, Florida is where NASA launches rockets into space. As modeled in the diagram below, a person views the launch of a rocket from observation area $A, 3280$ feet away from launch $\operatorname{pad} B$. After launch, the rocket was sighted at $C$ with an angle of elevation of $15^{\circ}$. The rocket was later sighted at $D$ with an angle of elevation of $31^{\circ}$.


Determine and state, to the nearest foot, the distance the rocket traveled between the two sightings, $C$ and $D$.

$$
\operatorname{Tan} 31=\frac{x}{3280}=
$$

distance rocket traveled was 1971 ff .

Score 2: The student correctly determined the length of $\overline{D B}$.

## Question 32

32 Cape Canaveral, Florida is where NASA launches rockets into space. As modeled in the diagram below, a person views the launch of a rocket from observation area $A, 3280$ feet away from launch pad $B$. After launch, the rocket was sighted at $C$ with an angle of elevation of $15^{\circ}$. The rocket was later sighted at $D$ with an angle of elevation of $31^{\circ}$.


Determine and state, to the nearest foot, the distance the rocket traveled between the two sightings, $C$ and $D$.


Score 2: The student made a conceptual error in using the tangent function in a non-right triangle.

## Question 32

32 Cape Canaveral, Florida is where NASA launches rockets into space. As modeled in the diagram below, a person views the launch of a rocket from observation area $A, 3280$ feet away from launch pad $B$. After launch, the rocket was sighted at $C$ with an angle of elevation of $15^{\circ}$. The rocket was later sighted at $D$ with an angle of elevation of $31^{\circ}$.


Determine and state, to the nearest foot, the distance the rocket traveled between the two sightings, $C$ and $D$.

$$
\tan 15^{\circ}=\frac{x}{3280}
$$

$$
\tan 39^{\circ}=\frac{y}{3288^{\circ}}
$$

Score 1: The student wrote two correct relevant trigonometric equations, but no further correct work was shown.

## Question 32

32 Cape Canaveral, Florida is where NASA launches rockets into space. As modeled in the diagram below, a person views the launch of a rocket from observation area $A, 3280$ feet away from launch $\operatorname{pad} B$. After launch, the rocket was sighted at $C$ with an angle of elevation of $15^{\circ}$. The rocket was later sighted at $D$ with an angle of elevation of $31^{\circ}$.


Determine and state, to the nearest foot, the distance the rocket traveled between the two sightings, $C$ and $D$. Sohcahtoa


Score 0: The student did not show enough correct relevant work to receive any credit.

## Question 33

33 A small can of soup is a right circular cylinder with a base diameter of 7 cm and a height of 9 cm . A large container is also a right circular cylinder with a base diameter of 9 cm and a height of 13 cm .

Determine and state the volume of the small can and the volume of the large container to the nearest cubic centimeter.


What is the minimum number of small cans that must be opened to fill the large container? Justify your answer.

$$
\begin{aligned}
& 3 \text { cans, } 827 \div 346=2,39 \text { but. you need } 3 \\
& \text { cans to fill thu larger container }
\end{aligned}
$$

Score 4: The student gave a complete and correct response.

## Question 33

33 A small can of soup is a right circular cylinder with a base diameter of 7 cm and a height of 9 cm . A large container is also a right circular cylinder with a base diameter of 9 cm and a height of 13 cm .

Determine and state the volume of the small can and the volume of the large container to the nearest cubic centimeter.


What is the minimum number of small cans that must be opened to fill the large container? Justify your answer.


Score 3: The student made an error in determining the number of small cans needed.

## Question 33

33 A small can of soup is a right circular cylinder with a base diameter of 7 cm and a height of 9 cm . A large container is also a right circular cylinder with a base diameter of 9 cm and a height of 13 cm .

Determine and state the volume of the small can and the volume of the large container to the nearest cubic centimeter.

$$
V=\pi r^{2} h
$$

$$
\begin{array}{ll}
S_{\text {mall }}=\pi 3.5^{2} 9 & \text { Large }=\pi 4.5^{2} 13 \\
S_{\text {mall }}=110 \mathrm{~cm}^{3} \mathrm{G:i} & \text { Large }=827 \mathrm{~cm}^{3}
\end{array}
$$

What is the minimum number of small cans that must be opened to fill the large container? Justify your answer.

$$
\text { About } 8
$$

$$
\frac{824.0242}{110.25}=7.5
$$

Score 3: The student made an error in determining the volume of the small can.

## Question 33

33 A small can of soup is a right circular cylinder with a base diameter of 7 cm and a height of 9 cm . A large container is also a right circular cylinder with a base diameter of 9 cm and a height of 13 cm .

Determine and state the volume of the small can and the volume of the large container to the nearest cubic centimeter.


What is the minimum number of small cans that must be opened to fill the large container? Justify your answer.

Score 2: The student determined the volume of the small can and large container, but no further correct work was shown.

Question 33

33 A small can of soup is a right circular cylinder with a base diameter of 7 cm and a height of 9 cm . A large container is also a right circular cylinder with a base diameter of 9 cm and a height of 13 cm .

Determine and state the volume of the small can and the volume of the large container to the nearest cubic centimeter.


$=441 \pi$


$=1053 \pi$


What is the minimum number of small cans that must be opened to fill the large container? Justify your answer.


Score 2: The student made an error by using diameter for the volume of both cylinders and made an error in determining the number of small cans needed.

## Question 33

33 A small can of soup is a right circular cylinder with a base diameter of 7 cm and a height of 9 cm . A large container is also a right circular cylinder with a base diameter of 9 cm and a height of 13 cm .

Determine and state the volume of the small can and the volume of the large container to the nearest cubic centimeter.

$$
\begin{array}{ll}
V=\pi r^{2} h & V=\pi r^{2} h \\
V=\pi 3.5^{2}(9) & V=\pi(4.5)^{2}(13) \\
V=\pi(7)(9) & V=\pi(20.25)(13) \\
V=63 \pi & V=263.25 \pi \\
V=197.920 & V=827.024 \\
V=198 & V=827
\end{array}
$$

What is the minimum number of small cans that must be opened to fill the large container? Justify your answer.

Score 1: The student determined the correct volume of the large container, but no further correct work was shown.

## Question 33

33 A small can of soup is a right circular cylinder with a base diameter of 7 cm and a height of 9 cm . A large container is also a right circular cylinder with a base diameter of 9 cm and a height of 13 cm .

Determine and state the volume of the small can and the volume of the large container to the nearest cubic centimeter.


What is the minimum number of small cans that must be opened to fill the large container? Justify your answer.

Score 1: The student found the volumes of the small can and large container, but rounded to the nearest tenth of a cubic centimeter. No further correct work was shown.

## Question 33

33 A small can of soup is a right circular cylinder with a base diameter of 7 cm and a height of 9 cm . A large container is also a right circular cylinder with a base diameter of 9 cm and a height of 13 cm .

Determine and state the volume of the small can and the volume of the large container to the nearest cubic centimeter.


What is the minimum number of small cans that must be opened to fill the large container? Justify your answer.

$$
2
$$

Score 0: The student made errors in determining the volumes of both cylinders and did not show enough correct relevant work to receive additional credit.

## Question 33

33 A small can of soup is a right circular cylinder with a base diameter of 7 cm and a height of 9 cm . A large container is also a right circular cylinder with a base diameter of 9 cm and a height of 13 cm .

Determine and state the volume of the small can and the volume of the large container to the nearest cubic centimeter.


What is the minimum number of small cans that must be opened to fill the large container? Justify your answer.
58 small cans of soup are needed to fill the container because it you do 72.3 .14 That gave you 153.86 multiply that by 9 you got 1384.74 then you ald That up By 49 to get 1433.74 then you do 49.0 odder By your theight to get 58 .

Score 0: The student did not show enough correct relevant work to receive any credit.

## Question 34

34 Parallelogram MATH has vertices $M(-7,-2), A(0,4), T(9,2)$, and $H(2,-4)$.
Prove that parallelogram MATH is a rhombus.
[The use of the set of axes below is optional.]

$$
\begin{aligned}
& M A=\sqrt{7^{2}+b^{2}}=\sqrt{85} \quad \text { MATt is a rhombus because all side lengths } \\
& A T=\sqrt{9^{2}+2^{2}}=\sqrt{85} \quad \text { are equal, therefore all sides are } \cong \text { to each other }
\end{aligned}
$$

$$
T H=\sqrt{7^{2}+6^{2}}=\sqrt{85}
$$

$$
\overline{M H}=\sqrt{9^{2}+2^{2}}=\sqrt{85}
$$

Determine and state the area of MATH.

$$
\begin{aligned}
& A=16(8)=128 \\
& A=\frac{1}{2}(6)(7)=21 \\
& A=\frac{1}{2}(2)(9)=9
\end{aligned}
$$

$$
128-2(4+21)=68
$$



Score 4: The student gave a complete and correct response.

Question 34

34 Parallelogram MATH has vertices $M(-7,-2), A(0,4), T(9,2)$, and $H(2,-4)$.
Prove that parallelogram MATH is a rhombus.
[The use of the set of axes below is optional.]
All four sides are $\cong(\overline{M A} \cong \overline{A T} \cong \overline{T H} \cong \overrightarrow{H M})$ therefore, MATH is a rhombus

Determine and state the area of MATH.


Score 4: The student gave a complete and correct response.

## Question 34

34 Parallelogram MATH has vertices $M(-7,-2), A(0,4), T(9,2)$, and $H(2,-4)$.
Prove that parallelogram MATH is a rhombus.

$$
\begin{aligned}
& \text { [The use of the set of axes below is optional.] } \\
& M A=\sqrt{(0-(-7))^{2}+(4-(-2))^{2}}\left|\quad A T=\sqrt{(9-0)^{2}+(2-4)^{2}}\right| \quad T H=\sqrt{(2-9)^{2}+(-4-2)^{2}} \mid M H=\sqrt{(2-(-7))^{2}+(-4-(-2))^{2}} \\
& =\sqrt{7^{2}+6^{2}} \\
& =\sqrt{49+36} \\
& =\sqrt{85} \\
& \begin{array}{l}
=\sqrt{9^{2}+(-2)^{2}} \\
=\sqrt{81+4} \\
=\sqrt{85}
\end{array}\left|\begin{array}{l}
=\sqrt{(-7)^{2}+(-6)^{2}} \\
=\sqrt{49+36} \\
=\sqrt{85}
\end{array}\right| \begin{array}{l}
=\sqrt{9^{2}+(-2)^{2}} \\
=\sqrt{81+4} \\
=\sqrt{85}
\end{array} \\
& \text { MATH is a rhombus since the oppositesides are } \cong
\end{aligned}
$$

Determine and state the area of MATH.

$$
\begin{aligned}
A H & =\sqrt{(2 \cdot 0)^{2}+(-4-4)^{2}} \quad M T & =\sqrt{\left(9-(-7)^{2}+(2-(-2))^{2}\right.} & A
\end{aligned} \quad \begin{aligned}
A & =\frac{1}{2} d_{1} \cdot d 2 \\
& =\sqrt{16^{2}+4^{2}} \\
& =\sqrt{2^{2}+(-812} \\
& =\sqrt{256+16} \\
& =\sqrt{4+64} \\
& =\sqrt{68}
\end{aligned}
$$

Score 3: The student wrote an incorrect concluding statement when proving the rhombus.

## Question 34

34 Parallelogram MATH has vertices $M(-7,-2), A(0,4), T(9,2)$, and $H(2,-4)$.
Prove that parallelogram MATH is a rhombus.
[The use of the set of axes below is optional.]

$$
\begin{array}{ll}
d(M, A)=\sqrt{7^{2}+6^{2}}=\sqrt{85} & d(A, H)=\sqrt{8^{2}+2^{2}}=\sqrt{68} \\
d(A, T)=\sqrt{9^{2}+2^{2}}=\sqrt{85} & d(M, T)=\sqrt{16^{2}+4^{2}}=\sqrt{272} \\
d(T, H)=\sqrt{7^{2}+6^{2}}=\sqrt{85} & \\
d(H, M)=\sqrt{9^{2}+2^{2}}=\sqrt{85} &
\end{array}
$$

Determine and state the area of MATH.


Score 3: The student did not write a concluding statement when proving the rhombus.

## Question 34

34 Parallelogram MATH has vertices $M(-7,-2), A(0,4), T(9,2)$, and $H(2,-4)$.
Prove that parallelogram MATH is a rhombus.
[The use of the set of axes below is optional.]

$$
\left(Q_{a r}\right)^{2}=6^{2}+7^{2}
$$

A panallelogram with a pain $Q^{2}=36+49$ of consecutive side
eruce $s$ a


Rhombus.
So MMathw
A rhombus

$$
\begin{aligned}
& Q=\sqrt{85} \\
& \left(Q_{A T}\right)^{2}=9^{2}+2^{2} \\
& Q^{2}=81+4
\end{aligned}
$$

Determine and state the area of MATH.


Score 3: The student made an error in computing the area of triangle IV.

## Question 34

34 Parallelogram MATH has vertices $M(-7,-2), A(0,4), T(9,2)$, and $H(2,-4)$.
Prove that parallelogram MATH is a rhombus.
[The use of the set of axes below is optional.]


Determine and state the area of MATH.


Score 2: The student proved parallelogram MATH is a rhombus, but no further correct work was shown.

## Question 34

34 Parallelogram MATH has vertices $M(-7,-2), A(0,4), T(9,2)$, and $H(2,-4)$.
Prove that parallelogram MATH is a rhombus.
[The use of the set of axes below is optional.]

> MATH is a rhombus due to $\widehat{M A} \| \overrightarrow{H T}$ ! $\frac{A T \|}{M H}$,

Determine and state the area of MATH.


Score 2: The student determined the area of MATH, but no further correct work was shown.

## Question 34

34 Parallelogram MATH has vertices $M(-7,-2), A(0,4), T(9,2)$, and $H(2,-4)$.
Prove that parallelogram MATH is a rhombus.
[The use of the set of axes below is optional.]


Determine and state the area of MATH.

$\frac{8}{2}$
$\frac{4}{16} \quad \frac{1}{4}$

Score 2: The student proved parallelogram MATH is a rhombus, but no further correct work was shown.

## Question 34

34 Parallelogram MATH has vertices $M(-7,-2), A(0,4), T(9,2)$, and $H(2,-4)$.
Prove that parallelogram MATH is a rhombus.
[The use of the set of axes below is optional.]


Determine and state the area of MATH.

$$
\begin{aligned}
M A & =\sqrt{(0+7)^{2}+(4+2)^{2}} \\
D & =\sqrt{(7)^{2}+(6)^{2}} \\
D & =\sqrt{49+36} \\
D & =\sqrt{85} \\
D & =\sqrt{(2-9)^{2}+(-4-2)^{2}} \\
D & =\sqrt{(-7)^{2}+(-6)^{2}} \\
D & =\sqrt{49+36} \\
D & =\sqrt{85} \text { AT }
\end{aligned}
$$

$$
{ }^{A T}=\sqrt{(9-0)^{2}+(2-4)^{2}}
$$

$A=85(85)$


$$
D=\sqrt{(9)^{2}+(-2)^{2}}
$$

$$
D=\sqrt{(81)+(4)}
$$

$$
D=\sqrt{85}
$$

$$
M H
$$

$$
\begin{aligned}
& H \\
& D=\sqrt{(2+7)^{2}+(-4+2)^{2}}
\end{aligned}
$$

$$
D=\sqrt{(9)^{2}+(-2)^{2}}
$$

$$
D=\sqrt{81+4}
$$

$$
D=\sqrt{85}
$$

Score 1: The student found the lengths of the sides of MATH, but the concluding statement was incorrect. No further correct work was shown.

## Question 34

34 Parallelogram MATH has vertices $M(-7,-2), A(0,4), T(9,2)$, and $H(2,-4)$.
Prove that parallelogram MATH is a rhombus.
[The use of the set of axes below is optional.]

$$
\begin{array}{ll}
M A=\binom{4-2}{0-7}=\frac{6}{7} & \text { Parallelogram MATH is not a } \\
A T=\binom{\frac{2-4}{a-0}}{a}=\frac{-2}{9} & \text { Rhombus because the diagonals } \\
T H=\left(\frac{-4.2}{2-9}\right)=\frac{-6}{-7}=\frac{6}{7} & \text { do not have negative reciprocal } \\
\text { slopes }
\end{array}
$$

$$
M H=\left(\frac{-4-2}{2--7}\right)=\frac{-2}{9}
$$

Determine and state the area of MATH.
MA $d=\sqrt{(0-1)^{2}+(4-2)^{2}}=\sqrt{7^{2}+6^{2}}=\sqrt{85}$
AT $d=\sqrt{(9-0)^{2}+(2 \cdot 4)^{2}}=\sqrt{9^{2}+(-2)^{2}}=\sqrt{85}$
TH $d=\sqrt{(2-a)^{2}+(-4-2)^{2}}=\sqrt{-7^{2}+6^{2}}=\sqrt{5} 5_{\uparrow}^{y}$



Score 1: The student found the length of at least two consecutive sides of MATH. No further correct relevant work was shown.

## Question 34

34 Parallelogram MATH has vertices $M(-7,-2), A(0,4), T(9,2)$, and $H(2,-4)$.
Prove that parallelogram MATH is a rhombus.
[The use of the set of axes below is optional.]

$$
\begin{gathered}
A H=\sqrt{(8)^{2}+(2)^{2}} \\
\sqrt{68}
\end{gathered} \quad \sqrt{6 T}=\sqrt{(16)^{2}+(4)^{2}}
$$

Determine and state the area of MATH.
Area $=$ diagonal $\times$ diagonal


Score 1: The student made an error in determining the area of MATH. No further correct relevant work was shown.

## Question 34

## $y_{1} \frac{x_{2}}{x_{1}}$

34 Parallelogram MATH has vertices $M(-7,-2), A(0,4), T(9,2)$, and $H(2,-4)$.
Prove that parallelogram MATH is a rhombus.
[The use of the set of axes below is optional.]



Determine and state thearea of MATH angle Mcasulgo $\frac{0+7}{4+2} \quad \frac{9}{2} \quad-\frac{11}{-6}$

$$
\text { Since two }
$$ are congruent I know is a men mise $\frac{7}{6}$



Score 0: The student did not show enough correct relevant work to receive any credit.

## Question 34

34 Parallelogram MATH has vertices $M(-7,-2), A(0,4), T(9,2)$, and $H(2,-4)$.
Prove that parallelogram MATH is a rhombus.
[The use of the set of axes below is optional.]


Determine and state the area of MATH.


Score 0: The student did not show enough correct relevant work to receive any credit.

## Question 34

34 Parallelogram MATH has vertices $M(-7,-2), A(0,4), T(9,2)$, and $H(2,-4)$.
Prove that parallelogram MATH is a rhombus.
[The use of the set of axes below is optional.]

$$
\begin{aligned}
& \text { Parallelogram MATH isarhombes } \\
& \text { bécarse all Sides are Congruent. }
\end{aligned}
$$

Determine and state the area of MATH.


Score 0: The student did not show enough correct relevant work to receive any credit.

## Question 35

35 Given: Quadrilateral $A B C D, \overline{A B} \cong \overline{C D}, \overline{A B} \| \overline{C D}$, diagonal $\overline{A C}$ intersects $\overline{E F}$ at $G$, and $\overline{D E} \cong \overline{B F}$


Prove: $G$ is the midpoint of $\overline{E F}$

| STATEMENT | REASONS |
| :---: | :---: |
| 1. quadrilateral $A B C D$ <br> - $\overline{A B} \cong \overline{C D}, \overline{A B} \\| \overline{C D}$ <br> 2. $A B C D$ Ts a parallelogram | 1. Given <br> 2. If a quadricateral has a pair of opposite sides that are parallel and congruent then $7 t$ is a parallelogram |
| 3. $\overline{D E} \cong \overline{\# F F}$ | 3. Given |
| 4. $\overline{A D} \cong \overline{C B}$ | 4. Oppositestdes of a parallelogram are congruent |
| 5. $\overline{A F} \cong \overline{C F}$ | 5. Subtraction Postulate |
| 6. $\overline{A D} \\| \overline{C B}$ | 6. Opposite sides of a parallelogram are parallel |
| ๆ. $\angle E A G \cong \angle F C G$ | 7. If two parallel ines arecut by a tronsversal, then the alternict interior amples are conguent |
| 8. $\angle A G E \because \angle C G F$ | 8. Tf two lines intereect, they form vertical ongles that are |
| 9. $\triangle A E G \cong \triangle C F G$ | 9. AAs Pistulate |
| 10. $E G \cong F G$ | 10.CPCTC |
| (1. Gis the midpoint of EF | III. If a point diüdes "segment into two congruent segments then it is the midpoint if the segment |

Score 6: The student gave a complete and correct response.

Question 35

35 Given: Quadrilateral $A B C D, \overline{A B} \cong \overline{C D}, \overline{A B} \| \overline{C D}$, diagonal $\overline{A C}$ intersects $\overline{E F}$ at $G$, and $\overline{D E} \cong \overline{B F}$


Prove: $G$ is the midpoint of $\overline{E F}$
Since quad. $A B C D$ has one set of opposite sides $\cong$ and $/ /$, it is a parallelogram. Then $\overline{A D} \cong \overline{B C}$ and $\overline{A D} / / \overline{B C}$ b/c opposite sides of a $p$-gram are $\cong$ and $/ 1$.

Since $\overline{A D} \cong \overline{B C}$ and $\overline{E D} \cong \overline{B F}$ (given), $\overline{A E} \cong \overline{C F}$ by the subtraction property.
since $\overline{A D} / / \overline{B C}$, transversal $\overline{A C}$ and $\overline{E F}$ will make $\cong$ alternate interior angles, $50 \triangle E A G \cong \angle F C G$ and $\triangle A E G \cong \triangle C F G$.

Therefore $\triangle A E G \cong \triangle C F G$ by $A S A \cong$.
Then $\overline{E G} \cong \overline{F G}$ by $C P C T C$. So, since $G$ is a point on $\overline{E F}$ and is dividing it into $2 \cong$ parts, $G$ must be a midpoint.

Score 6: The student gave a complete and correct response.

## Question 35

35 Given: Quadrilateral $A B C D, \overline{A B} \cong \overline{C D}, \overline{A B} \| \overline{C D}$, diagonal $\overline{A C}$ intersects $\overline{E F}$ at $G$, and $\overline{D E} \cong \overline{B F}$


Prove: $G$ is the midpoint of $\overline{E F}$

a. Quadrilateral ABC) is a parallelogram
2. If a set of opposite
sides of a quadrialemal are $\cong$ and 11 it is a parallengian
3. $\overline{A D}$ is $\|$ and $\cong$ to $\overline{C B}$
4. XEAG $\cong X F C O$ $\Varangle A E G \cong \Varangle C F G$
5. $\overline{A D}-\overline{E D} \cong \overline{C B} \cdot \overline{F B}$
$\overline{A E} \cong \overline{C F}$
6. $\triangle A E G \cong \triangle C F G$
7. $\overline{E Q} \cong \overline{F G}$
8. $G$ is the midpoint
3. Opposite sides of a parallelogram
ane $\cong$ and 11 .
4. When lines are II Alt. interior 孔's are $\cong$
5. Subircotion
6. $A S A \cong$
7. cpctc
8. If a point splits a segment into two $\cong$ segments, it is a midpoint.

Score 6: The student gave a complete and correct response.

## Question 35

35 Given: Quadrilateral $A B C D, \overline{A B} \cong \overline{C D}, \overline{A B} \| \overline{C D}$, diagonal $\overline{A C}$ intersects $\overline{E F}$ at $G$, and $\overline{D E} \cong \overline{B F}$


Prove: $G$ is the midpoint of $\overline{E F}$

| 1. Quadrilateral $A B C D, \overline{A B}=\overline{C D}, \overline{A B} \\| \overline{C D}$ $\overline{D E} \cong \overline{B F}$ | 1. Given |
| :---: | :---: |
| 2. $4 B A C \cong \triangle D C A$ | 2. If 2 parallel lines are cut by a transversal, the alternate interion angles ore? |
| 3. $\overline{A C} \cong$ | 3. Reflexive |
| 4. $\triangle A B C \cong \triangle C D A$ | 4. 545 S E 45 |
| $\text { 5. } \begin{aligned} & x E A G \cong \triangle F C E \\ & \overline{A D} \cong \overline{B C} \end{aligned}$ | 5. CPCTC |
| 6. $\overline{A D}-\overline{D E} \cong \overline{B C}-\overline{B F}$ | 6. Subtraction |
| $\begin{array}{r} \overline{A E} \xlongequal{\cong} \overline{F C} \\ 7 . \& A G E \cong A C G F \end{array}$ | 7. Vertical $*^{3}$ are $\cong$ |
| 8. $\triangle A G E \equiv \triangle C F F$ | 8. AAS $=A A S$ |
| 9. $\overline{E G} \cong \overline{F G}$ | $9 . \mathrm{CPCTC}$ |

Score 5: The student had a missing concluding statement and reason after step 9.

## Question 35

35 Given: Quadrilateral $A B C D, \overline{A B} \cong \overline{C D}, \overline{A B} \| \overline{C D}$, diagonal $\overline{A C}$ intersects $\overline{E F}$ at $G$, and $\overline{D E} \cong \overline{B F}$


Prove: $G$ is the midpoint of $\overline{E F}$

| 1. Quadrilateral $A B C D, \overline{A B} \cong \overline{C D}, \overline{A B} \backslash \overline{C D}$, and $\overline{D E} \cong \overline{B F}$ | 1. Gruen |
| :---: | :---: |
| 2. Quadrilateral $A B C D$ is a parallelogram | 2. When one raur ot mositc sides is cor gtur: t 110 rtallel, a quadrilatian is a paracisiograin, |
| 3. $\angle A G E \cong \angle F G C$ | 3. Vertical augles are cangruent. |
| $\text { 4. } \begin{aligned} & \overline{A D}-\overline{E D} \cong \overline{B C}-\overline{B F} \\ & \overline{A E} \equiv \overline{F C} \end{aligned}$ | 4. Subtraction postulate |
| 5. $\angle E A G_{T}=\angle F C G$ | 5. When lines are paraile!, allernatic intenor angies are erngruent. |
| 6. $\triangle A E G \cong \triangle C G$ | 6. AAS $=A M S$ |
| 1. $\overline{E G_{1}} \cong \overline{F_{1}}$ | 7. CPCTC |
| 8. $G$ is the midpoint of $\overline{E F}$ | 8. When two segments on a line segment are congruent, tis forion intersecting llem is lie raipmant. |

Score 4: The student had a missing statement and reason to prove step 4 and a missing statement and reason to prove step 5 .

## Question 35

35 Given: Quadrilateral $A B C D, \overline{A B} \cong \overline{C D}, \overline{A B} \| \overline{C D}$, diagonal $\overline{A C}$ intersects $\overline{E F}$ at $G$, and $\overline{D E} \cong \overline{B F}$


Prove: $G$ is the midpoint of $\overline{E F}$


DE $=\stackrel{\rightharpoonup}{B F}$
2) $\angle 1 \&<2$ are 2) Vent. is $\cong$
3) $A B C D$ is a parallelogram is a parallelogram.
4) $\overline{A A} \| \overline{C B}$ (D) A parallelogram has opp sides parallel
5) $\angle 3 \cong \angle 4$, 5) when troll Lines are
b) $\triangle E A G \approx \triangle F G G$ alt intis $i s \cong$
7) $\overline{E G} \cong \overline{G F} \quad$ 万) $\triangle$ AAA
8) (wis mid point 8) it $G$ cuts $\overline{E F}$ into of it is a midpoint

Score 4: The student made a conceptual error in proving $\triangle E A G \cong \triangle F C G$.

Question 35

35 Given: Quadrilateral $A B C D, \overline{A B} \cong \overline{C D}, \overline{A B} \| \overline{C D}$, diagonal $\overline{A C}$ intersects $\overline{E F}$ at $G$, and $\overline{D E} \cong \overline{B F}$


Prove: $G$ is the midpoint of $\overline{E F}$

2) Quad ABCD is it $\square$
3) $\overline{A D}-\overline{E D} \cong \overline{B C}-\overline{B P}$ $\overline{A \varepsilon} \underline{\underline{A}} \overline{F C}$
4) $\overline{A D} \| \overline{B C}$
5) $\bar{厶} \varepsilon_{A E G} A G \angle C G G$
6) $\triangle A G \varepsilon \cong \triangle C G F$
7) $\overline{\varepsilon G} \cong \overline{F G}$

Reasons

1) Given
2) when one prof app side
3) Subtraction Post.
4) A $\square$ has oppsideo //
5) wetint L's are $\underline{Z}^{2}$
6) $A S A$
) © © Ct

Score 3: The student had a missing statement and reason to prove step 3, an incomplete reason in step 5 , and a missing concluding statement and reason after step 7.

## Question 35

35 Given: Quadrilateral $A B C D, \overline{A B} \cong \overline{C D}, \overline{A B} \| \overline{C D}$, diagonal $\overline{A C}$ intersects $\overline{E F}$ at $G$, and $\overline{D E} \cong \overline{B F}$


Prove: $G$ is the midpoint of $\overline{E F}$

$$
\begin{aligned}
& \text { Quadrilateral } A B C D \\
& \text { 1. } \overline{A B} \cong \overline{C D}, \overline{A B} \| \overline{C D} \quad \text { 1. given } \\
& \text { 2. quadrilateral } A B C D \quad \text { 2. A quad with one pair of } \\
& \text { is a parallogram } \\
& \text { 3. } \angle 1 \text { and } \angle 2 \text { are } \cong \text { 3. vert. } \angle s \cong \\
& \text { 4. } \angle E A G \cong \angle G C F \quad \text { 4. af alt. int } \angle 5 \text { are parallel, then } \\
& \angle A E G \cong \angle C F G \\
& \text { 5. } \triangle A E G \cong \triangle C F G \text { 5. } A A A \simeq \\
& \text { 6. GAs the midpoint } 6 . C P C T C
\end{aligned}
$$

Score 2: The student made some correct relevant statements and reasons about the proof.

## Question 35

35 Given: Quadrilateral $A B C D, \overline{A B} \cong \overline{C D}, \overline{A B} \| \overline{C D}$, diagonal $\overline{A C}$ intersects $\overline{E F}$ at $G$, and $\overline{D E} \cong \overline{B F}$


Prove: $G$ is the midpoint of $\overline{E F}$

$$
\begin{aligned}
& \text { (2) } \angle E A G \cong \angle F C C \text { (2) } A I A \\
& \text { (3) } \overline{A E} \cong \overline{C F} \text { (3) Subtraction Postulate } \\
& \text { (4) } \angle A G E \cong \angle C G F \text { (4)vertical } \angle \text { s are } \cong \\
& \text { (5) } \triangle A E G \cong \triangle C F B \text { (5) } A S A \\
& \text { (6) Gis midpointaAF } 6 \text { CPCTC }
\end{aligned}
$$

Score 1: The student had only one correct relevant statement and reason in step 4.

## Question 35

35 Given: Quadrilateral $A B C D, \overline{A B} \cong \overline{C D}, \overline{A B} \| \overline{C D}$, diagonal $\overline{A C}$ intersects $\overline{E F}$ at $G$, and $\overline{D E} \cong \overline{B F}$


Prove: $G$ is the midpoint of $\overline{E F}$


Score 1: The student had only one correct relevant statement and reason in step 2.

## Question 35

35 Given: Quadrilateral $A B C D, \overline{A B} \cong \overline{C D}, \overline{A B} \| \overline{C D}$, diagonal $\overline{A C}$ intersects $\overline{E F}$ at $G$, and $\overline{D E} \cong \overline{B F}$


Prove: $G$ is the midpoint of $\overline{E F}$


Score 0: The student had a completely incorrect response.

