The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

GEOMETRY

Friday, June 21, 2024 — 9:15 a.m. to 12:15 p.m., only

MODEL RESPONSE SET

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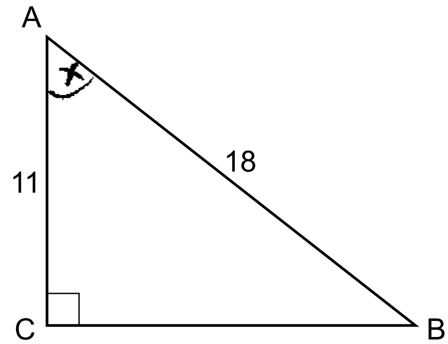
Question 25

25 In $\triangle ABC$ below, $m\angle C = 90°$, $AC = 11$, and $AB = 18$.



Determine and state the measure of angle A, to the *nearest degree*.

$\cos x = \frac{11}{18}$

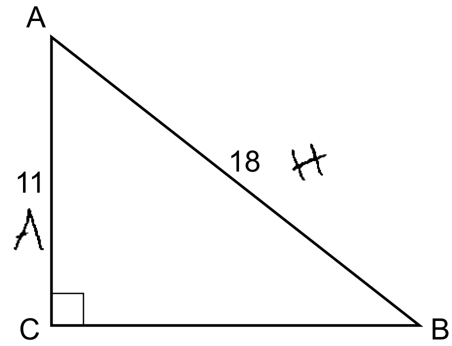
$x = 52.33011304$

$\boxed{52°}$

Score 2: The student gave a complete and correct response.

Question 25

25 In $\triangle ABC$ below, $m\angle C = 90°$, $AC = 11$, and $AB = 18$.



Determine and state the measure of angle A, to the *nearest degree*.

$A = \cos^{-1}\left(\frac{11}{18}\right)$

$A = 52.33611...$

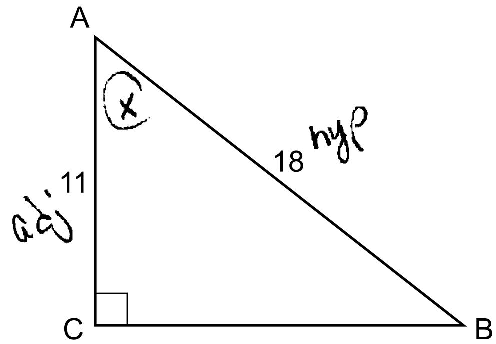
$A \approx 52$

$m\angle A = 52$

Score 2: The student gave a complete and correct response.

Question 25

25 In $\triangle ABC$ below, $m\angle C = 90°$, $AC = 11$, and $AB = 18$.



Determine and state the measure of angle A, to the *nearest degree*.

$$\cos x = \frac{11}{18}$$

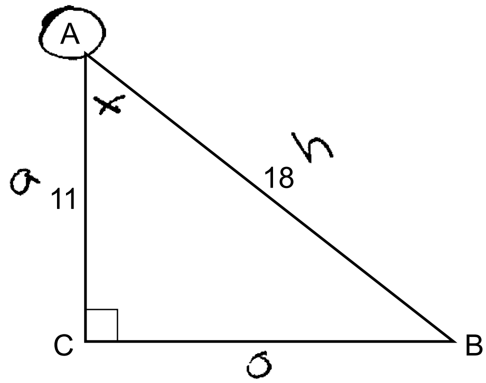
$$\cancel{\cos}^{-1}(\cancel{\cos} x) = \cos^{-1}\left(\frac{11}{18}\right)$$

$$x = 52°$$

Score 2: The student gave a complete and correct response.

Question 25

25 In $\triangle ABC$ below, $m\angle C = 90°$, $AC = 11$, and $AB = 18$.



Determine and state the measure of angle A, to the *nearest degree*.

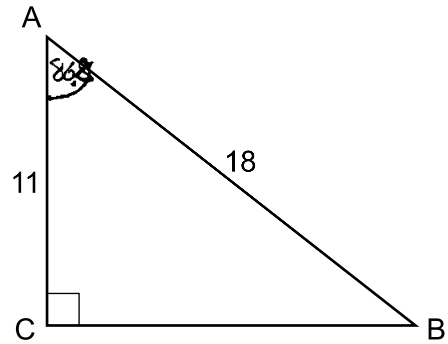
$\cos = a/h$

$\cos(x) = 11/18$

Score 1: The student wrote a correct relevant trigonometric equation, but no further correct work was shown.

Question 25

25 In $\triangle ABC$ below, $m\angle C = 90°$, $AC = 11$, and $AB = 18$.



Determine and state the measure of angle A, to the *nearest degree*.

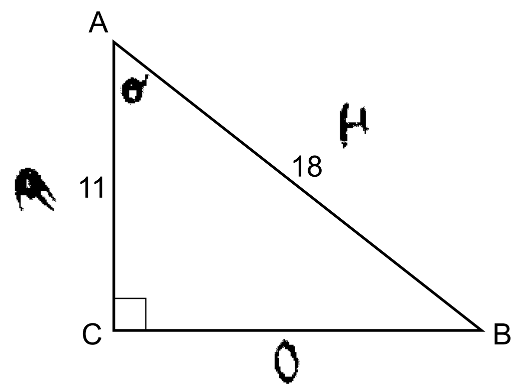
$\cos^{-1}\left(\frac{11}{18}\right) =$

$A = 86.8$

Score 1: The student wrote a correct relevant trigonometric equation, but no further correct work is shown.

Question 25

25 In $\triangle ABC$ below, $m\angle C = 90°$, $AC = 11$, and $AB = 18$.



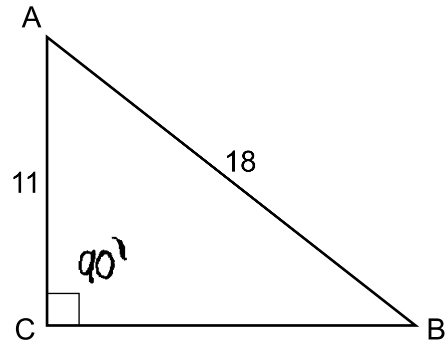
Determine and state the measure of angle A, to the *nearest degree*.

52°

Score 1: The student correctly determined the measure of $\angle A$, but showed no work.

Question 25

25 In $\triangle ABC$ below, $m\angle C = 90°$, $AC = 11$, and $AB = 18$.



Determine and state the measure of angle A, to the *nearest degree*.

$\cos(\angle BAC = \frac{\overline{AC}}{\overline{AB}}$

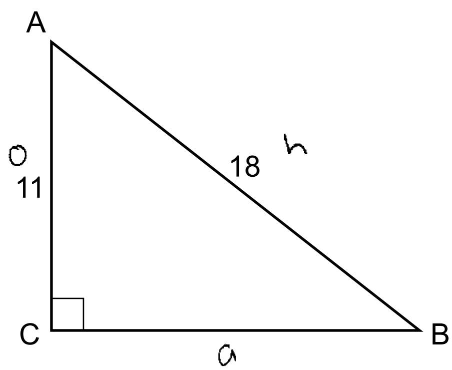
$\cos(\angle BAC) = \frac{11}{18}$

$\angle BAC = 52.3$

Score 1: The student made a rounding error.

Question 25

25 In $\triangle ABC$ below, $m\angle C = 90°$, $AC = 11$, and $AB = 18$.



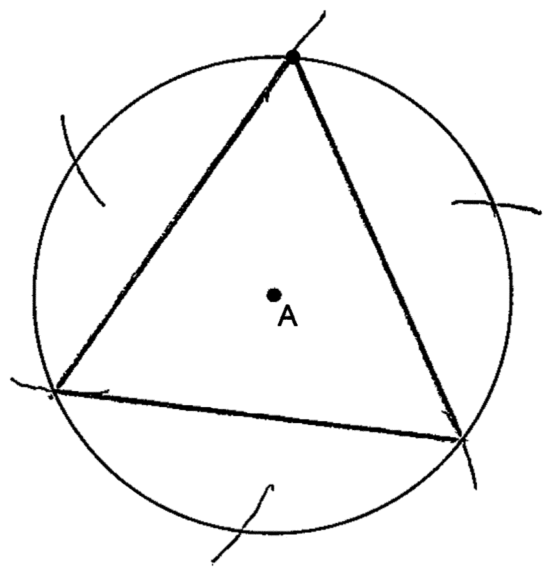
Determine and state the measure of angle A, to the *nearest degree*.

$S\frac{o}{h} = 11/\sin(18)$
$= 35.59674775$
$m\angle A = 36$

Score 0: The student gave a completely incorrect response.

Question 26

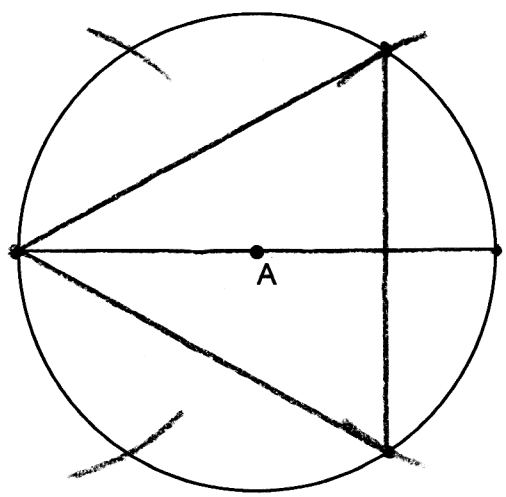
26 Use a compass and straightedge to construct an equilateral triangle inscribed in circle *A* below. [Leave all construction marks.]



Score 2: The student gave a complete and correct response.

Question 26

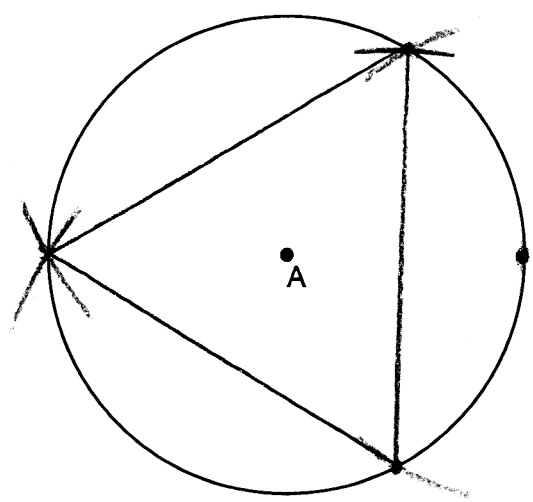
26 Use a compass and straightedge to construct an equilateral triangle inscribed in circle A below. [Leave all construction marks.]



Score 2: The student gave a complete and correct response.

Question 26

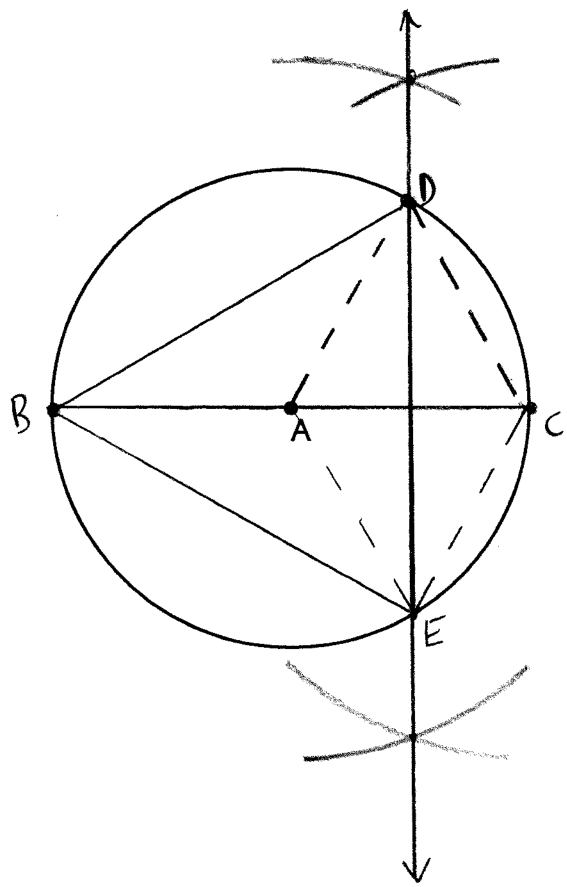
26 Use a compass and straightedge to construct an equilateral triangle inscribed in circle *A* below. [Leave all construction marks.]



Score 2: The student gave a complete and correct response. Using a compass the student measured the length of the radius and from a point on the circle, two arcs were drawn intersecting the circle forming two endpoints of one side of the triangle. Copying the length of the first side, two intersecting arcs were drawn intersecting the circle, forming the third vertex. The equilateral triangle was drawn.

Question 26

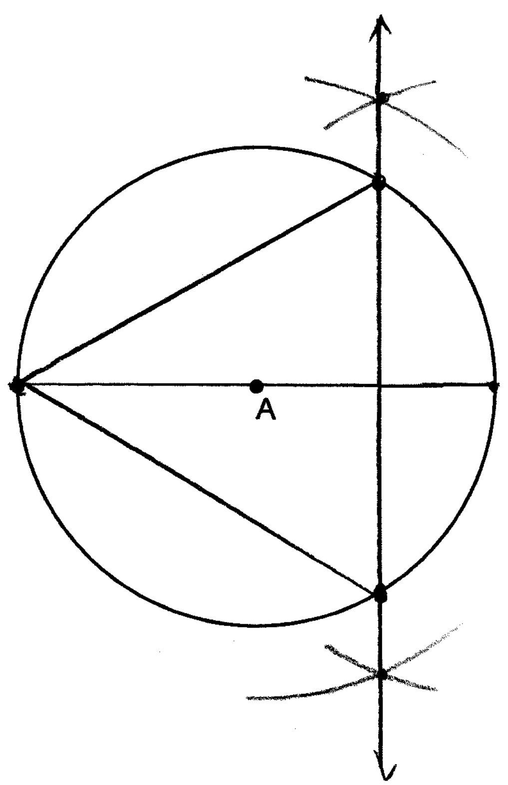
26 Use a compass and straightedge to construct an equilateral triangle inscribed in circle *A* below. [Leave all construction marks.]



Score 2: The student gave a complete and correct response. The student drew diameter $\overline{BC}$ and constructed the perpendicular bisector of radius $\overline{AC}$ resulting in equilateral triangles *ADC* and *AEC*. Central angles *DAE*, *DAB*, and *BAE* each measure 120° resulting in arcs $\overset{\frown}{BD}$, $\overset{\frown}{DCE}$, and $\overset{\frown}{BE}$ each measuring 120°. Equilateral triangle *BDE* was then drawn.

Question 26

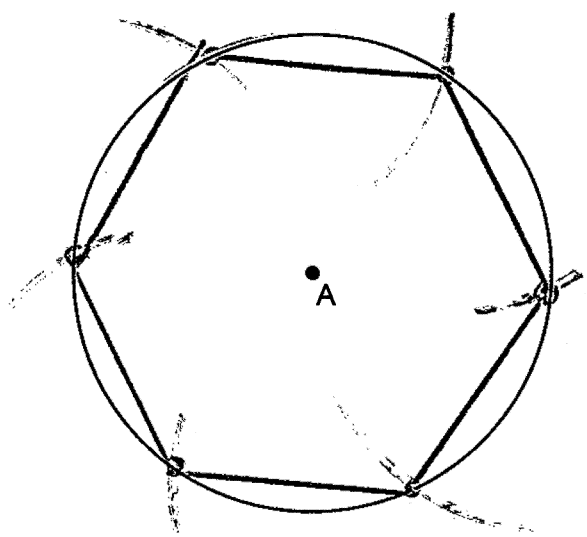
26 Use a compass and straightedge to construct an equilateral triangle inscribed in circle *A* below. [Leave all construction marks.]



Score 2: The student gave a complete and correct response.

Question 26

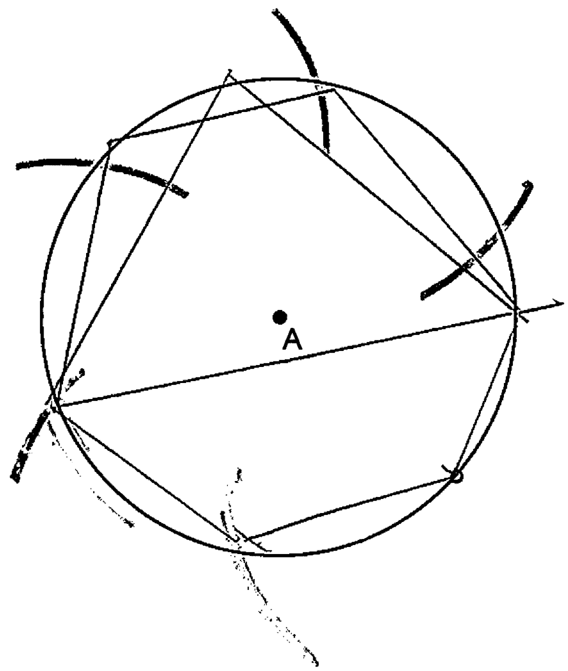
26 Use a compass and straightedge to construct an equilateral triangle inscribed in circle *A* below. [Leave all construction marks.]



Score 1: The student constructed all appropriate arcs, but the equilateral triangle was not drawn.

Question 26

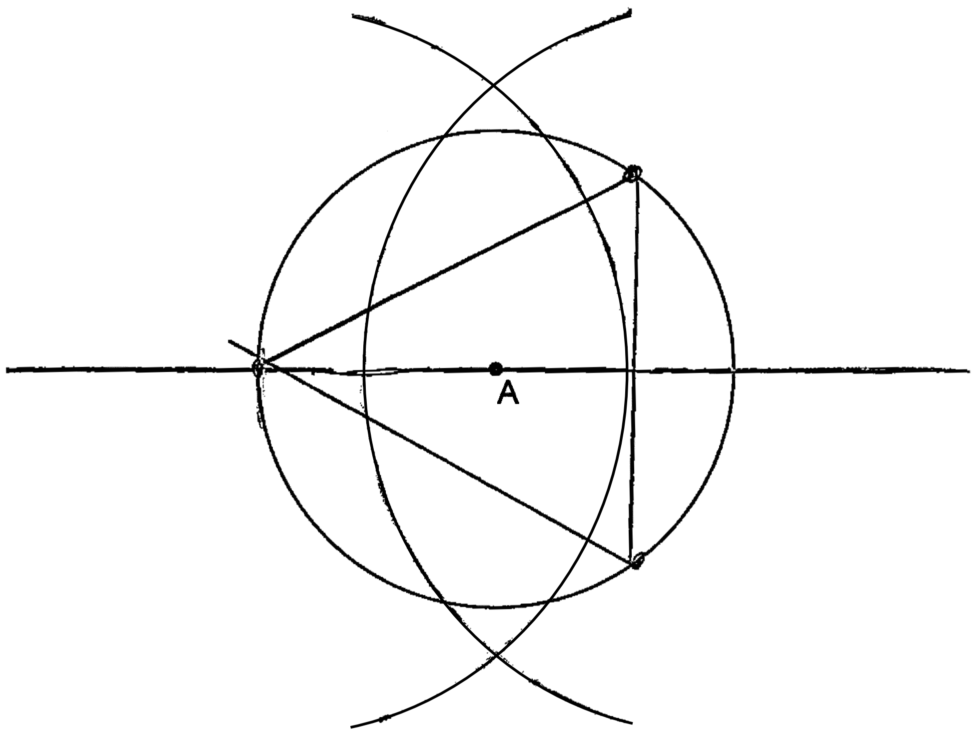
26 Use a compass and straightedge to construct an equilateral triangle inscribed in circle A below. [Leave all construction marks.]



Score 1: The student constructed all appropriate arcs, but made an error drawing the triangle.

Question 26

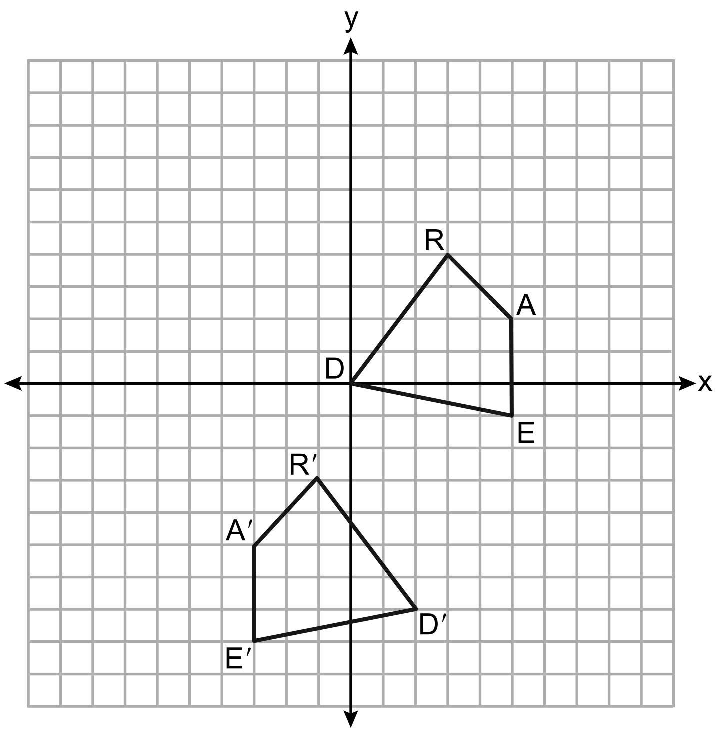
26 Use a compass and straightedge to construct an equilateral triangle inscribed in circle A below. [Leave all construction marks.]



Score 0: The student did not show enough correct relevant work to receive any credit.

Question 27

27 Quadrilateral *DEAR* and its image, quadrilateral *D′E′A′R′*, are graphed on the set of axes below.



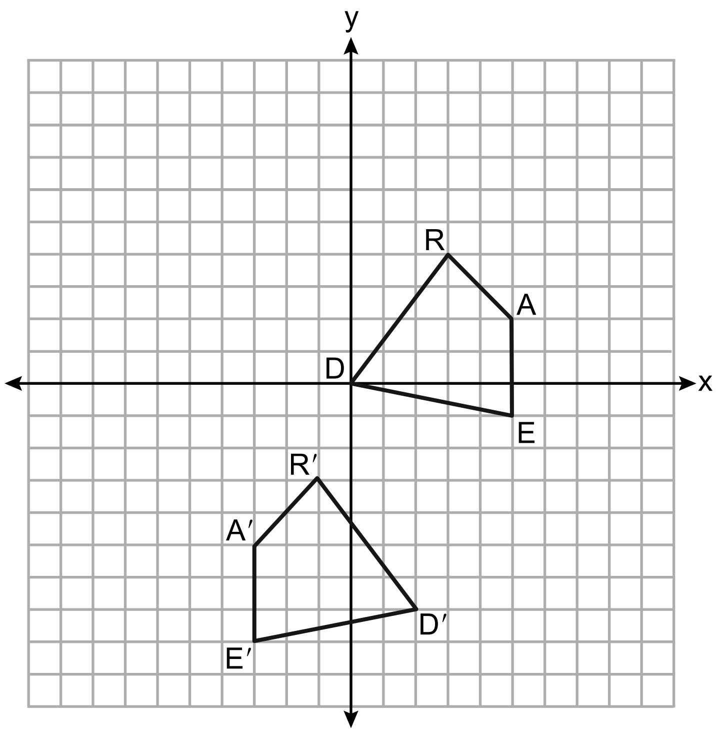
Describe a sequence of transformations that maps quadrilateral *DEAR* onto quadrilateral *D′E′A′R′*.

Reflection in the y-axis followed by a translation right 2 and down 7.

Score 2: The student gave a complete and correct response.

Question 27

27 Quadrilateral *DEAR* and its image, quadrilateral *D′E′A′R′*, are graphed on the set of axes below.



Describe a sequence of transformations that maps quadrilateral *DEAR* onto quadrilateral *D′E′A′R′*.

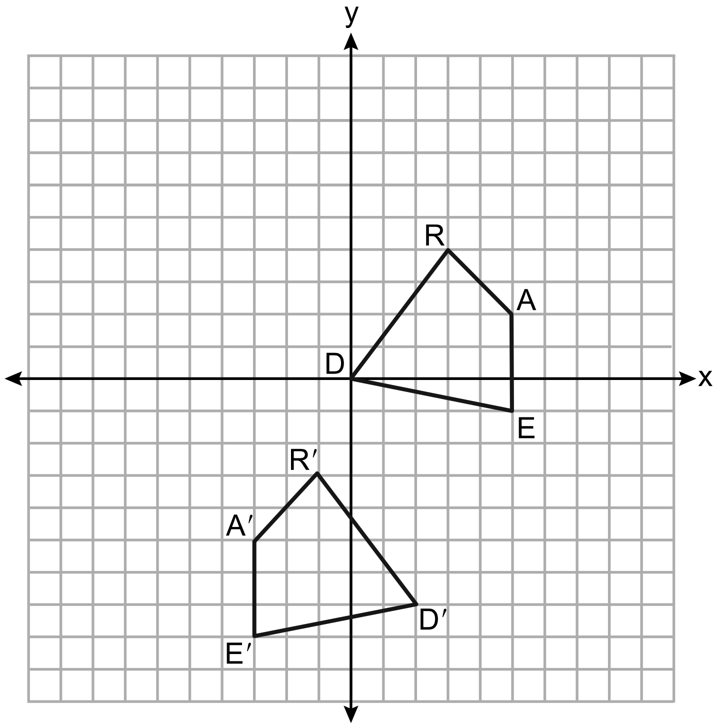
Reflection over line x=1

Translation of 0,-7

Score 2: The student gave a complete and correct response.

Question 27

27 Quadrilateral *DEAR* and its image, quadrilateral *D′E′A′R′*, are graphed on the set of axes below.



Describe a sequence of transformations that maps quadrilateral *DEAR* onto quadrilateral *D′E′A′R′*.

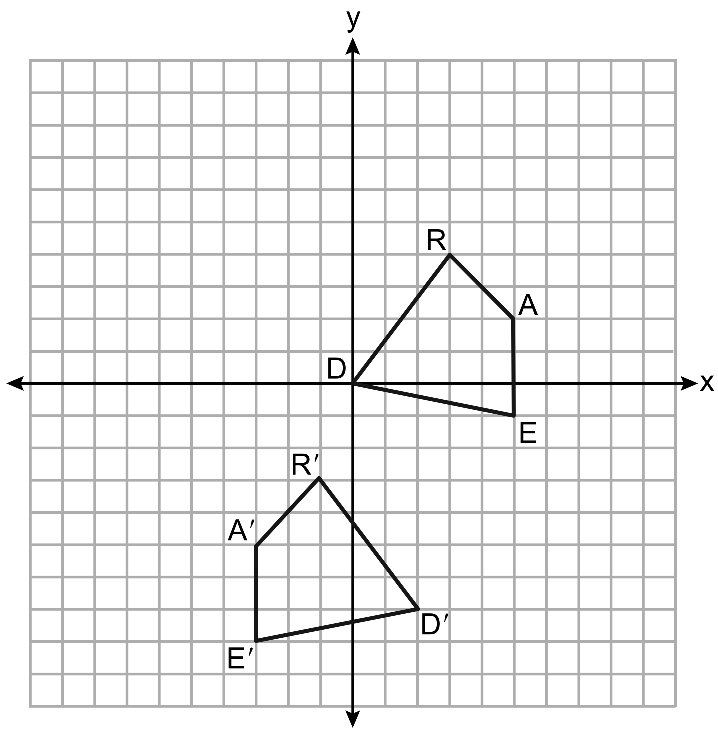
Reflect over y-axis

Translate 7 units down, 2 units Left

Score 1: The student wrote a correct reflection, but wrote an incorrect translation.

Question 27

27 Quadrilateral *DEAR* and its image, quadrilateral *D′E′A′R′*, are graphed on the set of axes below.



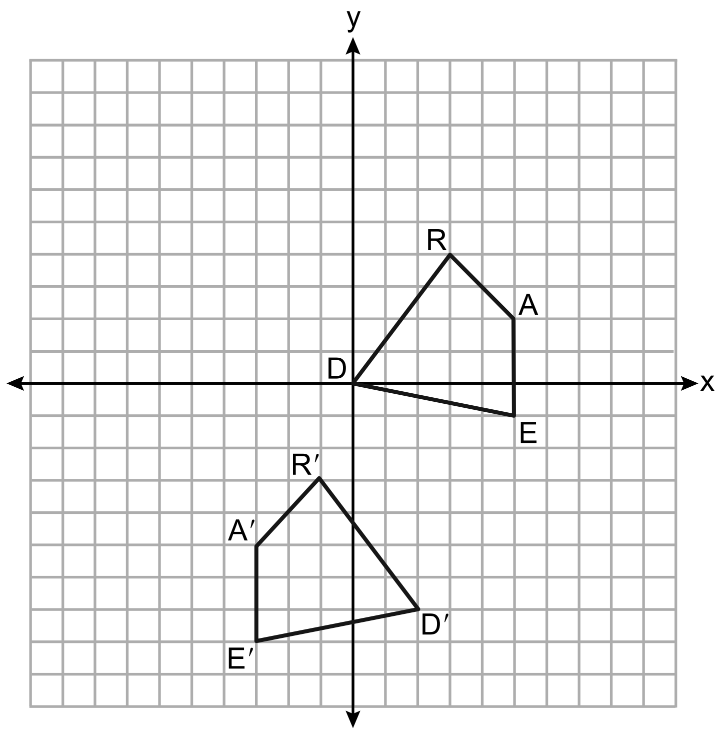
Describe a sequence of transformations that maps quadrilateral *DEAR* onto quadrilateral *D′E′A′R′*.

Reflection over the y-axes followed by transalion over the x-axes

Score 1: The student wrote a correct reflection, but wrote an incorrect translation.

Question 27

27 Quadrilateral *DEAR* and its image, quadrilateral *D′E′A′R′*, are graphed on the set of axes below.



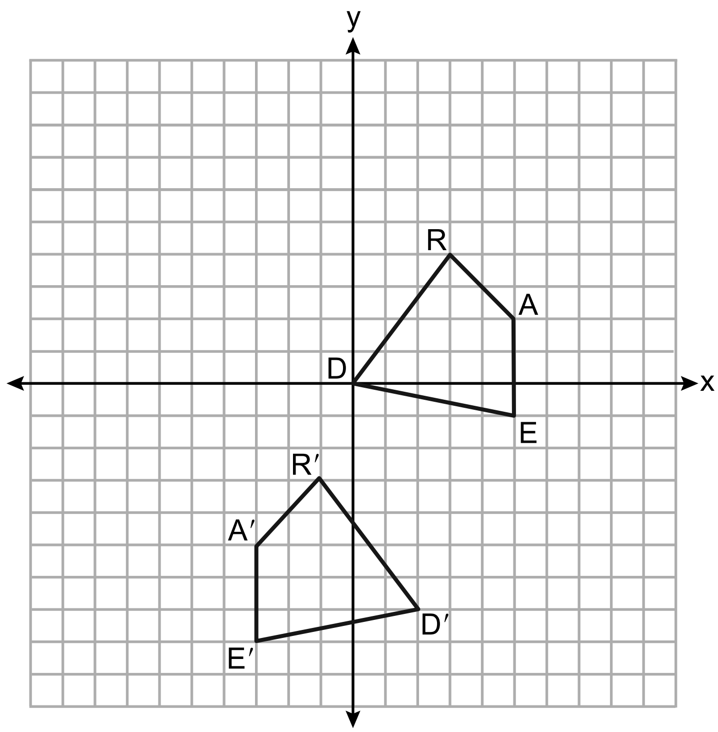
Describe a sequence of transformations that maps quadrilateral *DEAR* onto quadrilateral *D′E′A′R′*.

To map D'E'A'R' onto DEAR you would need a reflection on the y axis and a translation 7 units up and 3 units left

Score 0: The student made an error mapping D′E′A′R′ onto DEAR, and stated an incorrect translation.

Question 27

27 Quadrilateral *DEAR* and its image, quadrilateral *D′E′A′R′*, are graphed on the set of axes below.



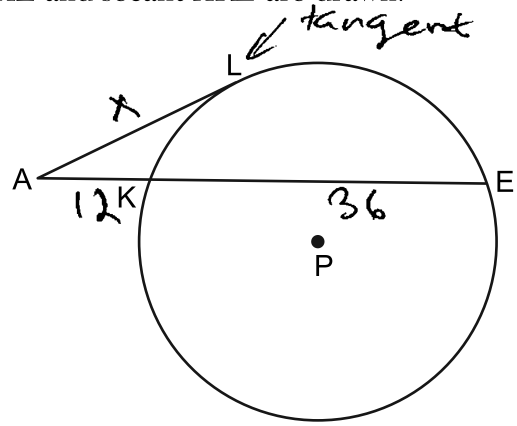
Describe a sequence of transformations that maps quadrilateral *DEAR* onto quadrilateral *D′E′A′R′*.

rotation 180° clockwise

Score 0: The student did not show enough correct relevant work to receive any credit.

Question 28

28 In circle P below, tangent $\overline{AL}$ and secant $\overline{AKE}$ are drawn.



If $AK = 12$ and $KE = 36$, determine and state the length of $\overline{AL}$.

outside × whole = tangent^2

$12 \times 48 = x^2$

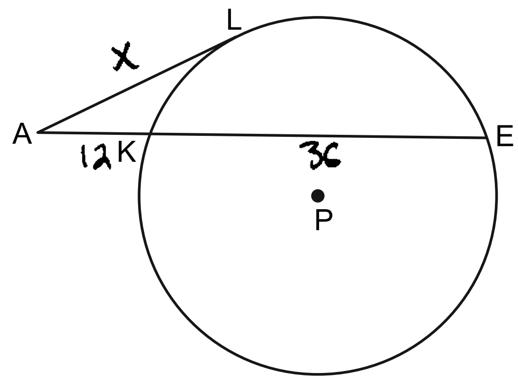
$\sqrt{576} = \sqrt{x^2}$

$\boxed{x = 24}$

Score 2: The student gave a complete and correct response.

Question 28

28 In circle P below, tangent $\overline{AL}$ and secant $\overline{AKE}$ are drawn.



If $AK = 12$ and $KE = 36$, determine and state the length of $\overline{AL}$.

$12 + 36 = 48$

$12(48) = x^2$

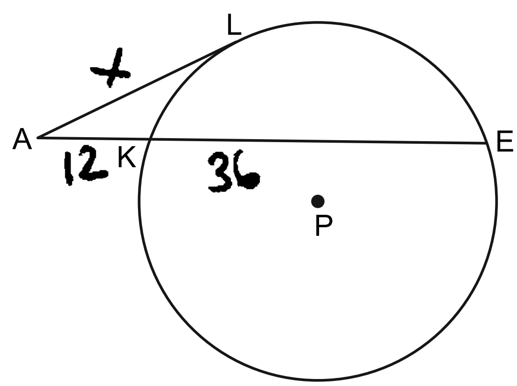
$\sqrt{576} = \sqrt{x^2}$

$x = 24$

$\boxed{AL = 24}$

Score 2: The student gave a complete and correct response.

28 In circle P below, tangent $\overline{AL}$ and secant $\overline{AKE}$ are drawn.



If $AK = 12$ and $KE = 36$, determine and state the length of $\overline{AL}$.

$36+12=48$

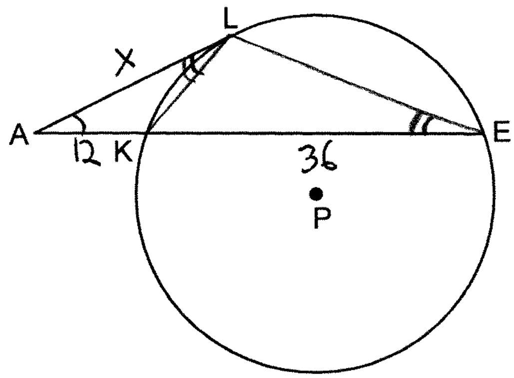
$48 \cdot 12 = x$

$576 = x$

Score 1: The student wrote an incorrect equation in not squaring the tangent length.

Question 28

28 In circle P below, tangent $\overline{AL}$ and secant $\overline{AKE}$ are drawn.



If $AK = 12$ and $KE = 36$, determine and state the length of $\overline{AL}$.

$\triangle ALE \sim \triangle AKL$

$$\frac{x}{12} = \frac{12}{48}$$

$$48x = 144$$

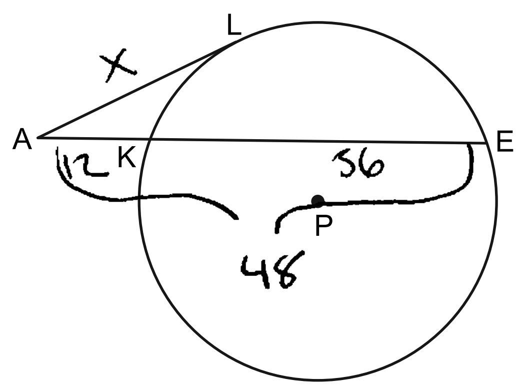
$$\frac{48x}{48} = \frac{144}{48}$$

$$x = 3$$

Score 1: The student wrote an incorrect proportion using 12 as the geometric mean.

Question 28

28 In circle P below, tangent $\overline{AL}$ and secant $\overline{AKE}$ are drawn.



If $AK = 12$ and $KE = 36$, determine and state the length of $\overline{AL}$.

$12 + 36 = 48$

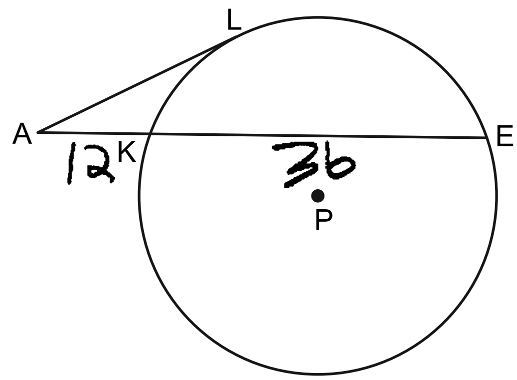
$\frac{48}{2} = 24$

$m\overline{AL} = 24$

Score 0: The student determined a correct answer by an obviously incorrect procedure.

Question 28

28 In circle P below, tangent $\overline{AL}$ and secant $\overline{AKE}$ are drawn.



If $AK = 12$ and $KE = 36$, determine and state the length of $\overline{AL}$.

$(\text{out})(\text{whole}) = \text{tangent}^2$

$(12)(36) = x$

$432 = x$

$\overline{AL} \cong 432$

Score 0: The student made two errors in not using the length of the entire secant and not taking the square root.

Question 29

29 The equation of a circle is $x^2 + y^2 + 8x - 6y + 7 = 0$. Determine and state the coordinates of the center and the length of the radius of the circle.

$$x^2 + y^2 + 8x - 6y \cancel{+7} = 0$$
$$\cancel{-7} \; -7$$

$$x^2 + 8x + 16 + y^2 - 6y + 9 = -7 + 16 + 9$$

$$(x+4)(x+4) + (y-3)(y-3) = 18$$

$$(x+4)^2 + (y-3)^2 = 18$$

Center: $(-4, 3)$

$r: \sqrt{18}$

Score 2: The student gave a complete and correct response.

Question 29

29 The equation of a circle is $x^2 + y^2 + 8x - 6y + 7 = 0$. Determine and state the coordinates of the center and the length of the radius of the circle.

$$x^2+y^2+8x-6y+\cancel{7}=0$$
$$\cancel{-7} \quad -7$$

$$\left(\frac{b}{2}\right)^2$$

$$x^2+8x+y^2-6y=-7$$
$$+16 \quad +9 \quad +25$$

$$x^2+8x+16+y^2-6y+9=18$$
$$(x+4)^2+(y-3)^2=(3\sqrt{2})^2$$

$$\boxed{\begin{array}{l}\text{center: } (-4,3) \\ \text{radius: } 3\sqrt{2}\end{array}}$$

Score 2: The student gave a complete and correct response.

Question 29

29 The equation of a circle is $x^2 + y^2 + 8x - 6y + 7 = 0$. Determine and state the coordinates of the center and the length of the radius of the circle.

$\frac{8}{2} = 4 \quad 4^2 = 16$

$\frac{-6}{2} = -3$

$(-3)^2 = 9$

$x^2 + 8x + \boxed{16} + y^2 - 6y + \boxed{9} = -7 + 16 + 9$

$(x+4)^2 + (y-3)^2 = 18$

$(-4, 3)$ ↑ Center

$\sqrt{18}$ ↑ Radius

Score 2: The student gave a complete and correct response.

Question 29

29 The equation of a circle is $x^2 + y^2 + 8x - 6y + 7 = 0$. Determine and state the coordinates of the center and the length of the radius of the circle.

$$x^2+y^2+8x-6y+7=0$$
$$\underline{\qquad\qquad\qquad -7\ -7}$$
$$x^2+y^2+8x-6y=-7$$
$$x^2+y^2+8x+16-6y+9=-7+16+9$$
$$(x+4)(x+4)+(y-3)(y-3)=18$$

Center $(-4,3)$ Radius : 9

Score 1: The student made an error when determining the length of the radius.

Question 29

29 The equation of a circle is $x^2 + y^2 + 8x - 6y + 7 = 0$. Determine and state the coordinates of the center and the length of the radius of the circle.

$$x^2 + y^2 + 8x - 6y + 7 = 0$$

$$x^2 + 8x \quad + y^2 - 6y = -7$$

$$x^2 + 8x + 16 \qquad y^2 - 6y + 9$$

$$(x+4)(x+4) + (y-3)(y-3) = -7$$

$$(x+4)^2 + (y-3)^2 = -7$$

center = $(-4, 3)$

radius = 7

Score 1: The student determined the center of the circle correctly.

Question 29

29 The equation of a circle is $x^2 + y^2 + 8x - 6y + 7 = 0$. Determine and state the coordinates of the center and the length of the radius of the circle.

$x^2 + y^2 + 8x - 6y + 7 = 0$

$\quad\quad\quad\quad\quad\quad\quad - 7 \quad -7$

$x^2 + 8x + y^2 - 6y = -7 + 64 + 36$

$(x+4)(x-3)$ $\quad \sqrt{93}$

center $= (4, -3)$

RADIUS $= 9$

Score 0: The student student did not show enough correct relevant work to receive any credit.

Question 29

29 The equation of a circle is $x^2 + y^2 + 8x - 6y + 7 = 0$. Determine and state the coordinates of the center and the length of the radius of the circle.

$$x^2 + y^2 + \frac{8x}{2} - \frac{6y}{2} + 7 = 0$$

$$4^2 = 16 \quad -3^2 = 9 \quad -7 \quad -7$$

$$(x + 16)^2 + (y + 9)^2 = -7 + 16 + 9$$

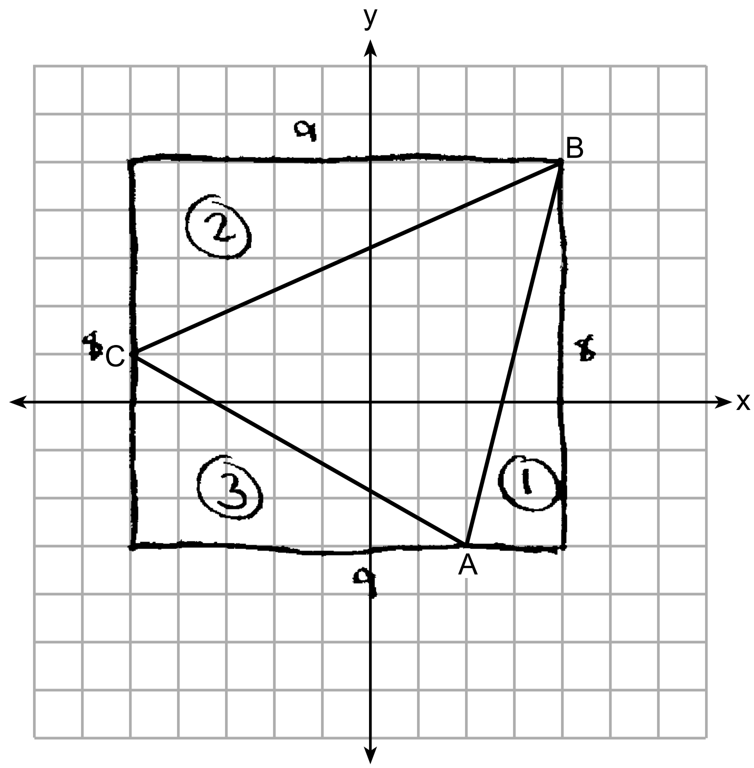
$$(x+16)^2 + (y+9)^2 = 18$$

$$\boxed{\begin{array}{l}\text{Coordinates} = (-16, -9) \\ \text{Radius} = 18\end{array}}$$

Score 0: The student did not show enough correct relevant course-level work to receive any credit.

Question 30

30 On the set of axes below, $\triangle ABC$ is drawn with vertices that have coordinates $A(2,-3)$, $B(4,5)$, and $C(-5,1)$.



Determine and state the area of $\triangle ABC$.

$A = bh$
$A = 9 \cdot 8$
$A = 72$

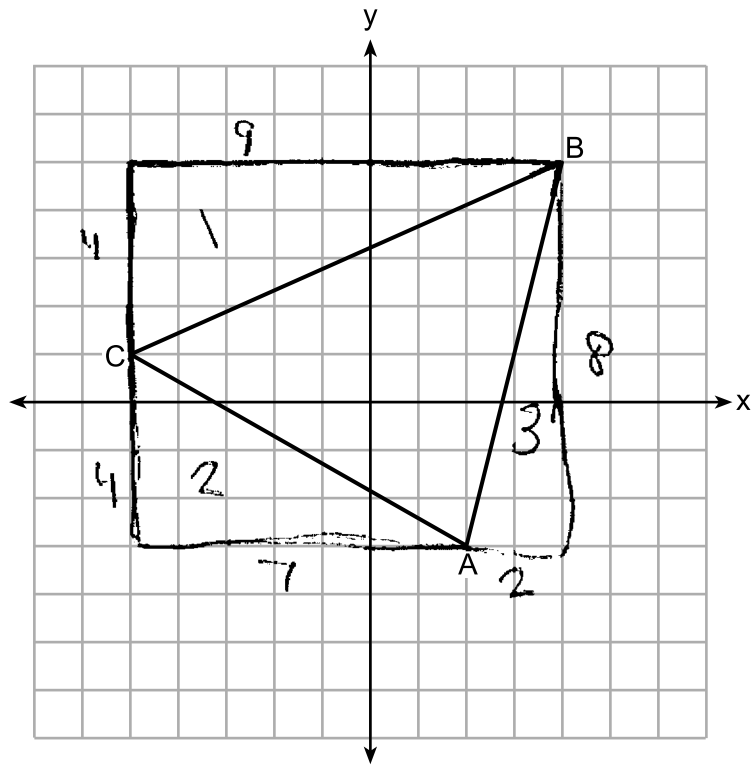
$\Delta 1 = \frac{1}{2} 2(8)$
$= 8$
$\Delta 2 = \frac{1}{2} 4(9)$
$= 18$
$\Delta 3 = \frac{1}{2} 7(4)$
$= 14$

$72 - 8 - 18 - 14$
$\boxed{= 32}$

Score 2: The student gave a complete and correct response.

Question 30

30 On the set of axes below, $\triangle ABC$ is drawn with vertices that have coordinates $A(2,-3)$, $B(4,5)$, and $C(-5,1)$.



Determine and state the area of $\triangle ABC$.

$A_\triangle = A_\square - (A_1 + A_2 + A_3)$

$A = 9 \cdot 8$
$= 72$

$A_\triangle = 72 - (18 + 14 + 8)$

$\boxed{32}$

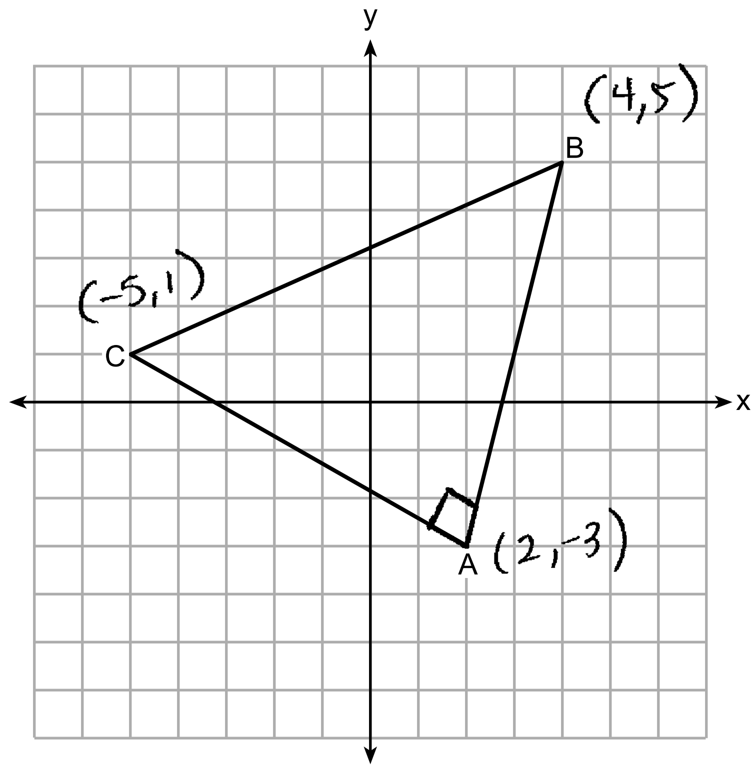
$\triangle 1 - \frac{1}{2} 4(9)$

$\triangle 2 - \frac{1}{2} 7(4)$

$\triangle 3 - \frac{1}{2} 8(2)$

Score 2: The student gave a complete and correct response.

30 On the set of axes below, $\triangle ABC$ is drawn with vertices that have coordinates $A(2,-3)$, $B(4,5)$, and $C(-5,1)$.



Determine and state the area of $\triangle ABC$.

$d\,AB = \sqrt{(4-2)^2 + (5+3)^2}$

$\sqrt{68}$

$d\,AC = \sqrt{(-5-2)^2 + (1+3)^2}$

$= \sqrt{65}$

$A = \frac{1}{2}bh$

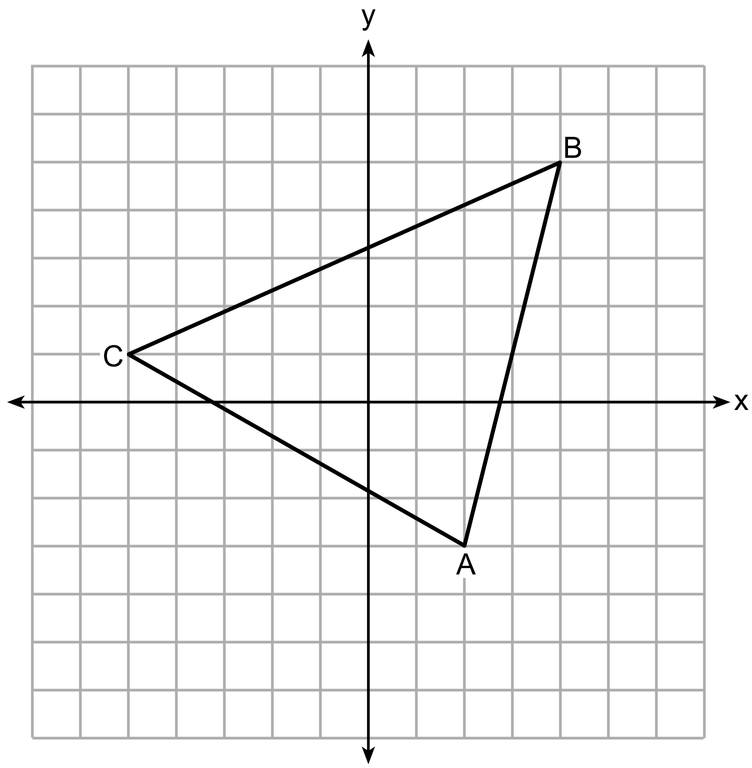
$= \frac{1}{2}(\sqrt{65})(\sqrt{68})$

$= 33.24154028$

Score 1: The student made an error in thinking $\overline{AC}$ was an altitude to $\overline{AB}$.

Question 30

30 On the set of axes below, $\triangle ABC$ is drawn with vertices that have coordinates $A(2,-3)$, $B(4,5)$, and $C(-5,1)$.



Determine and state the area of $\triangle ABC$.

AC

$D = \sqrt{(2+5)^2 + (-3+1)^2}$

$D = \sqrt{(7)^2 + (-2)^2}$

$D = \sqrt{49+4}$

$D = \sqrt{53}$

AB

$D = \sqrt{(4-2)^2 + (5+3)^2}$

$D = \sqrt{(2)^2 + (8)^2}$

$D = \sqrt{4+64}$

$D = \sqrt{68}$

$A = 1/2\sqrt{53}(\sqrt{68})$

$A = 60.033 \div 2$

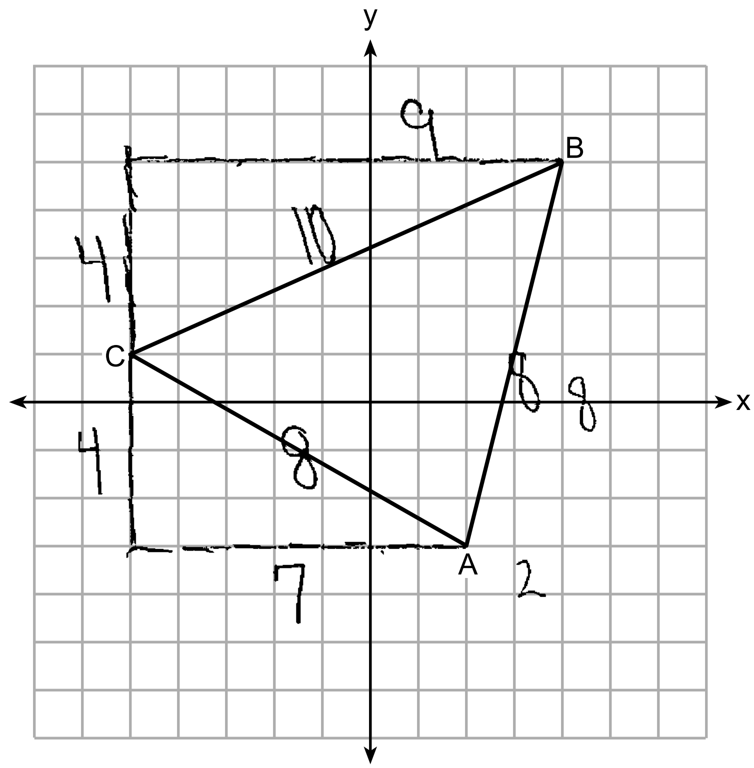
$A = 30.01$

$\boxed{A = 30}$

Score 0: The student made an error in determining the length of $\overline{AC}$. The student made an error in thinking $\overline{AC}$ was an altitude to $\overline{AB}$.

Question 30

30 On the set of axes below, $\triangle ABC$ is drawn with vertices that have coordinates $A(2,-3)$, $B(4,5)$, and $C(-5,1)$.



Determine and state the area of $\triangle ABC$.

$4^2+9^2=c^2$
$16+81=c^2$

$4^2+7^2=c^2$
$16+49=c^2$

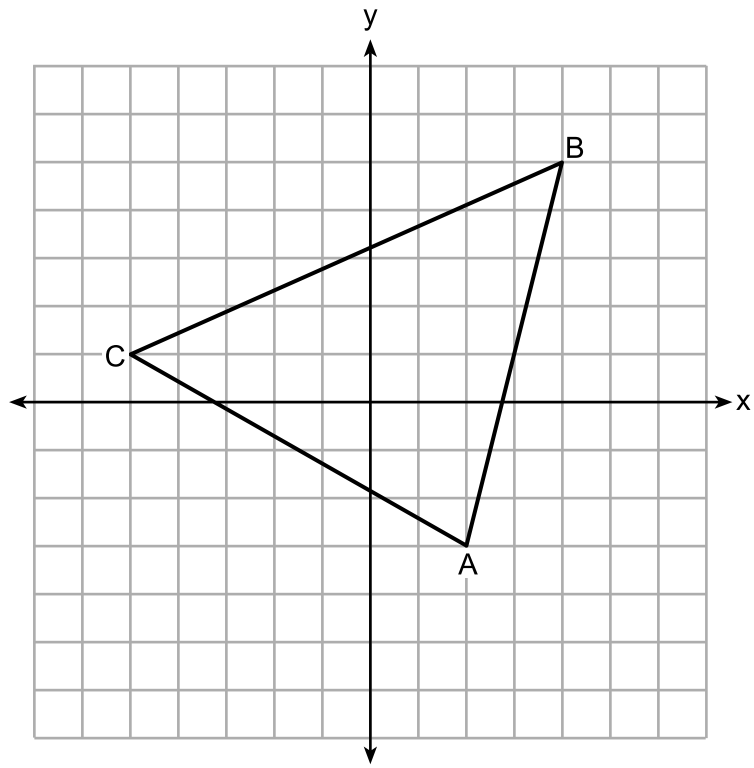
$2^2+8^2=c^2$
$4+64=c^2$

$A=\frac{1}{2}bh$
$A=\frac{1}{2}(8)(8)$
A32

Score 0: The student made an error in thinking $\overline{AC}$ was an altitude to $\overline{AB}$. The student made rounding errors in determining the lengths of $\overline{AC}$ and $\overline{AB}$.

Question 30

30 On the set of axes below, $\triangle ABC$ is drawn with vertices that have coordinates $A(2,-3)$, $B(4,5)$, and $C(-5,1)$.



Determine and state the area of $\triangle ABC$.

$A = \frac{1}{h} Bh$

$B = 7$

$h = 8$

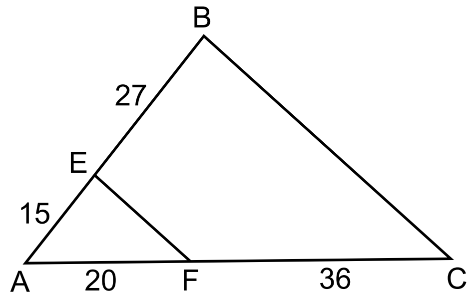
$A = \frac{1}{2}(7)(8)$

$A = 28$

Score 0: The student did not show enough correct relevant grade-level work to receive any credit.

Question 31

31 In the diagram below, $AE = 15$, $EB = 27$, $AF = 20$, and $FC = 36$.



Explain why $\overline{EF} \parallel \overline{BC}$.

$$\frac{15}{27} = \frac{20}{36}$$

$$\frac{5}{9} = \frac{5}{9} \checkmark$$

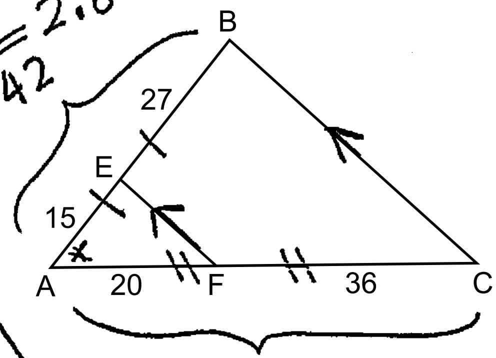
A line segment parallel to one side of a $\triangle$ divides the other 2 sides proportionally.

Since $\overline{EF}$ divides $\overline{AB}$ + $\overline{AC}$ proportionally, $\overline{EF} \parallel \overline{BC}$.

Score 2: The student gave a complete and correct response.

31 In the diagram below, $AE = 15$, $EB = 27$, $AF = 20$, and $FC = 36$.

$\frac{42}{15} = 2.8$ 42



SAS

Explain why $\overline{EF} \parallel \overline{BC}$.

$\frac{56}{20} = 2.8$

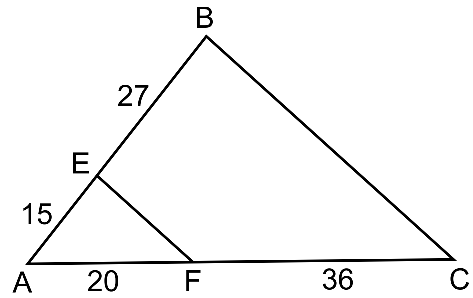
proportional

statements	Reasons
(1) $AE = 15$, $EB = 27$, $AF = 20$, $FC = 36$	(1) Given
(2) $\frac{AB}{AE} = \frac{AC}{AF}$	(2) $\overline{AE} = 15$, $\overline{AB} = 42$, and $\overline{AF} = 20$, $\overline{AC} = 56$, $42 \div 15 = 2.8$, $56 \div 20 = 2.8$, making both sides proportionate.
(3) $\angle EAF \cong \angle BAC$	(3) reflexive property
(4) $\triangle EAF \sim \triangle BAC$	(4) SAS $\sim$
(5) $AEF \cong \angle B$	(5) corresponding angles in similar $\triangle$'s are congruent
(6) $\overline{EF} \parallel \overline{BC}$	(6) congruent corresponding angles gives parallel lines

Score 2: The student gave a complete and correct response.

Question 31

31 In the diagram below, $AE = 15$, $EB = 27$, $AF = 20$, and $FC = 36$.



Explain why $\overline{EF} \parallel \overline{BC}$.

$\frac{15}{27} = \frac{20}{36}$, so $\frac{AE}{EB} = \frac{AF}{FC}$ are proportional.

$540 = 540$

$\angle A$ is a shared angle, so $\angle A \cong \angle A$.

$\triangle AEF \sim \triangle ABC$ by SAS$\sim$

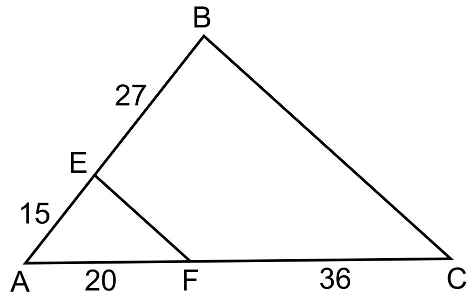
$\angle AEF \cong \angle ABC$, as corresponding $\angle$'s of similar $\triangle$'s are $\cong$.

Since these angles are corresponding congruent angles, $\overline{EF} \parallel \overline{BC}$

Score 2: The student gave a complete and correct response.

Question 31

31 In the diagram below, $AE = 15$, $EB = 27$, $AF = 20$, and $FC = 36$.



Explain why $\overline{EF} \parallel \overline{BC}$.

$\overline{EF}$ + $\overline{BC}$ are parallel because the side splitter theorm proves that the sides are proportional, resulting in parallel lines

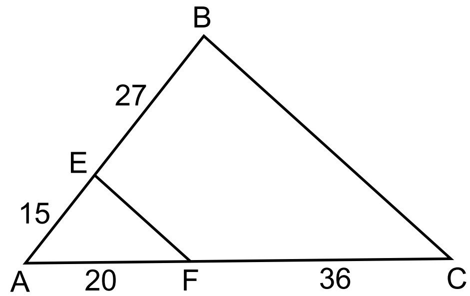
$$\frac{20}{36} = \frac{15}{27}$$

$$540 = 540$$

Score 2: The student gave a complete and correct response.

Question 31

31 In the diagram below, $AE = 15$, $EB = 27$, $AF = 20$, and $FC = 36$.



Explain why $\overline{EF} \parallel \overline{BC}$.

$\frac{15}{20} = \frac{27}{36}$

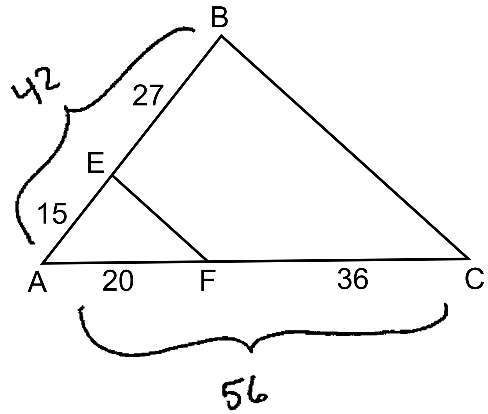
$\frac{3}{4} = \frac{3}{4}$

since the ratio $\frac{15}{20}$ is equal to $\frac{27}{36}$, $\overline{EF}$ and $\overline{BC}$ are congruent.

Score 1: The student wrote a partially correct explanation.

Question 31

31 In the diagram below, $AE = 15$, $EB = 27$, $AF = 20$, and $FC = 36$.



Explain why $\overline{EF} \parallel \overline{BC}$.

because two sides of
$\triangle ABC$ an $\triangle AEF$ are
proportional. Meaning,
the third side is in the
same proportion. ~~xxxx~~
Similar triangles have
parrallel sides.

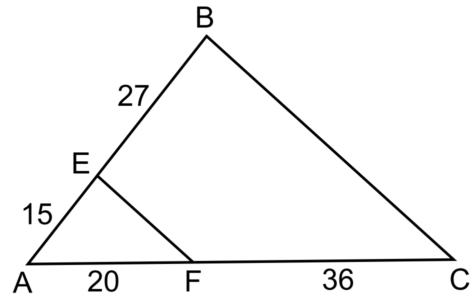
$$\frac{15}{42} = \frac{5}{14}$$

$$\frac{20}{56} = \frac{5}{14}$$

Score 1: The student wrote a partially correct explanation.

Question 31

31 In the diagram below, $AE = 15$, $EB = 27$, $AF = 20$, and $FC = 36$.



Explain why $\overline{EF} \parallel \overline{BC}$.

$\frac{42}{15} = 2.8$

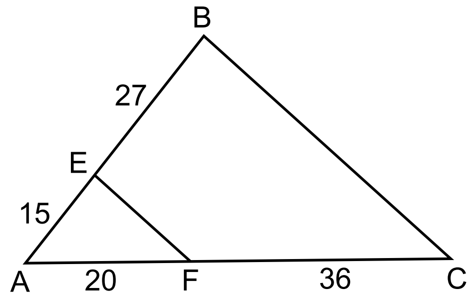
$\frac{56}{20} = 2.8$

$\triangle ABC$ is a dilation of $\triangle AEF$ of scale factor 2.8 Centered at Point A.

Score 1: The student wrote an incomplete explanation.

Question 31

31 In the diagram below, $AE = 15$, $EB = 27$, $AF = 20$, and $FC = 36$.



$$\frac{15}{20} = \frac{42}{56}$$

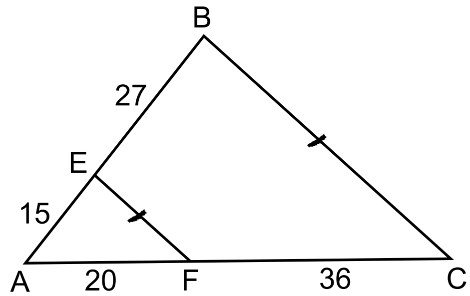
Explain why $\overline{EF} \parallel \overline{BC}$.

$\overline{EF} \parallel \overline{BC}$ because $\triangle AEF \cong \triangle ABC$. so $\overline{EF} \parallel \overline{BC}$ through simular sides of a simular triangle.

Score 0: The student wrote a correct proportion, but no further correct work is shown.

Question 31

31 In the diagram below, $AE = 15$, $EB = 27$, $AF = 20$, and $FC = 36$.



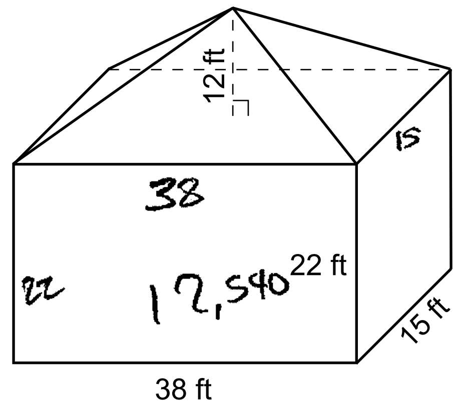
Explain why $\overline{EF} \parallel \overline{BC}$.

$\overline{EF}$ is parallel to $\overline{BC}$ because these two line never cross eachother no matter what.

Score 0: The student did not show enough correct relevant grade-level work to receive any credit.

Question 32

32 A building is composed of a rectangular pyramid on top of a rectangular prism, as shown in the diagram below. The rectangular prism has a length of 38 feet, a width of 15 feet, and a height of 22 feet. The rectangular pyramid sits directly on top of the rectangular prism, and its height is 12 feet.



An air purification filter was installed that will clean all the air in the building at a rate of 2400 cubic feet per minute. Determine and state how long it will take, to the *nearest tenth of a minute*, for the filter to clean the air contained in the building.

Volume - rectangular prism - lwh

$(22)(38)(15) = 12540$

Volume - pyramid - $\frac{1}{3}Bh$

$\left(\frac{1}{3}\right)(38 \times 15)(12) = 2280$

$$\begin{array}{r} 12540 \\ +\ 2280 \\ \hline 14{,}820\ ft^3 \end{array}$$

$\frac{14{,}820\ ft^3}{2{,}400} = 6.175$

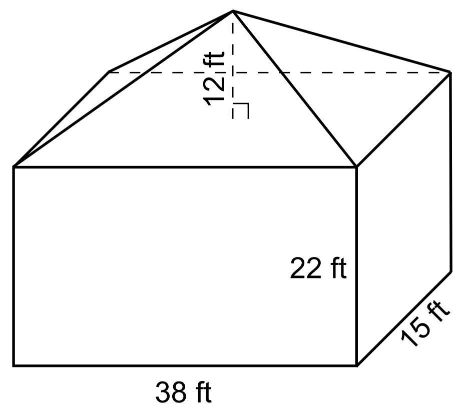
6.2

$\boxed{6.2 \text{ minutes}}$

Score 4: The student gave a complete and correct response.

Question 32

32 A building is composed of a rectangular pyramid on top of a rectangular prism, as shown in the diagram below. The rectangular prism has a length of 38 feet, a width of 15 feet, and a height of 22 feet. The rectangular pyramid sits directly on top of the rectangular prism, and its height is 12 feet.



An air purification filter was installed that will clean all the air in the building at a rate of 2400 cubic feet per minute. Determine and state how long it will take, to the *nearest tenth of a minute*, for the filter to clean the air contained in the building.

$V = lwh$

$V = 38(15)(22)$

$V = 12540$

$V = \frac{1}{3}BH$

$V = \frac{1}{3}(38 \cdot 15)(12)$

$V = 2280$

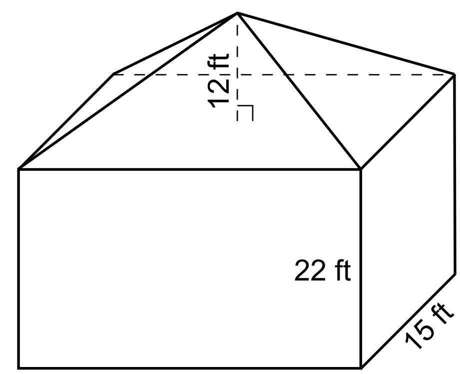
$V = 14820 \text{ ft}^3$

$\frac{14820}{2400} \longrightarrow \boxed{6.2 \text{ minutes}}$

Score 4: The student gave a complete and correct response.

Question 32

32 A building is composed of a rectangular pyramid on top of a rectangular prism, as shown in the diagram below. The rectangular prism has a length of 38 feet, a width of 15 feet, and a height of 22 feet. The rectangular pyramid sits directly on top of the rectangular prism, and its height is 12 feet.



An air purification filter was installed that will clean all the air in the building at a rate of 2400 cubic feet per minute. Determine and state how long it will take, to the *nearest tenth of a minute*, for the filter to clean the air contained in the building.

$V = lwh$

$= 38 \times 22 \times 15$

$= 12{,}540$

$\boxed{6.18 \text{ minutes}}$

~~$V = \frac{1}{3}Bh$~~

$V = \frac{1}{3}Bh$

$= \frac{1}{3}(570)(12)$

$= \frac{1}{3}(6840)$

$= 2280$

$\begin{array}{r} 2280 \\ + \ 12540 \\ \hline 14820 \end{array}$

$\frac{1}{2400} = \frac{x}{14820}$

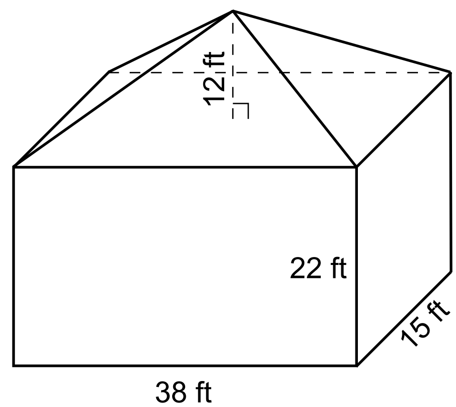
$\frac{2400x}{2400} = \frac{14820}{2400}$

$\boxed{x = 6.175 \text{ minutes}}$

Score 3: The student made one rounding error in determining the time.

Question 32

32 A building is composed of a rectangular pyramid on top of a rectangular prism, as shown in the diagram below. The rectangular prism has a length of 38 feet, a width of 15 feet, and a height of 22 feet. The rectangular pyramid sits directly on top of the rectangular prism, and its height is 12 feet.



An air purification filter was installed that will clean all the air in the building at a rate of 2400 cubic feet per minute. Determine and state how long it will take, to the *nearest tenth of a minute*, for the filter to clean the air contained in the building.

Volume of rect. prism + rect. pyramid

Prism
$V = lwh$
$V = (38)(15)(22)$
$V = 12540\ ft^3$

Pyramid
$V = \frac{1}{3}bh$
$V = \frac{1}{3}(38)(12)$
$V = 152\ ft^3$

$12540 + 152 = 12692\ ft^3$

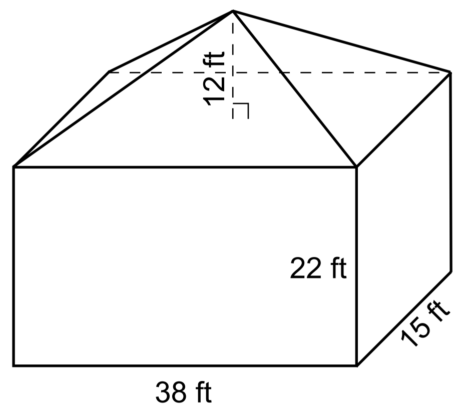
$\frac{12692}{2400} = 5.3$ min

It will take 5.3 minutes to clean the volume of air in the building

Score 3: The student made an error in determining the volume of the pyramid.

Question 32

32 A building is composed of a rectangular pyramid on top of a rectangular prism, as shown in the diagram below. The rectangular prism has a length of 38 feet, a width of 15 feet, and a height of 22 feet. The rectangular pyramid sits directly on top of the rectangular prism, and its height is 12 feet.



An air purification filter was installed that will clean all the air in the building at a rate of 2400 cubic feet per minute. Determine and state how long it will take, to the *nearest tenth of a minute*, for the filter to clean the air contained in the building.

$v=\frac{1}{3}(Bh)$

$v=\frac{1}{3}(l\cdot w\cdot h)$

$v=\frac{1}{3}(38,15\cdot 22)$

$v=\frac{1}{3}(12540)$

$v=4,180 \longrightarrow v=6460$

$v=\frac{1}{3}(Bh)$

$v=\frac{1}{3}(l\cdot w\cdot h)$

$v=\frac{1}{3}(38\cdot 15\cdot 12)$

$v=\frac{1}{3}(6840)$

$v=2280$

$\frac{1}{2400}=\frac{x}{6,460}$

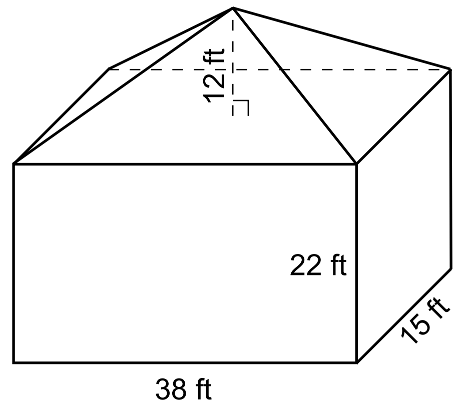
$\frac{2400x}{2,400}=\frac{6460}{2,400}$

$x=2.691$

$x=2.7$

Score 3: The student made an error in determining the volume of the pyramid.

32 A building is composed of a rectangular pyramid on top of a rectangular prism, as shown in the diagram below. The rectangular prism has a length of 38 feet, a width of 15 feet, and a height of 22 feet. The rectangular pyramid sits directly on top of the rectangular prism, and its height is 12 feet.



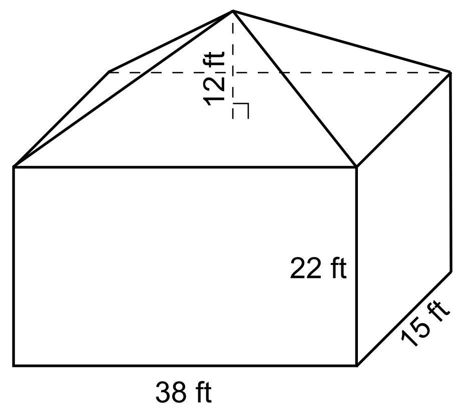
An air purification filter was installed that will clean all the air in the building at a rate of 2400 cubic feet per minute. Determine and state how long it will take, to the *nearest tenth of a minute*, for the filter to clean the air contained in the building.

$V = lwh$	$V = \frac{1}{3}Bh$
$38 \cdot 15 \cdot 22$	$V = \frac{1}{3}lwh$
$V = 12540$	$V = \frac{1}{3}38 \cdot 15 \cdot 12$
	$V = 2280$

Score 2: The student correctly determined the volumes of the prism and the pyramid.

Question 32

32 A building is composed of a rectangular pyramid on top of a rectangular prism, as shown in the diagram below. The rectangular prism has a length of 38 feet, a width of 15 feet, and a height of 22 feet. The rectangular pyramid sits directly on top of the rectangular prism, and its height is 12 feet.



An air purification filter was installed that will clean all the air in the building at a rate of 2400 cubic feet per minute. Determine and state how long it will take, to the *nearest tenth of a minute*, for the filter to clean the air contained in the building.

V= lwh

V=(38 ft)(15 ft)(22 ft)

V=12540

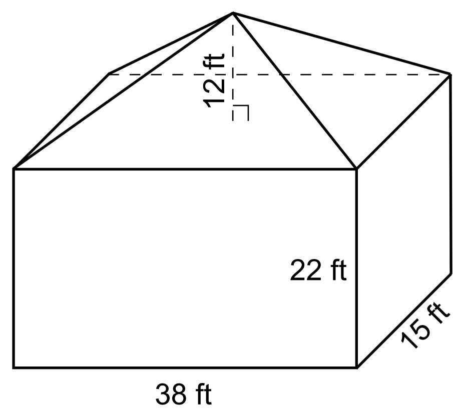
12540/2400

5.2 minutes

Score 2: The student found an appropriate time for the volume of the prism only.

Question 32

32 A building is composed of a rectangular pyramid on top of a rectangular prism, as shown in the diagram below. The rectangular prism has a length of 38 feet, a width of 15 feet, and a height of 22 feet. The rectangular pyramid sits directly on top of the rectangular prism, and its height is 12 feet.



An air purification filter was installed that will clean all the air in the building at a rate of 2400 cubic feet per minute. Determine and state how long it will take, to the *nearest tenth of a minute*, for the filter to clean the air contained in the building.

$V = lwh$

$V = (38)(22)(15)$

$V = 12540$

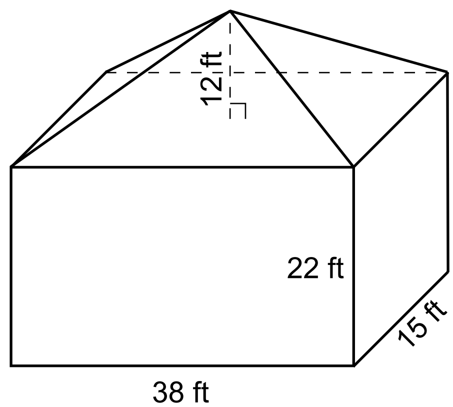
$60 \overline{)12540}$ = 209

$\boxed{\text{209 minutes}}$

Score 1: The student correctly determined the volume of the prism.

Question 32

32 A building is composed of a rectangular pyramid on top of a rectangular prism, as shown in the diagram below. The rectangular prism has a length of 38 feet, a width of 15 feet, and a height of 22 feet. The rectangular pyramid sits directly on top of the rectangular prism, and its height is 12 feet.



An air purification filter was installed that will clean all the air in the building at a rate of 2400 cubic feet per minute. Determine and state how long it will take, to the *nearest tenth of a minute*, for the filter to clean the air contained in the building.

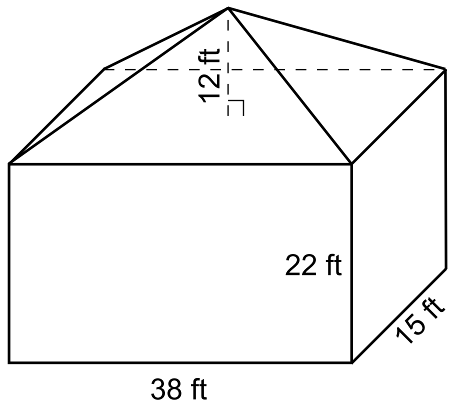
$V = l \cdot w \cdot h$

$2400 = 38 \cdot 15 \cdot 22$

Score 0: The student did not show enough correct relevant course-level work to receive any credit.

Question 32

32 A building is composed of a rectangular pyramid on top of a rectangular prism, as shown in the diagram below. The rectangular prism has a length of 38 feet, a width of 15 feet, and a height of 22 feet. The rectangular pyramid sits directly on top of the rectangular prism, and its height is 12 feet.



An air purification filter was installed that will clean all the air in the building at a rate of 2400 cubic feet per minute. Determine and state how long it will take, to the *nearest tenth of a minute*, for the filter to clean the air contained in the building.

$A = bh$
$= 38(15)$
$= 570$

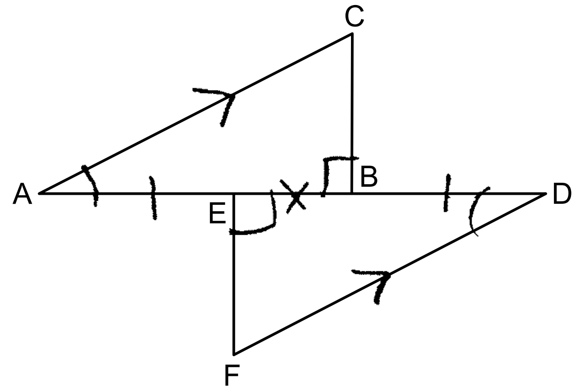
$A = \frac{1}{2}bh$
$= \frac{1}{2}(38)(12)$
$= 228$

598

Score 0: The student did not show enough course-level work to receive any credit.

Question 33

33 Given: $\triangle ABC$, $\triangle DEF$, $\overline{AB} \perp \overline{BC}$, $\overline{DE} \perp \overline{EF}$, $\overline{AE} \cong \overline{DB}$, and $\overline{AC} \parallel \overline{FD}$



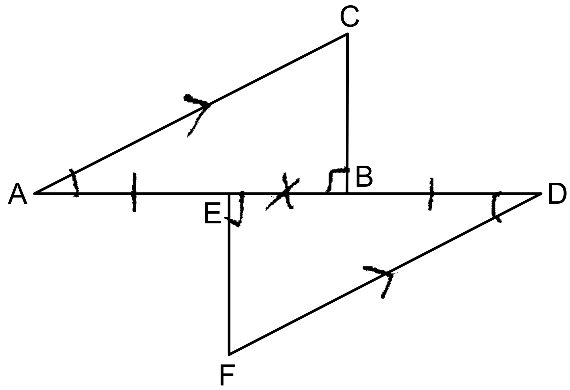
Prove: $\triangle ABC \cong \triangle DEF$

Statement	Reasons
1. $\triangle ABC$, $\triangle DEF$, $\overline{AB} \perp \overline{BC}$, $\overline{DE} \perp \overline{EF}$, $\overline{AE} \cong \overline{DB}$, and $\overline{AC} \parallel \overline{FD}$	1. Given
2. $\angle DEF \cong \angle CBA$	2. Perpendicular lines form congruent right angles.
3. $\angle CAB \cong \angle EDF$	3. If lines are parallel alternate interior angles are congruent.
4. $\overline{EB} \cong \overline{EB}$	4. Reflexive Property
5. $\overline{AE} + \overline{EB} \cong \overline{DB} + \overline{BE}$ $\overline{AB} \cong \overline{ED}$	5. Addition
6. $\triangle ABC \cong \triangle DEF$	6. ASA

Score 4: The student gave a complete and correct response.

Question 33

33 Given: $\triangle ABC$, $\triangle DEF$, $\overline{AB} \perp \overline{BC}$, $\overline{DE} \perp \overline{EF}$, $\overline{AE} \cong \overline{DB}$, and $\overline{AC} \parallel \overline{FD}$



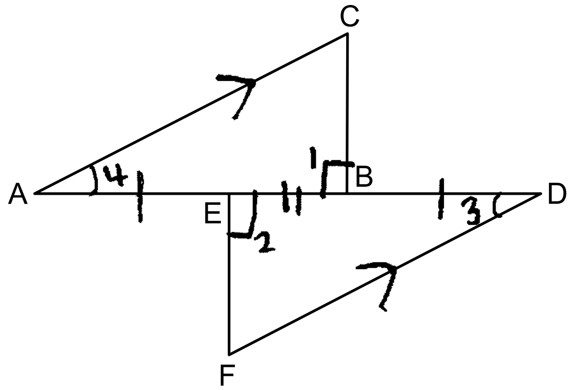
Prove: $\triangle ABC \cong \triangle DEF$

Statements	Reasons
1. $\overline{AB} \perp \overline{BC}$, $\overline{DE} \perp \overline{EF}$, $\overline{AE} \cong \overline{DB}$, $\overline{AC} \parallel \overline{FD}$	1. Given
2. $\overline{EB} \cong \overline{EB}$	2. Reflexive
3. $\overline{AB} \cong \overline{ED}$	3. Addition
4. $\angle CBA$ and $\angle FED$ are right $\angle$'s	4. Def. of $\perp$ lines
5. $\angle CBA \cong \angle FED$	5. All right $\angle$'s $\cong$
6. $\angle CAB \cong \angle EDF$	6. When $\parallel$ lines w/ transversal, alt. int. $\angle$'s $\cong$
7. $\triangle ABC \cong \triangle DEF$	7. ASA

Score 4: The student gave a complete and correct response.

Question 33

33 Given: $\triangle ABC$, $\triangle DEF$, $\overline{AB} \perp \overline{BC}$, $\overline{DE} \perp \overline{EF}$, $\overline{AE} \cong \overline{DB}$, and $\overline{AC} \parallel \overline{FD}$



Prove: $\triangle ABC \cong \triangle DEF$

$\overline{AB} \perp \overline{BC}$, $\overline{DE} \perp \overline{EF}$ — Given

$\angle 1$ and $\angle 2$ are right — $\perp$ lines form right $\angle$s

$\angle 1 \cong \angle 2$ — all right $\angle$s are $\cong$

$\overline{AC} \parallel \overline{FD}$ — given

$\angle 3 \cong \angle 4$ — $\parallel$ lines $\rightarrow \cong$ alt int. $\angle$s

$\overline{AE} \cong \overline{DB}$ — given

$\overline{BE} \cong \overline{BE}$ — reflexive

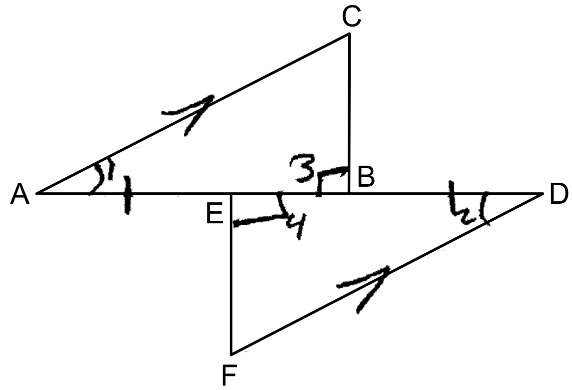
$\overline{AB} \cong \overline{DE}$ — addition

$\triangle ABC \cong \triangle DEF$ — AAS

Score 3: The student had an incorrect reason in proving $\triangle ABC \cong \triangle DEF$.

Question 33

33 Given: $\triangle ABC$, $\triangle DEF$, $\overline{AB} \perp \overline{BC}$, $\overline{DE} \perp \overline{EF}$, $\overline{AE} \cong \overline{DB}$, and $\overline{AC} \parallel \overline{FD}$



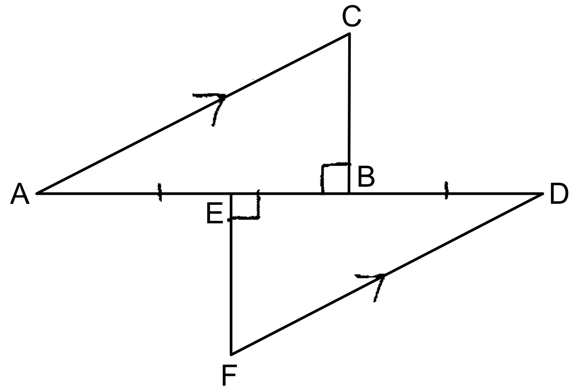
Prove: $\triangle ABC \cong \triangle DEF$

	Statements	Reasons
1	$\triangle ABC$, $\triangle DEF$, $\overline{AB} \perp \overline{BC}$, $\overline{DE} \perp \overline{EF}$, $\overline{AE} \cong \overline{DB}$ and $\overline{AC} \parallel \overline{FD}$	1 given
2	$\measuredangle 1 \cong \measuredangle 2$	2 // Lines → ≅ Alt int ∡s
3	$\measuredangle 3 \cong \measuredangle 4$	3 ⊥ Lines → ≅ Right ∡s
4	$\overline{AB} \cong \overline{ED}$	4 Addition
5	$\triangle ABC \cong \triangle DEF$	5 ASA ≅

Score 3: The student had one missing statement and reason to prove step 4.

Question 33

33 Given: $\triangle ABC$, $\triangle DEF$, $\overline{AB} \perp \overline{BC}$, $\overline{DE} \perp \overline{EF}$, $\overline{AE} \cong \overline{DB}$, and $\overline{AC} \parallel \overline{FD}$



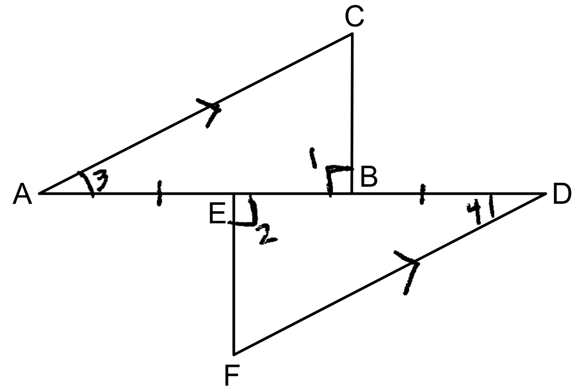
Prove: $\triangle ABC \cong \triangle DEF$

Statement	Reason
① $\overline{AB} \perp \overline{BC}$, $\overline{DE} \perp \overline{EF}$, $\overline{AE} \cong \overline{DB}$, $\overline{AC} \parallel \overline{FD}$	① Given
② $\angle CBA \cong \angle FED$	② Perpendicular lines form ≅ right angles
③ $\overline{EB} \cong \overline{EB}$	③ Reflexive
④ $\overline{AE} + \overline{EB} \cong \overline{EB} + \overline{BD}$	④ Addition
⑤ $\overline{AB} \cong \overline{ED}$	⑤ Partition
⑥ $\angle CAB \cong \angle FDE$	⑥ Alternate Interior Angles
⑦ $\triangle ABC \cong \triangle DEF$	⑦ ASA

Score 2: The student had an incorrect reason in step 5 and an incomplete reason in step 6.

Question 33

33 Given: $\triangle ABC$, $\triangle DEF$, $\overline{AB} \perp \overline{BC}$, $\overline{DE} \perp \overline{EF}$, $\overline{AE} \cong \overline{DB}$, and $\overline{AC} \parallel \overline{FD}$



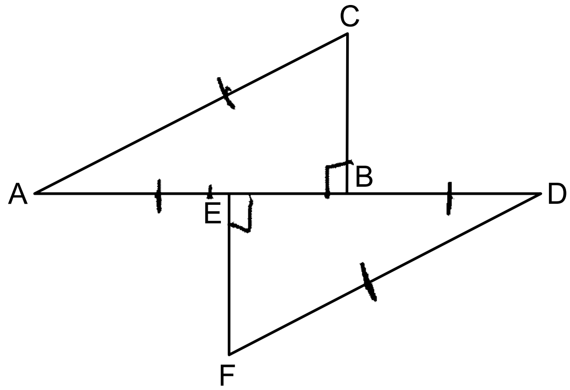
Prove: $\triangle ABC \cong \triangle DEF$

Statement	Reason
① $\triangle ABC, \triangle DEF, \overline{AB} \perp \overline{BC}$, $\overline{DE} \perp \overline{EF}$, $\overline{AE} \cong \overline{DB}$ and $\overline{AC} \parallel \overline{FD}$	① given
② $\angle 1 + \angle 2$ are right $\angle$s	② Def of $\perp$
③ $\angle 3 \cong \angle 4$	③ alt int $\angle$s
④ $\overline{EB} \cong \overline{EB}$	④ reflexive
⑤ $\overline{AE} + \overline{EB} \cong \overline{EB} + \overline{BD}$	⑤ addition
⑥ $\overline{AE} \cong \overline{BD}$	⑥ substitution
⑦ $\triangle ABC \cong \triangle DEF$	⑦ ASA
5a $\overline{AE} + \overline{EB} \cong \overline{AB}$, $\overline{EB} + \overline{BD} \cong \overline{ED}$	5a whole = sum parts

Score 2: The student did not prove $\angle 1 \cong \angle 2$ and had an incomplete reason in step 3.

Question 33

33 Given: $\triangle ABC$, $\triangle DEF$, $\overline{AB} \perp \overline{BC}$, $\overline{DE} \perp \overline{EF}$, $\overline{AE} \cong \overline{DB}$, and $\overline{AC} \parallel \overline{FD}$



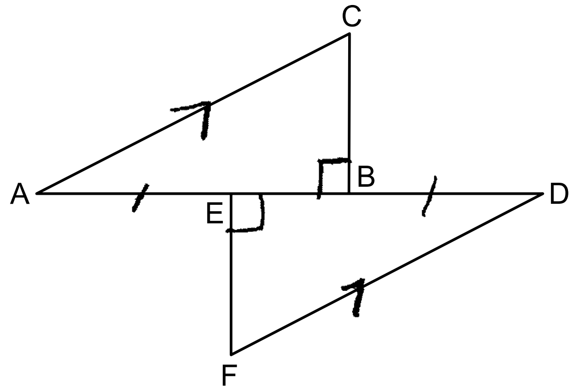
Prove: $\triangle ABC \cong \triangle DEF$

reason	explination
1 - $\overline{AE} \cong \overline{DB}$	1 - given
2 - $\angle CAB \cong \angle EDF$	2 - // lines form $\cong$ alternate interior angles.
3 - $\angle CBA \cong \angle DEF$	3 - $\perp$ lines form 90° angles
4 - $\triangle ABC \cong \triangle DEF$	4 - HL

Score 1: The student had only one correct statement and reason in step 2.

Question 33

33 Given: $\triangle ABC$, $\triangle DEF$, $\overline{AB} \perp \overline{BC}$, $\overline{DE} \perp \overline{EF}$, $\overline{AE} \cong \overline{DB}$, and $\overline{AC} \parallel \overline{FD}$

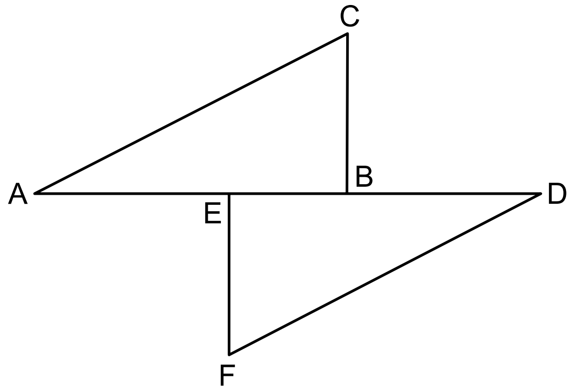


Prove: $\triangle ABC \cong \triangle DEF$

Statement	reason
① $\overline{AB} \perp \overline{BC}$, $\overline{DE} \perp \overline{EF}$, $\overline{AE} \cong \overline{DB}$ $\overline{AC} \parallel \overline{FD}$	① given
② $\angle CBA \cong \angle DEF$	② perp lines form right angles, all right angles are congruent
③	③

Score 1: The student had only one correct statement and reason in step 2.

33 Given: $\triangle ABC$, $\triangle DEF$, $\overline{AB} \perp \overline{BC}$, $\overline{DE} \perp \overline{EF}$, $\overline{AE} \cong \overline{DB}$, and $\overline{AC} \parallel \overline{FD}$



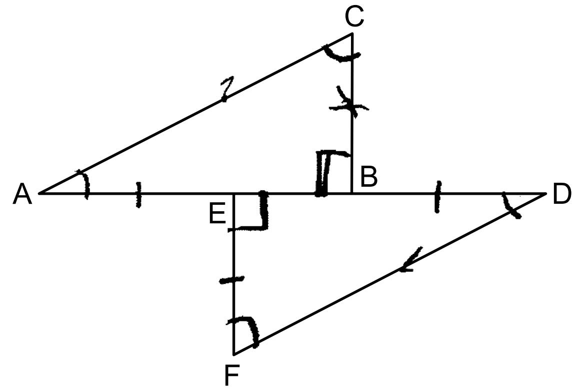
Prove: $\triangle ABC \cong \triangle DEF$

Statements	Reasons
1 $\overline{AB} \perp \overline{BC}$, $\overline{DE} \perp \overline{EF}$, $\overline{AE} \cong \overline{DB}$, $\overline{AC} \parallel \overline{FD}$	1 Given
2 $\overline{FE} \parallel \overline{BC}$	2 RT are $\cong$
3 $\overline{BE}$ bisects $\overline{AB}$	3
4 $\triangle ABC \cong \triangle DEF$	4 ASA

Score 0: The student gave a completely incorrect response.

Question 33

33 Given: $\triangle ABC$, $\triangle DEF$, $\overline{AB} \perp \overline{BC}$, $\overline{DE} \perp \overline{EF}$, $\overline{AE} \cong \overline{DB}$, and $\overline{AC} \parallel \overline{FD}$



Prove: $\triangle ABC \cong \triangle DEF$

s

∠A ≅ ∠D ∠EF ≅ ∠C

∠CB ≅ ∠FE

ABC ≅ EFD

p

alt int ∠'s

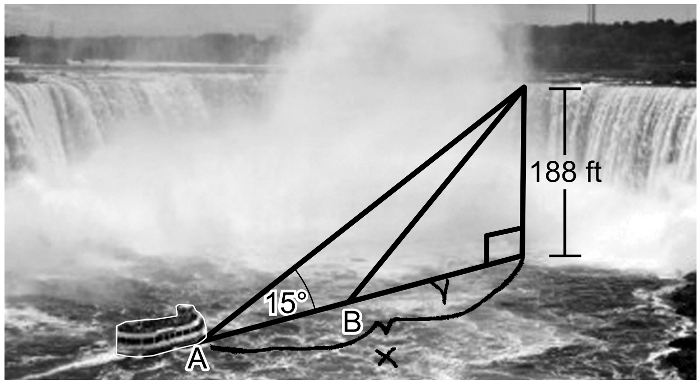
all right angles are ≅

SAS

Score 0: The student did not show enough correct relevant work to receive any credit.

Question 34

34 In the diagram below, a boat at point *A* is traveling toward the most powerful waterfall in North America, the Horseshoe Falls. The Horseshoe Falls has a vertical drop of 188 feet. The angle of elevation from point *A* to the top of the waterfall is 15°.



After the boat travels toward the falls, the angle of elevation at point *B* to the top of the waterfall is 23°. Determine and state, to the *nearest foot*, the distance the boat traveled from point *A* to point *B*.

$\tan 15° = \frac{188}{x}$

$x \tan 15° = 188$

$x = \frac{188}{\tan 15°}$

$= 701.6255$

$\tan 23 = \frac{188}{y}$

$y = 442.9$

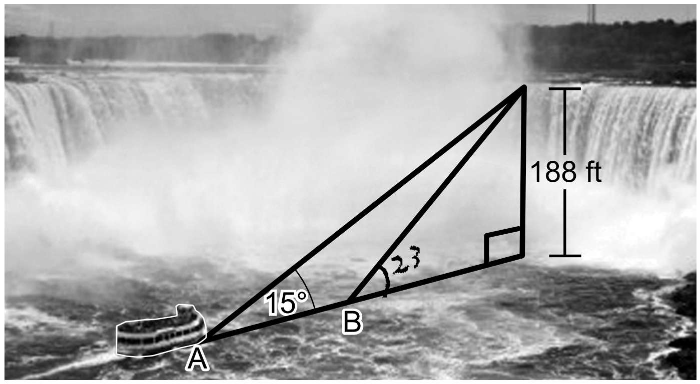
$701.6255 - 442.9002 = 258.7253$

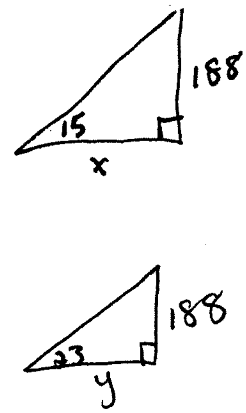
The distance is 259 ft.

Score 4: The student gave a complete and correct response.

Question 34

34 In the diagram below, a boat at point A is traveling toward the most powerful waterfall in North America, the Horseshoe Falls. The Horseshoe Falls has a vertical drop of 188 feet. The angle of elevation from point A to the top of the waterfall is 15°.





After the boat travels toward the falls, the angle of elevation at point B to the top of the waterfall is 23°. Determine and state, to the *nearest foot*, the distance the boat traveled from point A to point B.

$\tan 15° = \frac{188}{x}$

$\tan 15° x = 188$

$x = \frac{188}{\tan 15}$

$x = 701.6$

$\tan 23 = \frac{188}{y}$

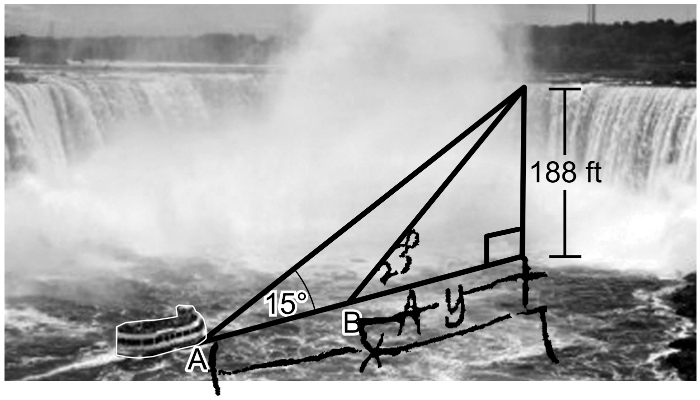
$y = 442.9$

$701.6 - 442.4 = \boxed{259 \text{ feet}}$

Score 4: The student gave a complete and correct response.

Question 34

34 In the diagram below, a boat at point A is traveling toward the most powerful waterfall in North America, the Horseshoe Falls. The Horseshoe Falls has a vertical drop of 188 feet. The angle of elevation from point A to the top of the waterfall is 15°.



After the boat travels toward the falls, the angle of elevation at point B to the top of the waterfall is 23°. Determine and state, to the *nearest foot*, the distance the boat traveled from point A to point B.

from point A

$\frac{\tan 15}{1} = \frac{188}{x}$

$\frac{188}{\tan 15} = \frac{\tan 15 x}{\tan 15}$

$x = 701.626\text{ft}$

$\frac{\tan 23}{1} = \frac{188}{y}$

$\frac{188}{\tan 23} = \frac{\tan 23 y}{\tan 23}$

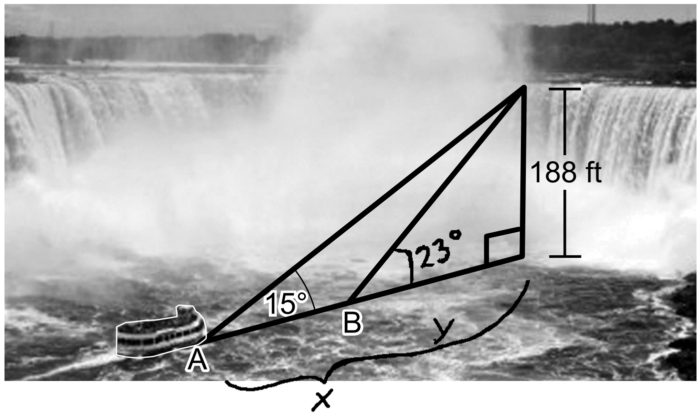
$y = 442.900\text{ft}$

$\begin{array}{r} 701.626 \\ -\,442.900 \\ \hline 258.726\text{ft} \end{array}$ from A to B

Score 3: The student made a rounding error.

Question 34

34 In the diagram below, a boat at point A is traveling toward the most powerful waterfall in North America, the Horseshoe Falls. The Horseshoe Falls has a vertical drop of 188 feet. The angle of elevation from point A to the top of the waterfall is 15°.



After the boat travels toward the falls, the angle of elevation at point B to the top of the waterfall is 23°. Determine and state, to the *nearest foot*, the distance the boat traveled from point A to point B.

$$\tan 15 = \frac{188}{x} = 701$$

$$\tan 23 = \frac{188}{y} = 443$$

$$701 - 443 = 258$$

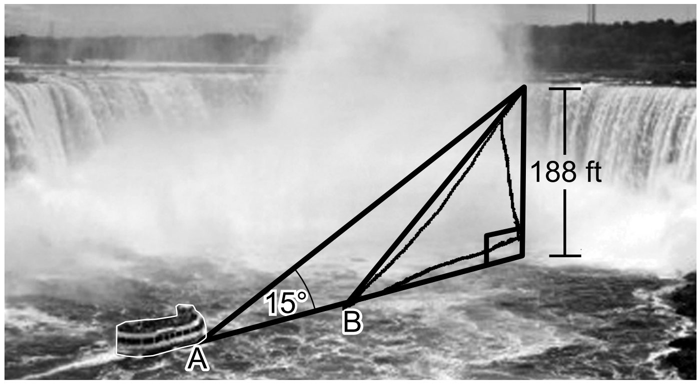
258 ft

Score 3: The student made a rounding error.

Question 34

34 In the diagram below, a boat at point *A* is traveling toward the most powerful waterfall in North America, the Horseshoe Falls. The Horseshoe Falls has a vertical drop of 188 feet. The angle of elevation from point *A* to the top of the waterfall is 15°.

$S\frac{O}{H}\ C\frac{A}{H}\ T\frac{O}{A}$



After the boat travels toward the falls, the angle of elevation at point *B* to the top of the waterfall is 23°. Determine and state, to the *nearest foot*, the distance the boat traveled from point *A* to point *B*.

$$\tan 15 = \frac{188}{x}$$

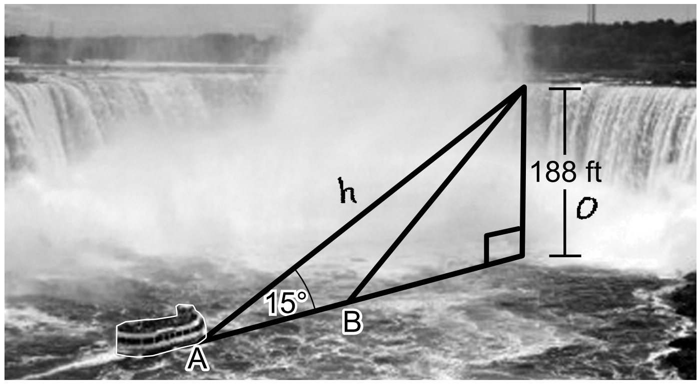
$$\frac{\tan 15\, x}{\tan 15} = \frac{188}{\tan 15}$$

$$x = 701.6255$$

Score 2: The student correctly determined the distance from point *A* to the base of the waterfall, but no further correct work is shown.

Question 34

34 In the diagram below, a boat at point A is traveling toward the most powerful waterfall in North America, the Horseshoe Falls. The Horseshoe Falls has a vertical drop of 188 feet. The angle of elevation from point A to the top of the waterfall is 15°.



After the boat travels toward the falls, the angle of elevation at point B to the top of the waterfall is 23°. Determine and state, to the *nearest foot*, the distance the boat traveled from point A to point B.

$$x \cdot \tan 15° = \frac{188}{x} \cdot x$$

$$\frac{x \tan 15°}{\tan 15°} = \frac{188}{\tan 15°}$$

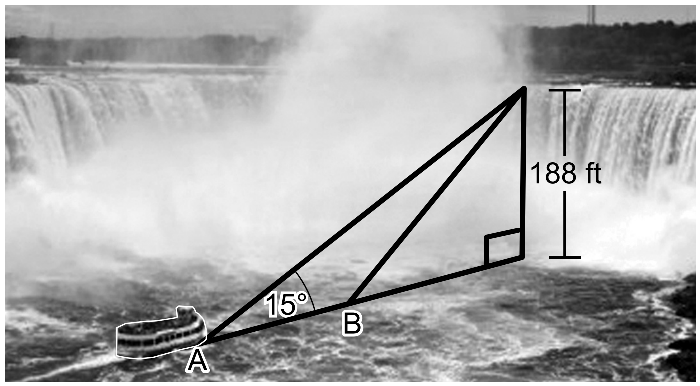
$$x = 701.625$$

702 ft from point A to B

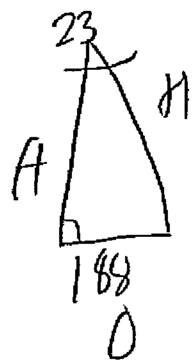
Score 2: The student correctly determined the distance from point A to the base of the waterfall, but no further correct work is shown.

Question 34

34 In the diagram below, a boat at point A is traveling toward the most powerful waterfall in North America, the Horseshoe Falls. The Horseshoe Falls has a vertical drop of 188 feet. The angle of elevation from point A to the top of the waterfall is 15°.



After the boat travels toward the falls, the angle of elevation at point B to the top of the waterfall is 23°. Determine and state, to the *nearest foot*, the distance the boat traveled from point A to point B.



$\boxed{x = 443ft}$

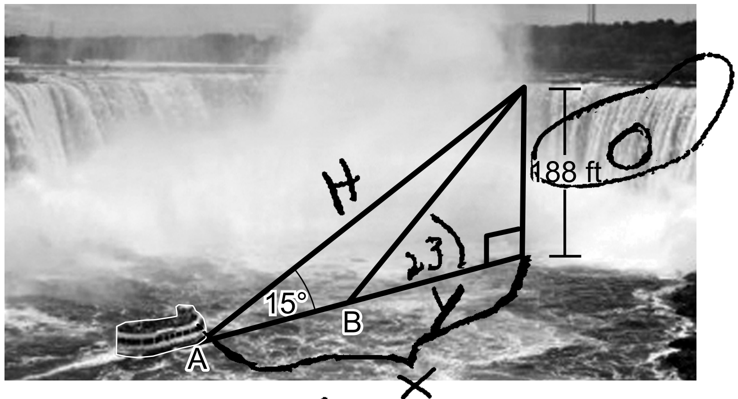
$$\frac{\tan 23}{1} = \frac{188}{x}$$

They are 443 ft away from the waterfall

Score 2: The student correctly determined the distance from point B to the base of the waterfall, but no further correct work is shown.

Question 34

34 In the diagram below, a boat at point *A* is traveling toward the most powerful waterfall in North America, the Horseshoe Falls. The Horseshoe Falls has a vertical drop of 188 feet. The angle of elevation from point *A* to the top of the waterfall is 15°.



A

After the boat travels toward the falls, the angle of elevation at point *B* to the top of the waterfall is 23°. Determine and state, to the *nearest foot*, the distance the boat traveled from point *A* to point *B*.

SOH CAH TOA

$\frac{\cancel{\tan 15}^{\,0.2679}}{1} \times \frac{118}{x}$

$\frac{\cancel{\tan 23}^{\,0.4245}}{1} \times \frac{118}{y}$

$\frac{0.2679x}{0.2679} = \frac{118}{0.2679}$

$\frac{0.4245y}{0.4245} = \frac{118}{0.4245}$

$x = 440.5$

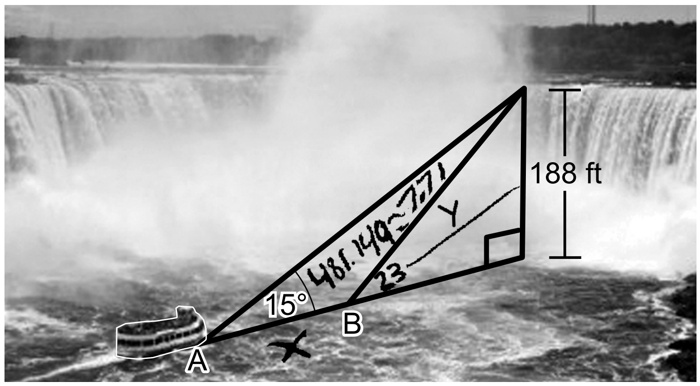
$y = 277.9$

$$\begin{array}{r} 440.5 \\ -277.9 \\ \hline \end{array}$$

$AB = 162.6$

Score 2: The student made a transposition error in stating the height was 118. The student made the same rounding error multiple times.

34 In the diagram below, a boat at point *A* is traveling toward the most powerful waterfall in North America, the Horseshoe Falls. The Horseshoe Falls has a vertical drop of 188 feet. The angle of elevation from point *A* to the top of the waterfall is 15°.



After the boat travels toward the falls, the angle of elevation at point *B* to the top of the waterfall is 23°. Determine and state, to the *nearest foot*, the distance the boat traveled from point *A* to point *B*.

$$\frac{\sin(23)}{1} = \frac{188}{Y}$$

$$\frac{\sin(23) \cdot Y}{\sin(23)} = \frac{188}{\sin(23)}$$

$$Y = 481.1492771$$

$$\frac{\tan 15}{1} = \frac{481.1492771}{X}$$

$$\frac{\tan 15 \cdot X}{\tan 15} = \frac{481.1492771}{\tan 15}$$

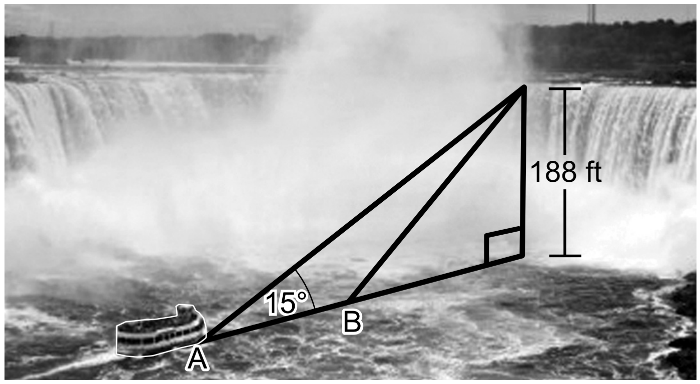
$$\boxed{1795.673548 \text{ ft}}$$

$$\boxed{X = 1796 \text{ ft from point } A - \text{pont } B}$$

Score 2: The student made a conceptual error in using right triangle trigonometry in a non-right triangle.

Question 34

34 In the diagram below, a boat at point A is traveling toward the most powerful waterfall in North America, the Horseshoe Falls. The Horseshoe Falls has a vertical drop of 188 feet. The angle of elevation from point A to the top of the waterfall is 15°.



After the boat travels toward the falls, the angle of elevation at point B to the top of the waterfall is 23°. Determine and state, to the *nearest foot*, the distance the boat traveled from point A to point B.

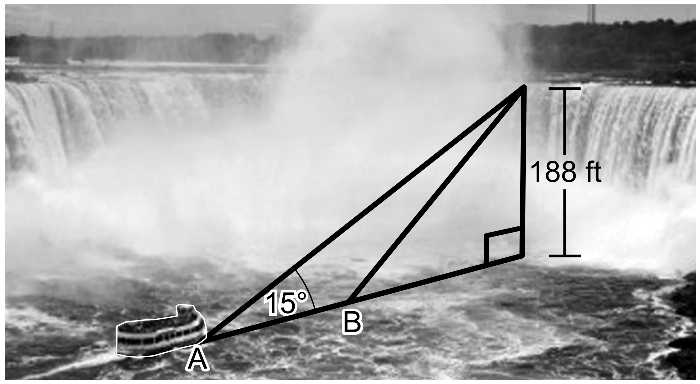
$$(188)\tan 15 = \frac{\cancel{188}}{X}\cancel{(188)}$$

$$50.3744 = X$$

Score 1: The student wrote one correct relevant trigonometric equation.

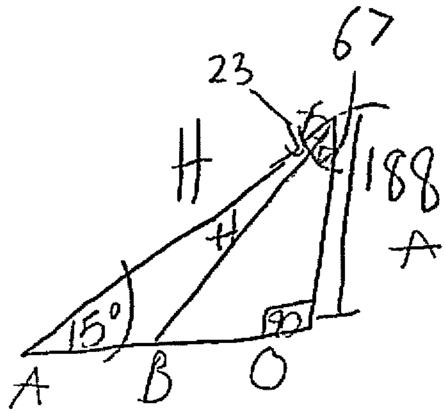
Question 34

34 In the diagram below, a boat at point *A* is traveling toward the most powerful waterfall in North America, the Horseshoe Falls. The Horseshoe Falls has a vertical drop of 188 feet. The angle of elevation from point *A* to the top of the waterfall is 15°.



After the boat travels toward the falls, the angle of elevation at point *B* to the top of the waterfall is 23°. Determine and state, to the *nearest foot*, the distance the boat traveled from point *A* to point *B*.

482 feet



39073 $\frac{\cos(67)}{1} = \frac{188}{H}$

Score 0: The student did not show enough correct relevant work to receive any credit.

35 Triangle *JOE* has vertices whose coordinates are *J*(4,6), *O*(−2,4), and *E*(6,0).

Prove that $\triangle JOE$ is isosceles.

[The use of the set of axes on the next page is optional.]

$D_{JO}: \sqrt{(-2-4)^2+(4-6)^2}$

$\sqrt{(-6)^2+(-2)^2}$

$\sqrt{36+4}$

$\sqrt{40}$

$D_{OE}: \sqrt{(6+2)^2+(-4)^2}$

$\sqrt{64+16}$

$\sqrt{80}$

$D_{JE}: \sqrt{(4-6)^2+(0-6)^2}$

$\sqrt{(-2)^2+(-6)^2}$

$\sqrt{4+36}$

$\sqrt{40}$

$\triangle JOE$ is an isosceles $\triangle$ because it has 2 congruent sides, $\overline{JO} \cong \overline{JE}$, therefore it follows the definition of an isosceles $\triangle$.

Question 35 is continued on the next page.

Score 6: The student gave a complete and correct response.

Point $Y(2,2)$ is on $\overline{OE}$.

Prove that $\overline{JY}$ is the perpendicular bisector of $\overline{OE}$.

$\overline{JY}$ is the perpendicular bisector of $\overline{OE}$ b/c y is the midpoint of $\overline{OE}$ & $\overline{JY}$ and $\overline{OE}$ have neg. reciprical slopes so $\overline{JY} \perp \overline{OE}$.

Slope $\overline{OE}$: $\frac{0-4}{6+2} \rightarrow -\frac{4}{8} \rightarrow -1/2$

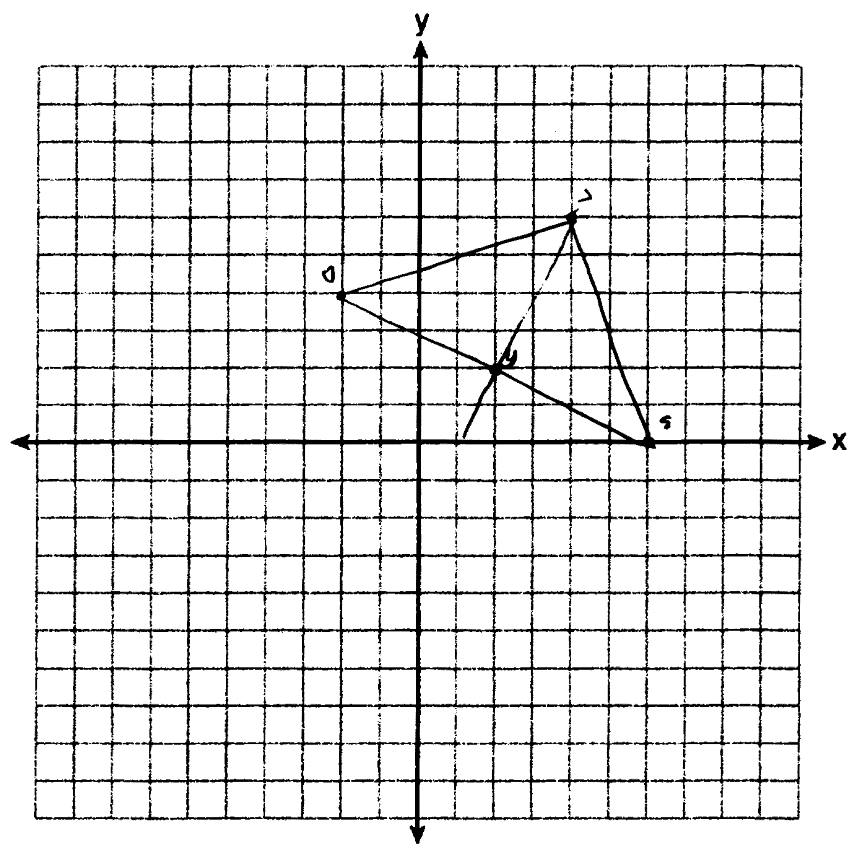
Slope $\overline{JY}$: $\frac{2-6}{2-4} \rightarrow \frac{-4}{-2} \rightarrow \rightarrow 2$

$\left(\frac{6-2}{2}, \frac{0+4}{2}\right)$

$\left(\frac{4}{2}, \frac{4}{2}\right)$

$\downarrow$

$y(2,2)$



Question 35

35 Triangle JOE has vertices whose coordinates are $J(4,6)$, $O(-2,4)$, and $E(6,0)$.

Prove that $\triangle JOE$ is isosceles.

[The use of the set of axes on the next page is optional.]

Plan: Find 2 sides with the same distance.

work:

$JO = \sqrt{(-2-4)^2 + (4-6)^2}$

$JO = \sqrt{(-6)^2 + (2)^2}$

$JO = \sqrt{36+4} = \sqrt{40}$

$JE = \sqrt{(6-4)^2 + (0-6)^2}$

$JE = \sqrt{(2)^2 + (-6)^2} =$

$JE = \sqrt{4+36}$

$JE = \sqrt{40}$

Conclusion Triangle JOE is isosceles because ~~JE~~ the distance of $\overline{JE}$ and $\overline{JO}$ are =.

Question 35 is continued on the next page.

Score 6: The student gave a complete and correct response.

Point $Y(2,2)$ is on $\overline{OE}$.

Prove that $\overline{JY}$ is the perpendicular bisector of $\overline{OE}$.

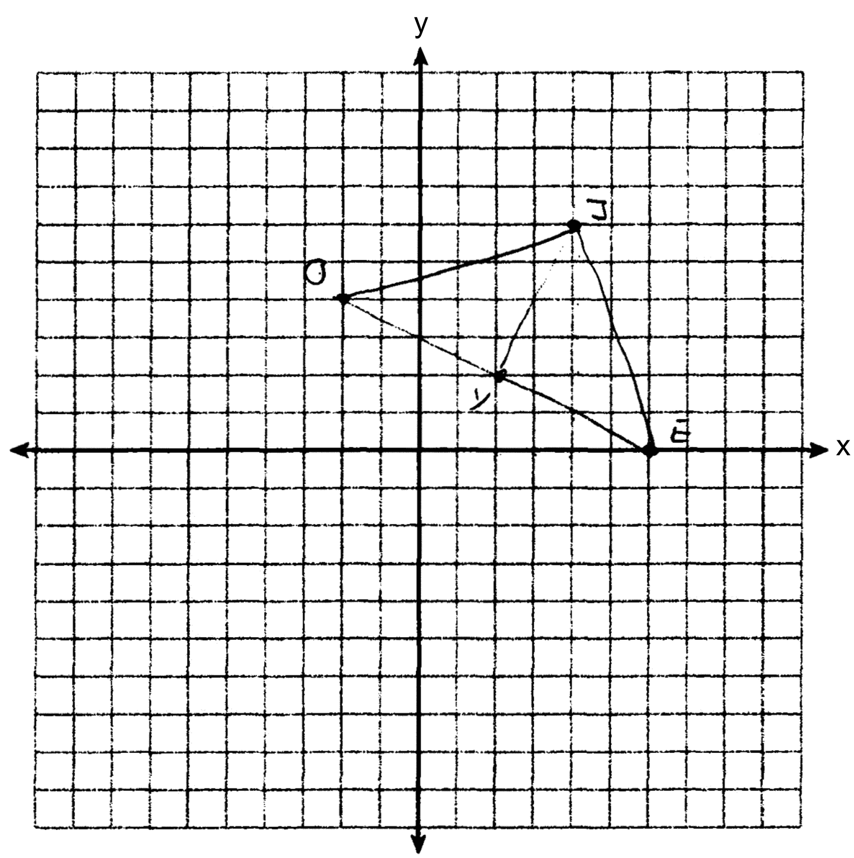
Plan: Find the distance of $\overline{OY}$ & $\overline{YE}$
Find the neg reciprical slope of $\overline{JY}$ & $\overline{YE}$

Work: $\sqrt{(2+2)^2+(2-4)^2} = \sqrt{(4)^2+(-2)^2} = \sqrt{16+4} = \sqrt{20} = OY$

$\sqrt{(6-2)^2+(0-2)^2} = \sqrt{(4)^2+(-2)^2} = \sqrt{16+4} = \sqrt{20} = YE$

Slope $\overline{JY}$: $\frac{2-6}{2-4} = \frac{4}{2}$ Slope $\overline{YE} = \frac{0-2}{6-2} = -\frac{2}{4}$

Conclusion: $\angle JYE$ is a right $\angle$ because $\overline{JY}$ and $\overline{YE}$'s slopes are negitive recipricals of each other making $\overline{JY}$ perpendicular to $\overline{YE}$. $\overline{JY}$ bisects $\overline{OE}$ because $\overline{OY}$ & $\overline{YE}$ are $\cong$, and bisectors split a segment into 2 $\cong$ parts.



Question 35

35 Triangle *JOE* has vertices whose coordinates are $J(4,6)$, $O(-2,4)$, and $E(6,0)$.

Prove that $\triangle JOE$ is isosceles.

[The use of the set of axes on the next page is optional.]

$d\,\overline{JO}$

$$2^2+6^2=x^2$$
$$4+36=x^2$$
$$\sqrt{40}=\sqrt{x^2}$$
$$\sqrt{40}=x$$
$$d\,\overline{JO}=\sqrt{40}$$

$d\,\overline{EJ}$

$$2^2+6^2=x^2$$
$$4+36=x^2$$
$$\sqrt{40}=\sqrt{x^2}$$
$$\sqrt{40}=x$$
$$d\,\overline{EJ}=\sqrt{40}$$

$\triangle$ JOE is an isosceles $\triangle$ because it has 2 $\cong$ sides.

Question 35 is continued on the next page.

Score 5: The student did not write a concluding statement when proving $\overline{JY}$ bisects $\overline{OE}$.

Point $Y(2,2)$ is on $\overline{OE}$.

Prove that $\overline{JY}$ is the perpendicular bisector of $\overline{OE}$.

$d\ \overline{OY} \Rightarrow \quad 2^2 + 4^2 = x^2$

$4 + 16 = x^2$

$\sqrt{20} = \sqrt{x^2}$

$\overline{OY} = \sqrt{20}$

$d\ \overline{EY} \Rightarrow \quad 2^2 + 4^2 = x^2$

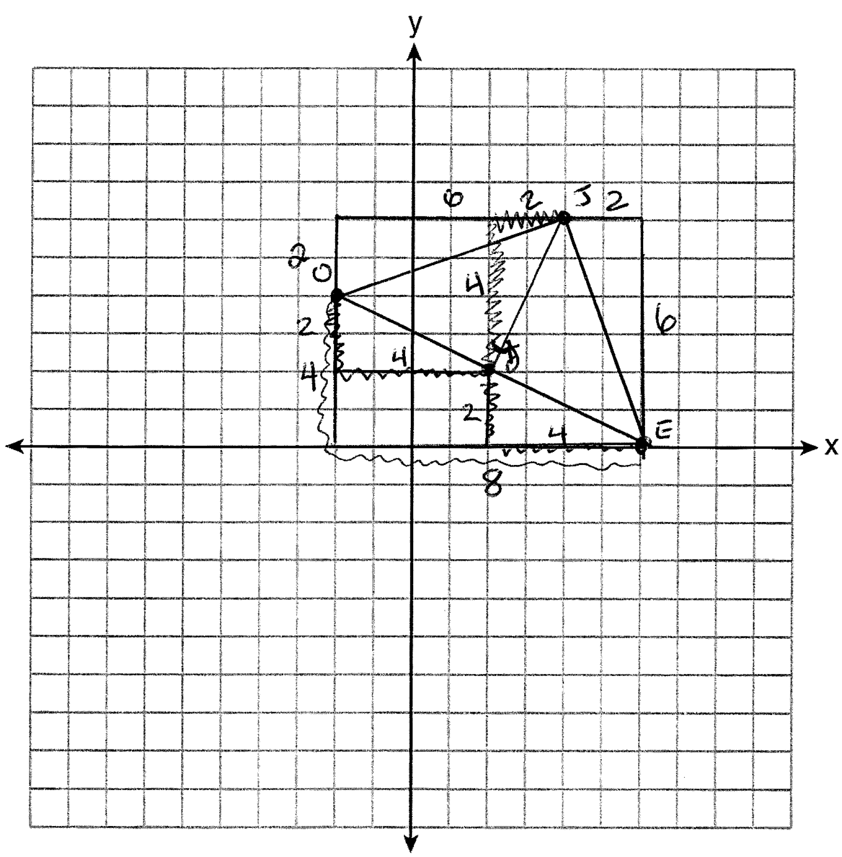
$4 + 16 = x^2$

$\sqrt{20} = \sqrt{x^2}$

$EY = \sqrt{20}$

$\overline{OE}$ is $\perp$ to $\overline{JY}$ because $\overline{OE}$ and $\overline{JY}$ have opposite signed reciprocal slopes.

$m\ \overline{OE} = \frac{-4}{8} = \frac{-1}{2} \quad m\ \overline{YJ} = \frac{4}{2} = 2$



Question 35

35 Triangle JOE has vertices whose coordinates are $J(4,6)$, $O(-2,4)$, and $E(6,0)$.

Prove that $\triangle JOE$ is isosceles.

[The use of the set of axes on the next page is optional.]

$$JO = \sqrt{2^2 + 6^2} = \sqrt{40}$$

$$JE = \sqrt{2^2 + 6^2} = \sqrt{40}$$

Question 35 is continued on the next page.

Score 5: The student did not write a concluding statement when proving $\triangle JOE$ was isosceles.

Point $Y(2,2)$ is on $\overline{OE}$.

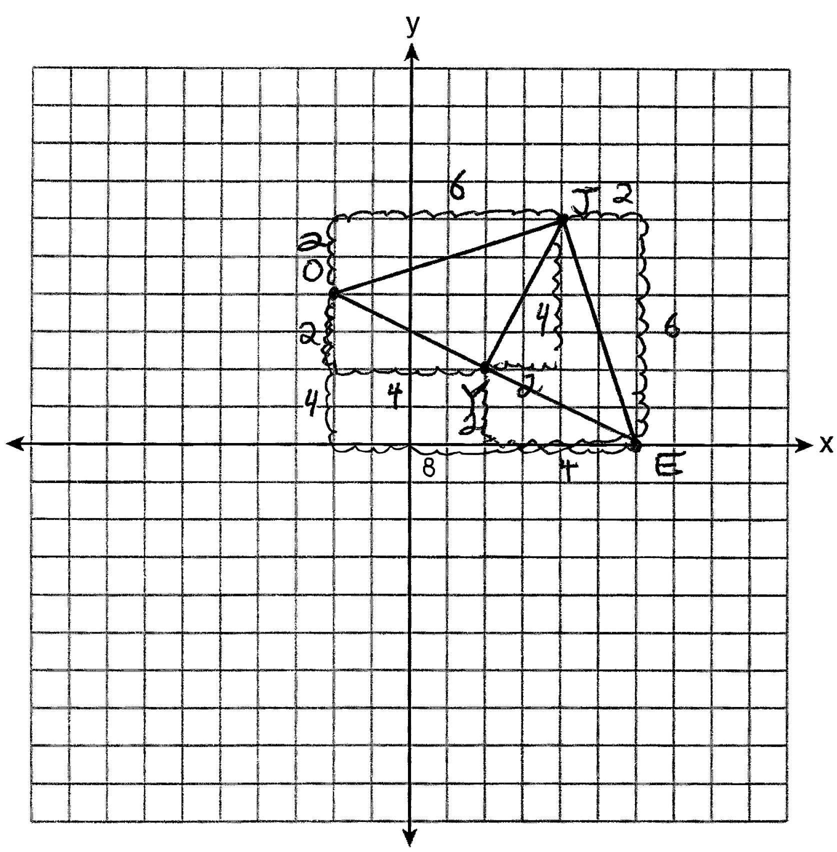
Prove that $\overline{JY}$ is the perpendicular bisector of $\overline{OE}$.

slope of $\overline{JY}$: $\frac{4}{2} = 2$ slope of $\overline{OE}$: $\frac{-4}{8} = -\frac{1}{2}$

$\overline{JY} \perp \overline{OE}$ because their slopes are neg. reciprocals.

$OY = \sqrt{2^2+4^2} = \sqrt{20}$ $EY = \sqrt{2^2+4^2} = \sqrt{20}$

$\overline{JY}$ is the perpendicular bisector of $\overline{OE}$ since $\overline{JY} \perp \overline{OE}$ and $\overline{OY} \cong \overline{EY}$.



Question 35

35 Triangle *JOE* has vertices whose coordinates are $J(4,6)$, $O(-2,4)$, and $E(6,0)$.

Prove that $\triangle JOE$ is isosceles.

[The use of the set of axes on the next page is optional.]

Distance formula 3x

$d \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$JO \rightarrow \sqrt{(-2-4)^2 + (4-6)^2}$

$\sqrt{36+4}$

$\sqrt{40}$

$JE \rightarrow \sqrt{(6-4)^2 + (0-6)^2}$

$\sqrt{4+36}$

$\sqrt{40}$

$OE \rightarrow \sqrt{(6+2)^2 + (0-4)^2}$

$\sqrt{64+16}$

$\sqrt{80}$

$\sqrt{40} = \sqrt{40} \neq \sqrt{80}$

$\triangle JOE$ is isosceles b/c it has one pair of congruent legs, lengths $\overline{JO}$ and $\overline{JE}$.

Question 35 is continued on the next page.

Score 4: The student made a computational error when determining the slope of $\overline{OE}$ and wrote an incorrect concluding statement when proving $\overline{JY} \perp \overline{OE}$.

Point $Y(\overset{x^1}{2},\overset{x^2}{2})$ is on $\overline{OE}$.

Prove that $\overline{JY}$ is the perpendicular bisector of $\overline{OE}$.

Distance 2x

$OY \rightarrow \sqrt{(2+2)^2+(2-4)^2}$

$\sqrt{16+4}$

$\sqrt{20}$

$YE \rightarrow \sqrt{(6-2)^2+(0-2)^2}$

$\sqrt{16+4}$

$\sqrt{20}$

Point Y is a midpoint of $\overline{OE}$ because

$\overline{OY} \cong \overline{YE}$

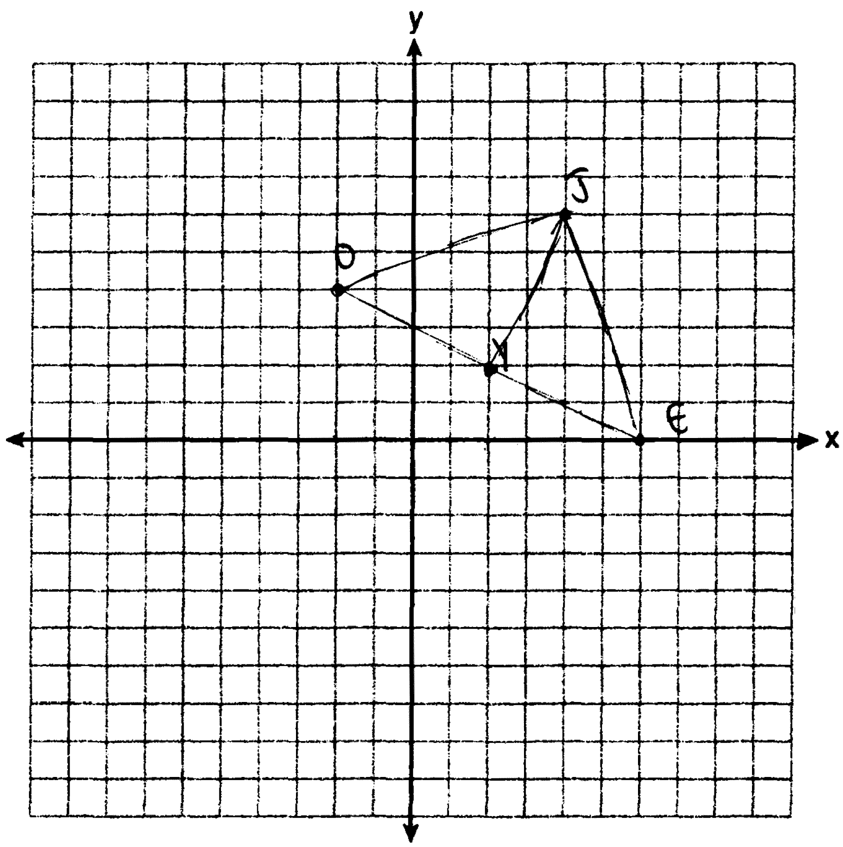
Slope formula 2x to show neg reciprocal

$\frac{y_2-y_1}{x_2-x_1}$

$\overline{JY} \rightarrow \frac{2-6}{2-4} : \frac{-4}{-2} \rightarrow 2$

$\overline{OE} \rightarrow \frac{0-4}{6-4}\ \frac{-4}{2} \rightarrow -2$

$\overline{JY}$ is perpendicular to $\overline{OE}$ b/c of negative reciprocal slopes.



$\overline{JY}$ is the perpendicular bisector of $\overline{OE}$ b/c point Y is a midpoint and the $\overline{JY}$ is perpendicular to $\overline{OE}$ b/c of negative reciprocal slopes

Question 35

35 Triangle JOE has vertices whose coordinates are $J(4,6)$, $O(-2,4)$, and $E(6,0)$.

Prove that $\triangle JOE$ is isosceles.

[The use of the set of axes on the next page is optional.]

JO distance: $\sqrt{(6-4)^2 + (4+2)^2} \rightarrow \sqrt{40}$

JE distance $= \sqrt{(6-4)^2 + (0-6)^2} \rightarrow \sqrt{40}$

OE distance $= \sqrt{(6+2)^2 + (0-4)^2} \rightarrow \sqrt{80}$

Δ JOE is isosceles b/c it has 2 congruent sides

Question 35 is continued on the next page.

Score 4: The student did not write concluding statements when proving $\overline{JY}$ is the perpendicular bisector of $\overline{OE}$.

Point $Y(2,2)$ is on $\overline{OE}$. $J(4,6)$ $O(-2,4)$ $E(6,0)$

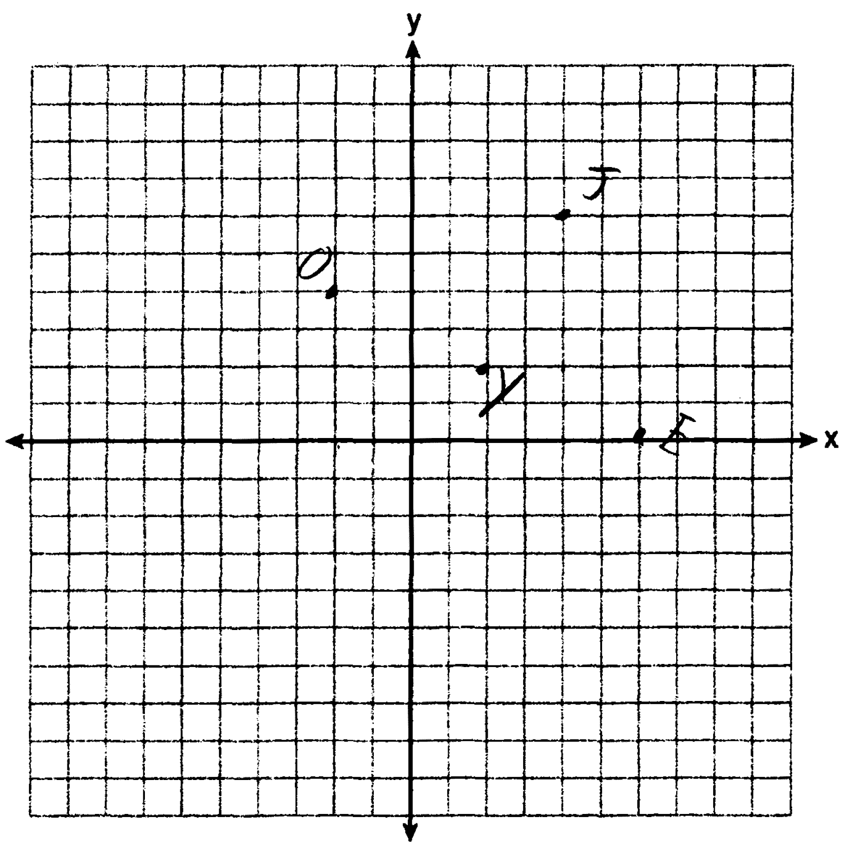
Prove that $\overline{JY}$ is the perpendicular bisector of $\overline{OE}$.

$\overline{JY}$ Slope $= \frac{6-2}{4-2} = 2$ $\overline{JY}$ is $\perp$ to $\overline{OE}$

$\overline{OE}$ Slope $= \frac{0-4}{6+2} = \frac{-4}{8} = -\frac{1}{2}$

$\overline{OY}$ distance $= \sqrt{(2+2)^2 + (2-4)^2} \rightarrow \sqrt{20}$ $\}$ Congruent

$\overline{YE}$ distance $= \sqrt{(6-2)^2 + (0-2)^2} \rightarrow \sqrt{20}$



Question 35

35 Triangle *JOE* has vertices whose coordinates are $J(4,6)$, $O(-2,4)$, and $E(6,0)$.

Prove that $\triangle JOE$ is isosceles.

[The use of the set of axes on the next page is optional.]

$JO\ \sqrt{(4--2)^2+(6-4)^2} = \sqrt{40}$

$OE\ \sqrt{(-2-6)^2+(4-0)^2} = \sqrt{80}$

$EJ\ \sqrt{(6-4)^2+(0-6)^2} = \sqrt{40}$

$\cong$

$\triangle JOE$ is isos b/c it has 2 $\cong$ sides.

Question 35 is continued on the next page.

Score 4: The student did not prove $\overline{JY}$ was perpendicular to $\overline{OE}$.

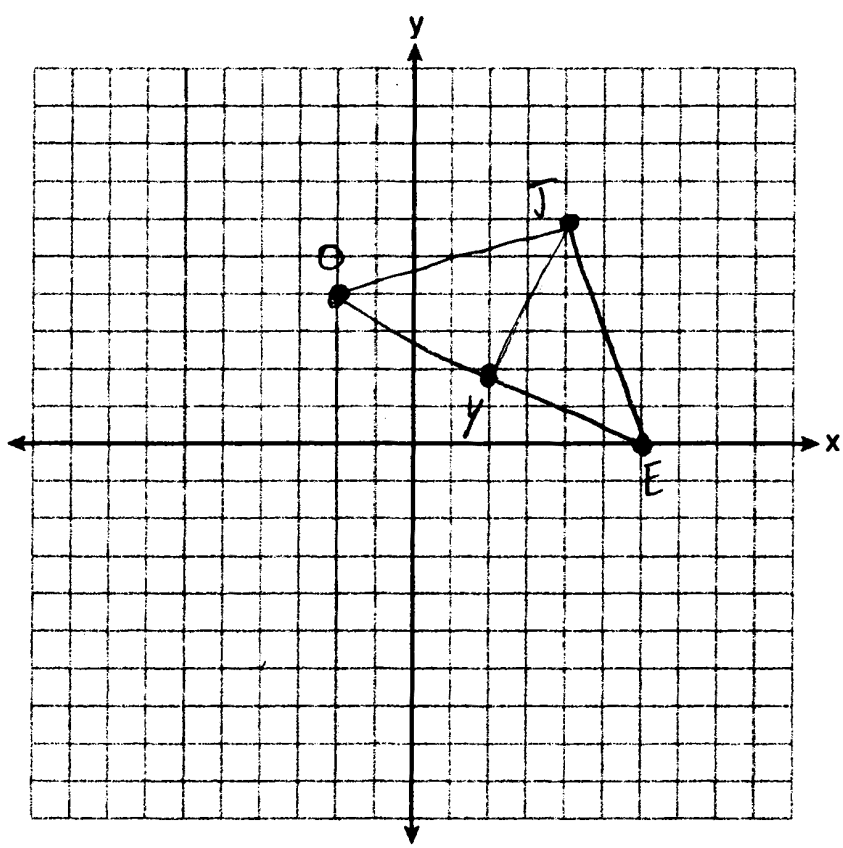
Question 35 continued.

Point $Y(2,2)$ is on $\overline{OE}$.

Prove that $\overline{JY}$ is the perpendicular bisector of $\overline{OE}$.

$$\left(\frac{-2+6}{2}, \frac{4+0}{2}\right) = (2,2)$$

$\overline{JY}$ is the perpendicular bisector of $\overline{OE}$ because $\overline{JY}$ connects to point Y which is the midpt. of $\overline{OE}$.



Question 35

35 Triangle JOE has vertices whose coordinates are $J(4,6)$, $O(-2,4)$, and $E(6,0)$.

Prove that $\triangle JOE$ is isosceles.

[The use of the set of axes on the next page is optional.]

$$M = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\overline{JO}: M = \frac{4-6}{-2-4} = \frac{-2}{-6} = \frac{1}{3}$$

$$\overline{JE}: m = \frac{0-6}{6-4} = \frac{-6}{2} = \frac{-3}{1}$$

ΔJOE is isosceles because when 2 sides of a triangle are negative reciprocals of eachother they are also $\cong$.

Question 35 is continued on the next page.

Score 4: The student did not prove $\triangle JOE$ was isosceles.

Point $Y(2,2)$ is on $\overline{OE}$.

Prove that $\overline{JY}$ is the perpendicular bisector of $\overline{OE}$.

$\overline{OE}$: $m = \frac{y_2 - y_1}{x_2 - x_1}$
(slope)
$m = \frac{0-4}{6-(-2)}$
$m = \frac{-4}{8}$
$m = -\frac{1}{2}$

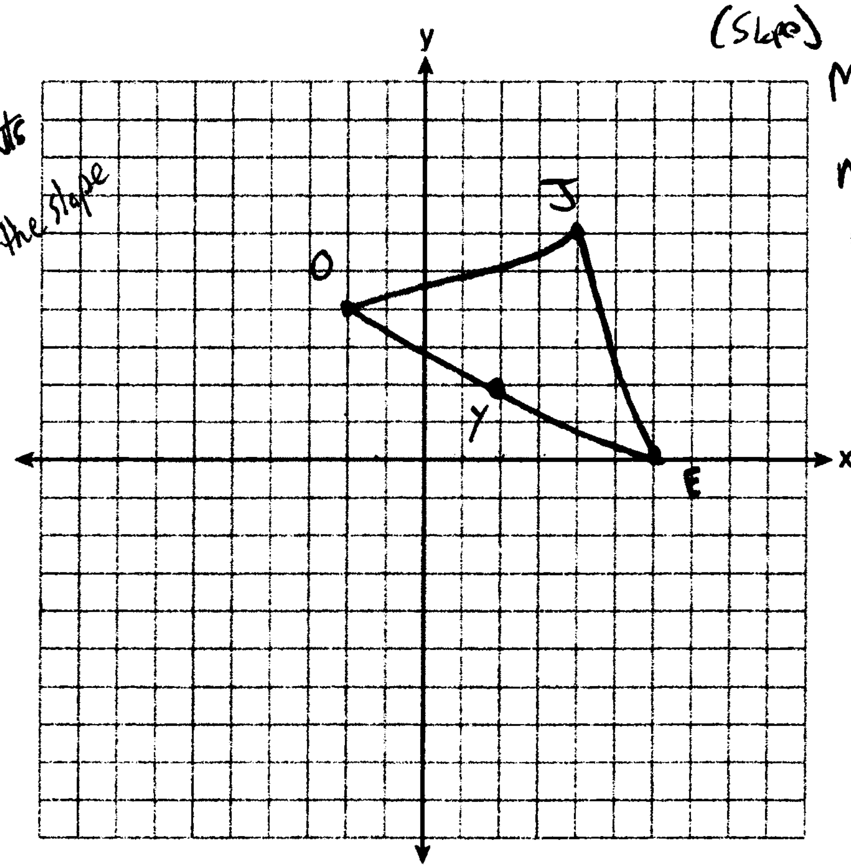
O(-2,4)
E (6,0)
J (4,6)

$\overline{OY}$: $d = \sqrt{(y_2-y_1)^2 + (x_2-x_1)^2}$
$d = \sqrt{(2-4)^2 + (2-(-2))^2}$
$d = \sqrt{(-2)^2 + (4)^2}$
$d = \sqrt{4+16}$
$d = \sqrt{20}$

$\overline{EY}$: $d = \sqrt{(2-0)^2 + (2-6)^2}$
$d = \sqrt{(2)^2 + (-4)^2}$
$d = \sqrt{4+16}$
$d = \sqrt{20}$

$\overline{JY}$: $m = \frac{y_2 - y_1}{x_2 - x_1}$
(slope)
$m = \frac{2-6}{2-4}$
$m = \frac{-4}{-2}$
$m = \frac{2}{1}$

When a point divides a line segment into 2 ≅ segments and the slope of $\overline{JY}$ is the negative reciprical of the slope of $\overline{OE}$, $\overline{JY}$ is a perp. bisector of $\overline{OE}$.



Question 35

35 Triangle JOE has vertices whose coordinates are $J(4,6)$, $O(-2,4)$, and $E(6,0)$.

Prove that $\triangle JOE$ is isosceles.

[The use of the set of axes on the next page is optional.]

$d=\sqrt{(4-6)^2+(-2-4)^2}$ $\overline{JO}$

$d=\sqrt{(4)+(36)}$

$d=\sqrt{40}$

$d=\sqrt{(0-4)^2+(6+2)^2}$ $\overline{OE}$

$d=\sqrt{(16)+(64)}$

$d=\sqrt{80}$

$d=\sqrt{(0-6)^2+(6-4)^2}$ $\overline{EJ}$

$d=\sqrt{(-6)^2+(2)^2}$

$d=\sqrt{36+4}$

$d=\sqrt{40}$

$\overline{JO} \cong \overline{EJ}$, therefore $\triangle JOE$ is an isosceles triangle.

Question 35 is continued on the next page.

Score 3: The student proved $\triangle JOE$ was isosceles and determined the slopes of $\overline{JY}$ and $\overline{OE}$. No further correct work was shown.

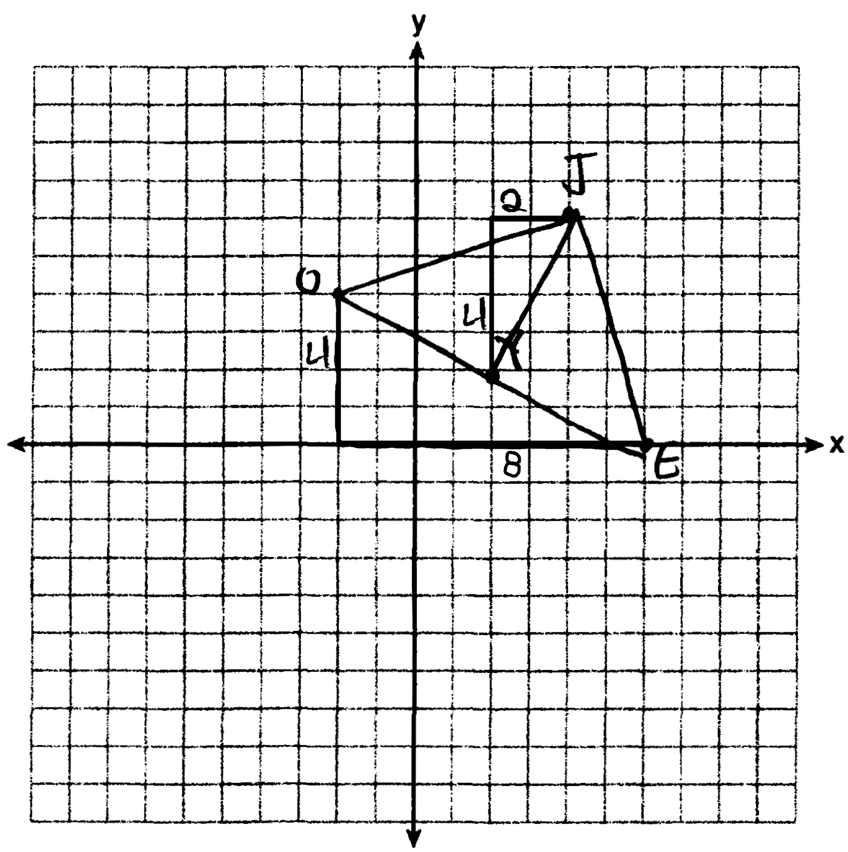
Point $Y(2,2)$ is on $\overline{OE}$.

Prove that $\overline{JY}$ is the perpendicular bisector of $\overline{OE}$.

Slope of $\overline{OE} = \frac{-4}{8} = -\frac{1}{2}$

Slope of $\overline{JY} = \frac{4}{2} = 2$

The slope of $\overline{JY}$ is perp. to the slope of $\overline{OE}$ so it is the perpendicular bisector.



Question 35

35 Triangle JOE has vertices whose coordinates are $J(4,6)$, $O(-2,4)$, and $E(6,0)$.

Prove that $\triangle JOE$ is isosceles.

[The use of the set of axes on the next page is optional.]

$$JO = \sqrt{(-2-4)^2+(4-6)^2}$$
$$= \sqrt{(-6)^2+(-2)^2}$$
$$= \sqrt{36+4}$$
$$= \sqrt{40}$$

$$JE = \sqrt{(6-4)^2+(0-6)^2}$$
$$= \sqrt{(2)^2+(-6)^2}$$
$$= \sqrt{4+36}$$
$$= \sqrt{40}$$

$$\overline{JO} \cong \overline{JE}$$

$2 \cong$ sides

$\therefore$ $\triangle JOE$ is isosceles

Question 35 is continued on the next page.

Score 3: The student proved $\triangle JOE$ was isosceles and found the midpoint of $\overline{OE}$, but no further correct work was shown.

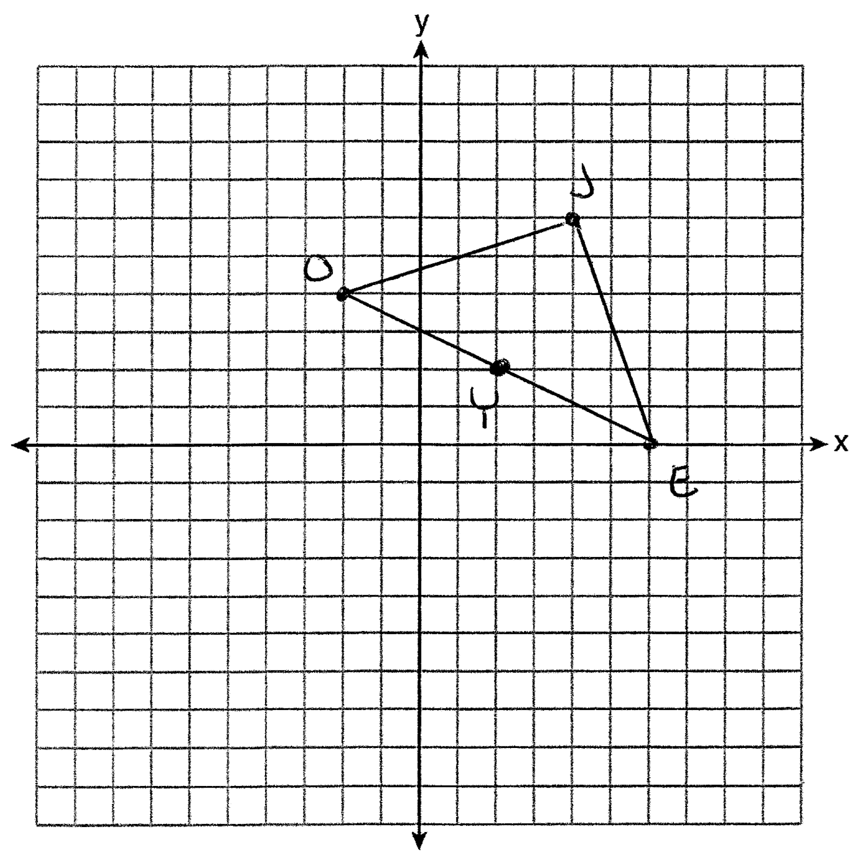
Point $Y(2,2)$ is on $\overline{OE}$.

Prove that $\overline{JY}$ is the perpendicular bisector of $\overline{OE}$.

Midpoint of $\overline{OE}$: $\left(\frac{-2+6}{2}, \frac{4+0}{2}\right)$

$\left(\frac{4}{2}, \frac{4}{2}\right)$

$(2,2)$



Question 35

35 Triangle JOE has vertices whose coordinates are $J(4,6)$, $O(-2,4)$, and $E(6,0)$.

Prove that $\triangle JOE$ is isosceles.

[The use of the set of axes on the next page is optional.]

$OJ = 2^2 + 6^2 = x^2$

$OJ = 4 + 36 = x^2$

$OJ = \sqrt{40}$

$JE = 6^2 + 2^2 = x^2$

$36 + 4 = x^2$

$JE = \sqrt{40}$

$\overline{OJ} \cong \overline{JE}$

$OE = 4^2 + 8^2 = x^2$

$= 16 + 64 = x^2$

$OE = \sqrt{80}$

Question 35 is continued on the next page.

Score 2: The student did not write a concluding statement when proving $\triangle JOE$ was isosceles. The student found the lengths of $\overline{OY}$ and $\overline{EY}$, but no further correct work was shown.

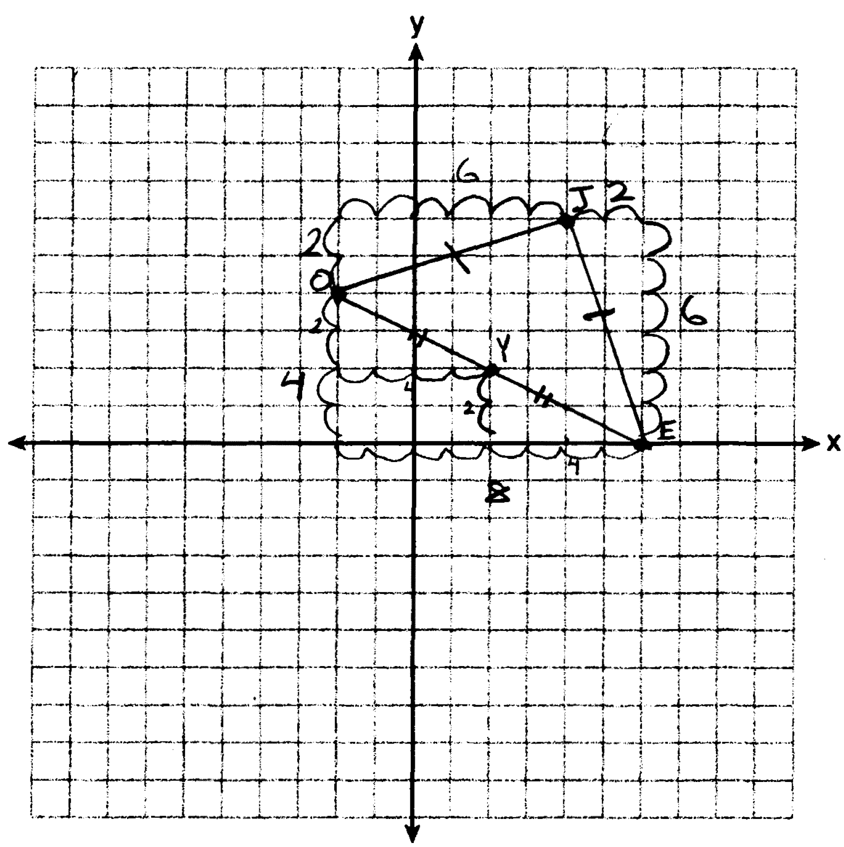
Point $Y(2,2)$ is on $\overline{OE}$.

Prove that $\overline{JY}$ is the perpendicular bisector of $\overline{OE}$.

$OY = 2^2 + 4^2 = x^2$
$= 4 + 16 = x^2$
$= \sqrt{20}$

$\overline{OY} \cong \overline{YE}$

$YE = 4^2 + 2^2 = x^2$
$= 16 + 4 = x^2$
$= \sqrt{20}$



35 Triangle JOE has vertices whose coordinates are $J(4,6)$, $O(-2,4)$, and $E(6,0)$.

Prove that $\triangle JOE$ is isosceles.

[The use of the set of axes on the next page is optional.]

$JO = \sqrt{(-2-4)^2+(4-6)^2}$

$\sqrt{(-6)^2+(-2)^2}$

$\sqrt{36+4} = \sqrt{40}$

$OE = \sqrt{(6+2)^2+(0-4)^2}$

$\sqrt{(8)^2+(-4)^2}$

$\sqrt{64+16} = \sqrt{80}$

$JE\ \sqrt{(6-4)^2+(0-6)^2}$

$\sqrt{(2)^2+(-6)^2}$

$\sqrt{4+36} = \sqrt{40}$

Δ JOE is isosceles because for a Δ to be isosceles two of its sides have to be equal. $\overline{JO}$ and $\overline{JE}$ are equal. Resulting in Δ JOE being an isosceles Δ.

Question 35 is continued on the next page.

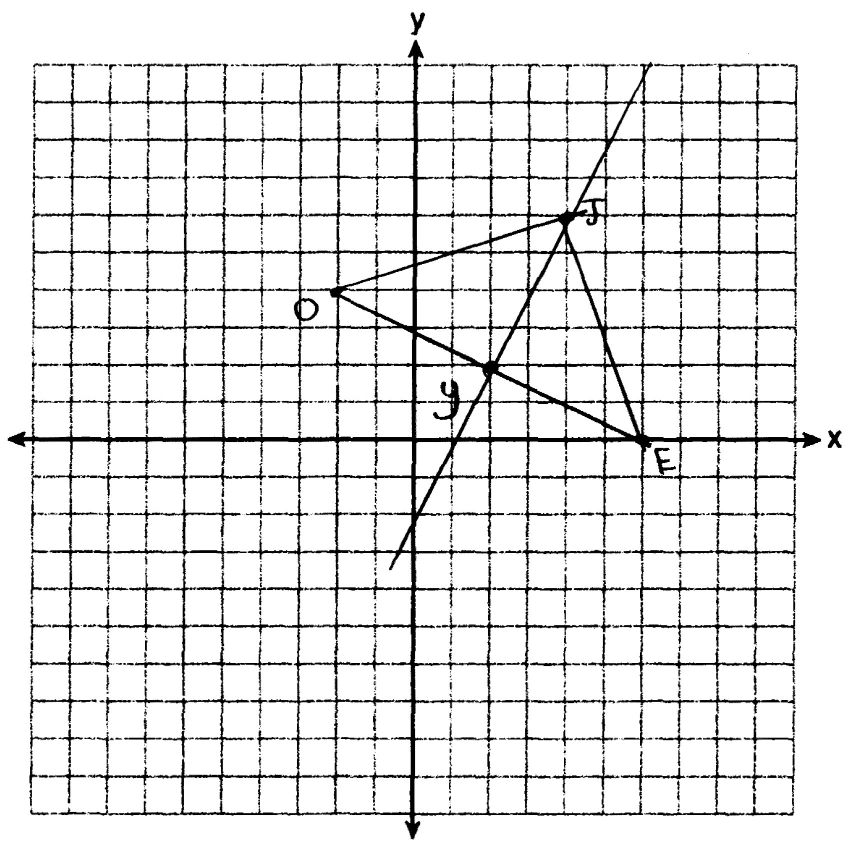
Score 2: The student proved $\triangle JOE$ was isosceles. No further correct work was shown.

Point $Y(2,2)$ is on $\overline{OE}$.

Prove that $\overline{JY}$ is the perpendicular bisector of $\overline{OE}$.

Statements	Reasons
① point Y(2,2) is on $\overline{OE}$	① given
② $\overline{JY}$ bisects $\overline{OE}$	② A bisector is a line that splits up a segment
③ $\overline{JY}$ is a ⊥ bisector of $\overline{OE}$	③ a ⊥ bisector splits a segment

J(4,6)
O(-2,4)
E(6,0)



Question 35

35 Triangle JOE has vertices whose coordinates are $J(4,6)$, $O(-2,4)$, and $E(6,0)$.

Prove that $\triangle JOE$ is isosceles.

[The use of the set of axes on the next page is optional.]

distance from O to E = $d = \sqrt{(0-(4))^2 + (6-(-2))^2}$

$d = \sqrt{(-4)^2 + 8^2}$

$d = \sqrt{80}$

distance of JO = $d = \sqrt{(4-6)^2 + (-2-4)^2}$

$d = \sqrt{(-2)^2 + (-6^2)}$

$d = \sqrt{40}$

distance of JE = $d = \sqrt{(0-6)^2 + (6-4)^2}$

$d = \sqrt{(-6)^2 + 2^2}$

$d = \sqrt{40}$

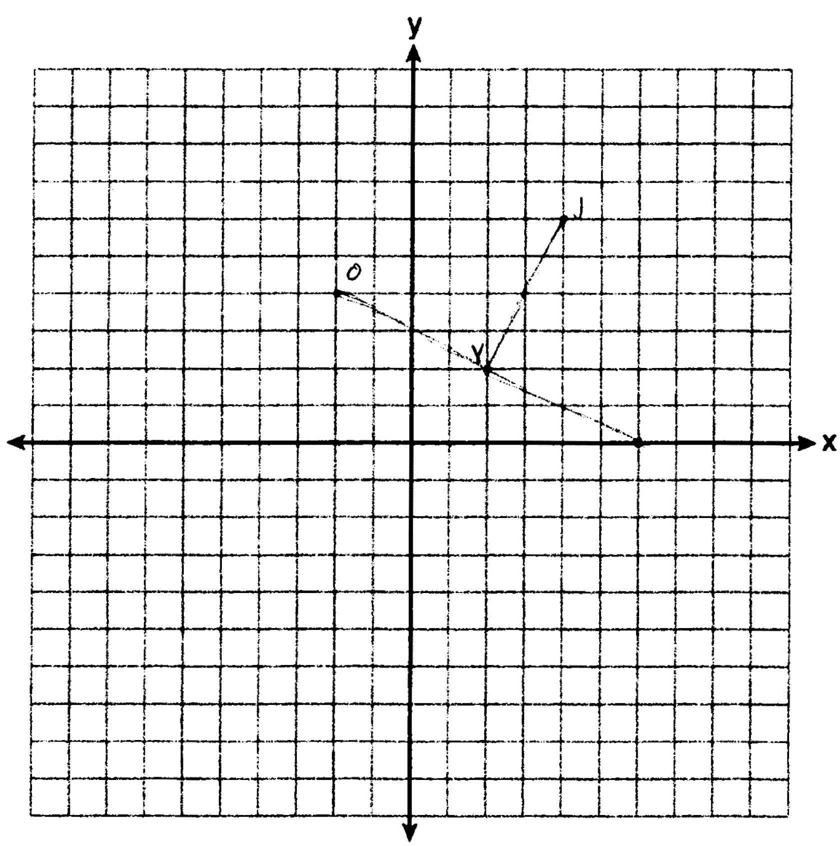
Two sides of JOE are equal but the last side isn't so its isosceles

Question 35 is continued on the next page.

Score 2: The student proved $\triangle JOE$ was isosceles. No further correct work was shown.

Point $Y(2,2)$ is on $\overline{OE}$.

Prove that $\overline{JY}$ is the perpendicular bisector of $\overline{OE}$.



Question 35

35 Triangle *JOE* has vertices whose coordinates are $J(4,6)$, $O(-2,4)$, and $E(6,0)$.

Prove that $\triangle JOE$ is isosceles.

[The use of the set of axes on the next page is optional.]

$JO = \sqrt{(-2-4)^2+(4-6)^2}$
$= \sqrt{(-6)^2+(-2)^2}$
$= \sqrt{36+4}$
$= \sqrt{40}$

$OE = \sqrt{(6-(-2))^2+(0-4)^2}$
$= \sqrt{8^2+(-4)^2}$
$= \sqrt{64+16}$
$= \sqrt{80}$

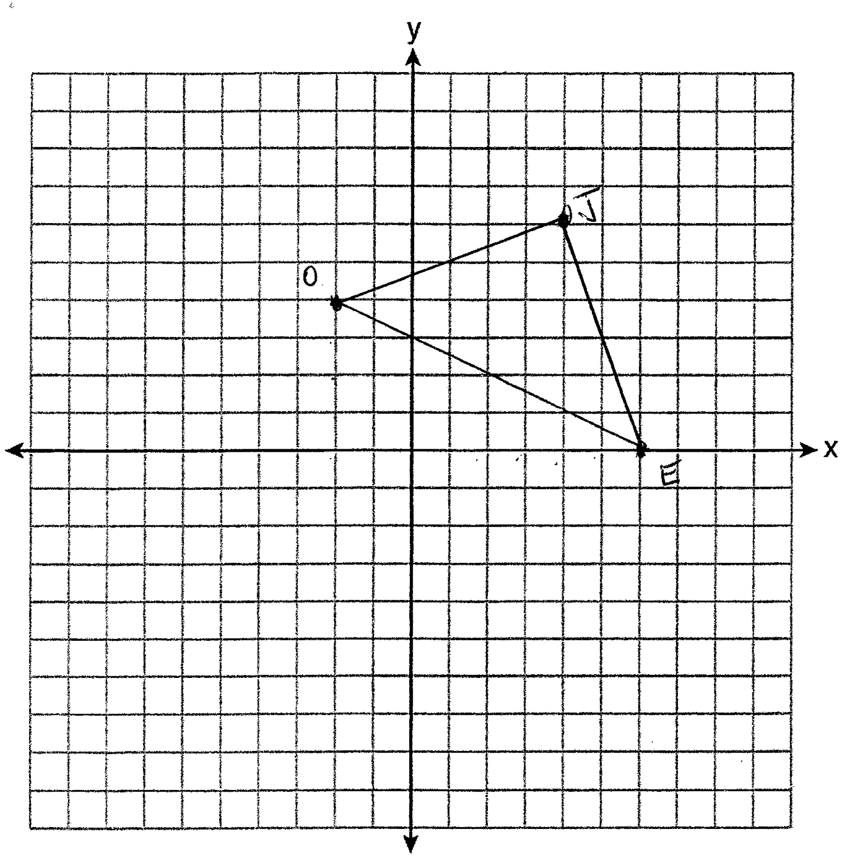
$JE = \sqrt{(6-4)^2+(0-6)^2}$
$= \sqrt{2^2+(-6)^2}$
$= \sqrt{4+36}$
$= \sqrt{40}$

Question 35 is continued on the next page.

Score 1: The student determined the lengths of the sides of $\triangle JOE$, but no further correct work was shown.

Point $Y(2,2)$ is on $\overline{OE}$.

Prove that $\overline{JY}$ is the perpendicular bisector of $\overline{OE}$.



Question 35

35 Triangle JOE has vertices whose coordinates are $J(4,6)$, $O(-2,4)$, and $E(6,0)$.

Prove that $\triangle JOE$ is isosceles.

[The use of the set of axes on the next page is optional.]

2 ≅ sides

∴ Joe is isosceles

Question 35 is continued on the next page.

Score 1: The student determined the midpoint of $\overline{OE}$, but no further correct work was shown.

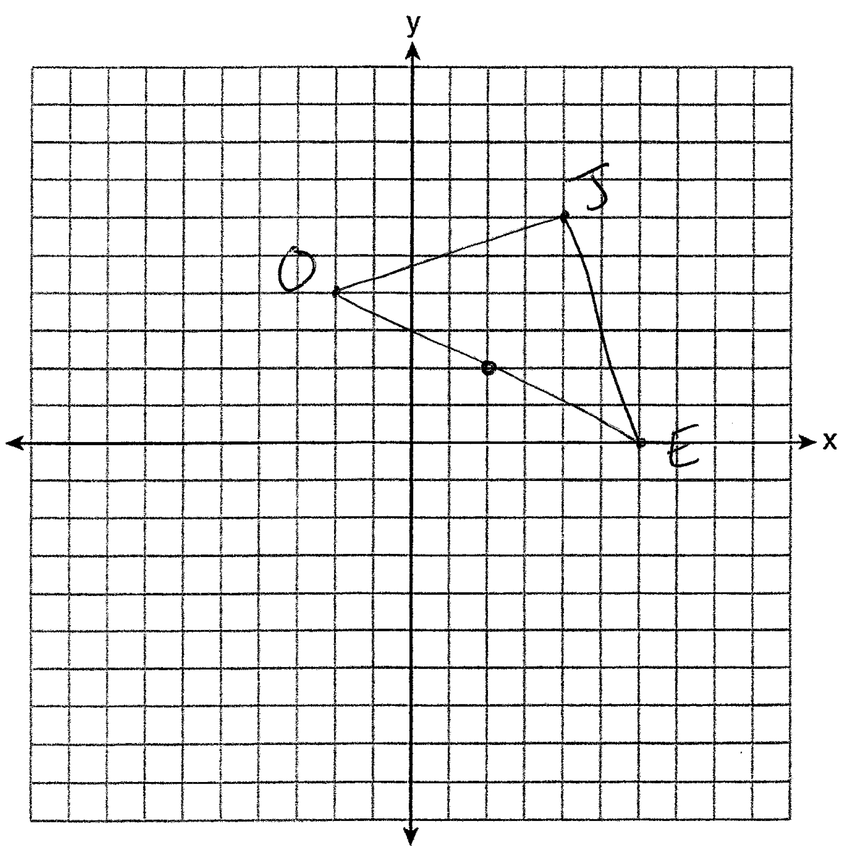
Point $Y(2,2)$ is on $\overline{OE}$.

Prove that $\overline{JY}$ is the perpendicular bisector of $\overline{OE}$.

$\overline{OE}$ midpoint!

$$\left(\frac{-2+6}{2}, \frac{4+0}{2}\right)$$

$$(2,2)$$



Question 35

35 Triangle JOE has vertices whose coordinates are $J(4,6)$, $O(-2,4)$, and $E(6,0)$.

Prove that $\triangle JOE$ is isosceles.

[The use of the set of axes on the next page is optional.]

$JO\ \sqrt{(4--2)^2+(6-4)^2}=\sqrt{40}$

$OE\ \sqrt{(-2-6)^2+(4-0)^2}=\sqrt{80}$

$EJ\ \sqrt{(6-4)^2+(0-4)^2}=\sqrt{20}$

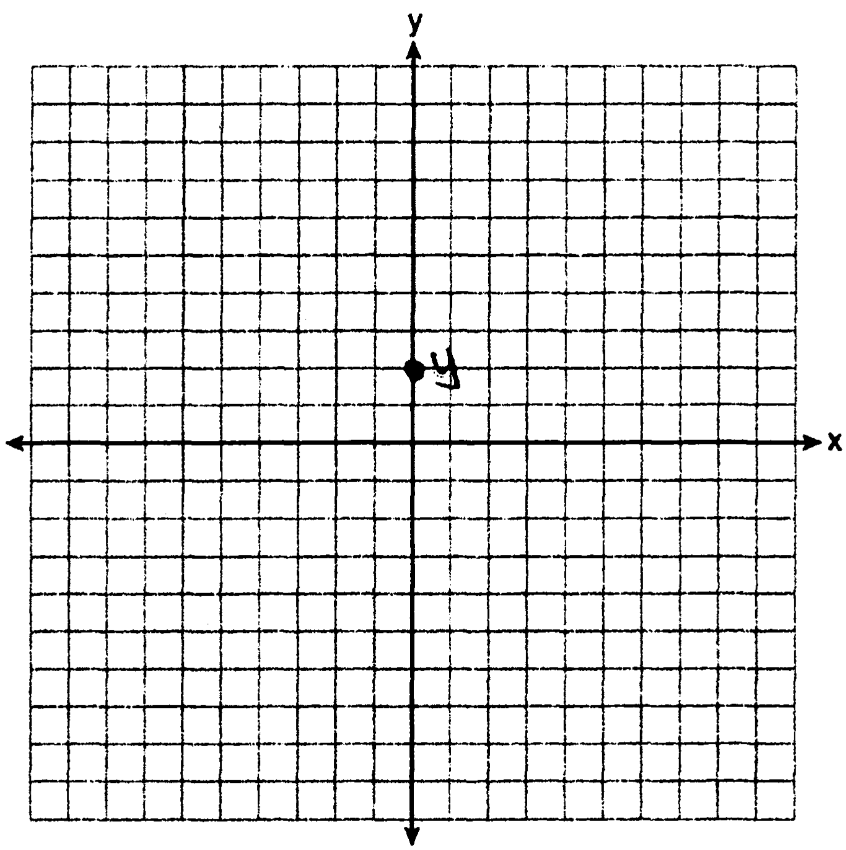
Question 35 is continued on the next page.

Score 0: The student did not show enough correct relevant work to receive any credit.

Question 35 continued.

Point $Y(2,2)$ is on $\overline{OE}$.

Prove that $\overline{JY}$ is the perpendicular bisector of $\overline{OE}$.



Question 35

35 Triangle *JOE* has vertices whose coordinates are $J(4,6)$, $O(-2,4)$, and $E(6,0)$.

Prove that $\triangle JOE$ is isosceles.

[The use of the set of axes on the next page is optional.]

$\overline{JO}$ & $\overline{JE}$ are congruent which makes the two corresponding ∠s be equal, which is only producing an isocles △'s

Question 35 is continued on the next page.

Score 0: The student did not show enough correct relevant work to receive any credit.

Question 35 continued.

Point $Y(2,2)$ is on $\overline{OE}$.

Prove that $\overline{JY}$ is the perpendicular bisector of $\overline{OE}$.

The resulting ∠s of JYE & ∠JYO are right Δ's which means they have to be made from ⊥ lines. Also, any bisected line in an isoceles Δ, that point connected to the top of a triangle will almost always make a ⊥ lines.