

**The University of the State of New York**  
**REGENTS HIGH SCHOOL EXAMINATION**

# **GEOMETRY**

**Tuesday, June 23, 2026 — 1:15 to 4:15 p.m., only**

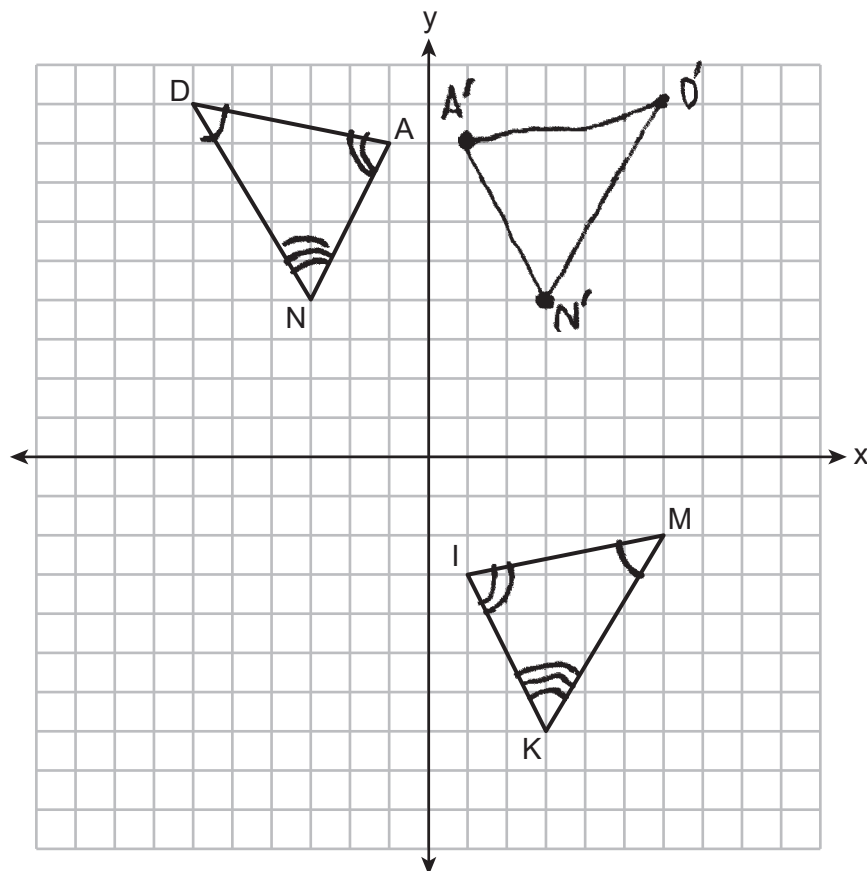
## **MODEL RESPONSE SET**

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Question 25

25 On the set of axes below,  $\triangle DAN \cong \triangle MIK$ .



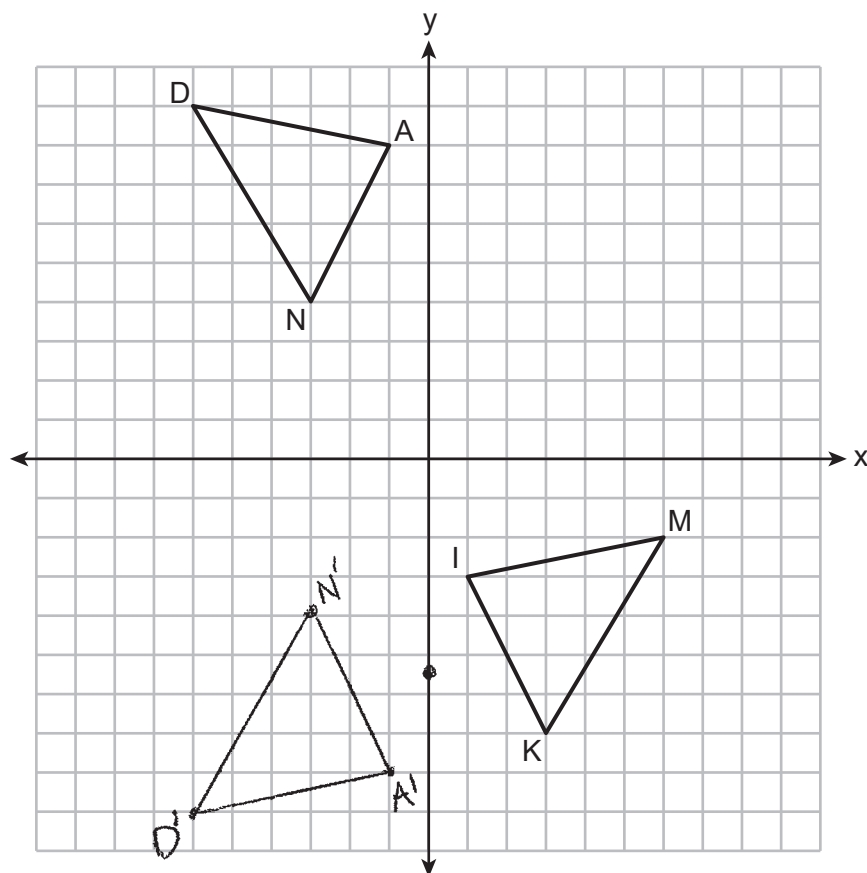
Describe a sequence of rigid motions that maps  $\triangle DAN$  onto  $\triangle MIK$ .

1<sup>st</sup>: Reflect  $\triangle DAN$  over the y-axis  
 2<sup>nd</sup>: Translate  $\triangle DAN$  down 11 units

**Score 2:** The student gave a complete and correct response.

Question 25

25 On the set of axes below,  $\triangle DAN \cong \triangle MIK$ .



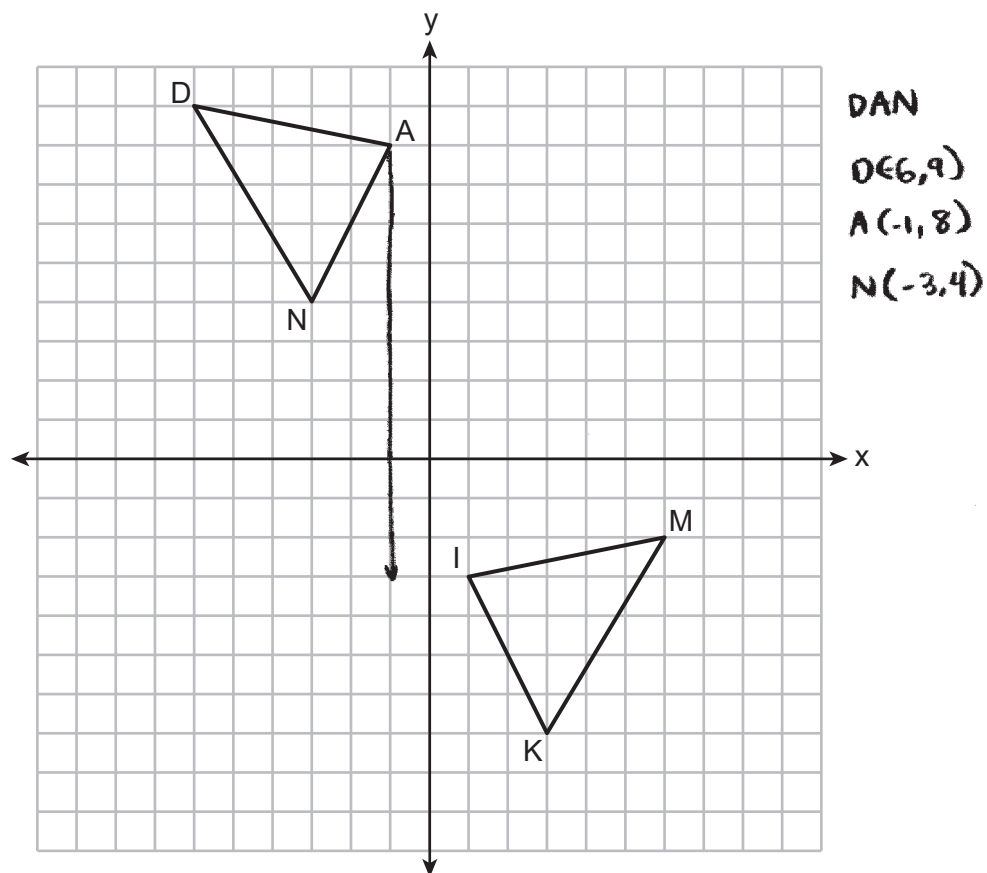
Describe a sequence of rigid motions that maps  $\triangle DAN$  onto  $\triangle MIK$ .

Reflect  $\triangle DAN$  over the  $x$ -axis  
Then reflect it through the point  $(0, -5.5)$ .

**Score 2:** The student gave a complete and correct response.

Question 25

25 On the set of axes below,  $\triangle DAN \cong \triangle MIK$ .



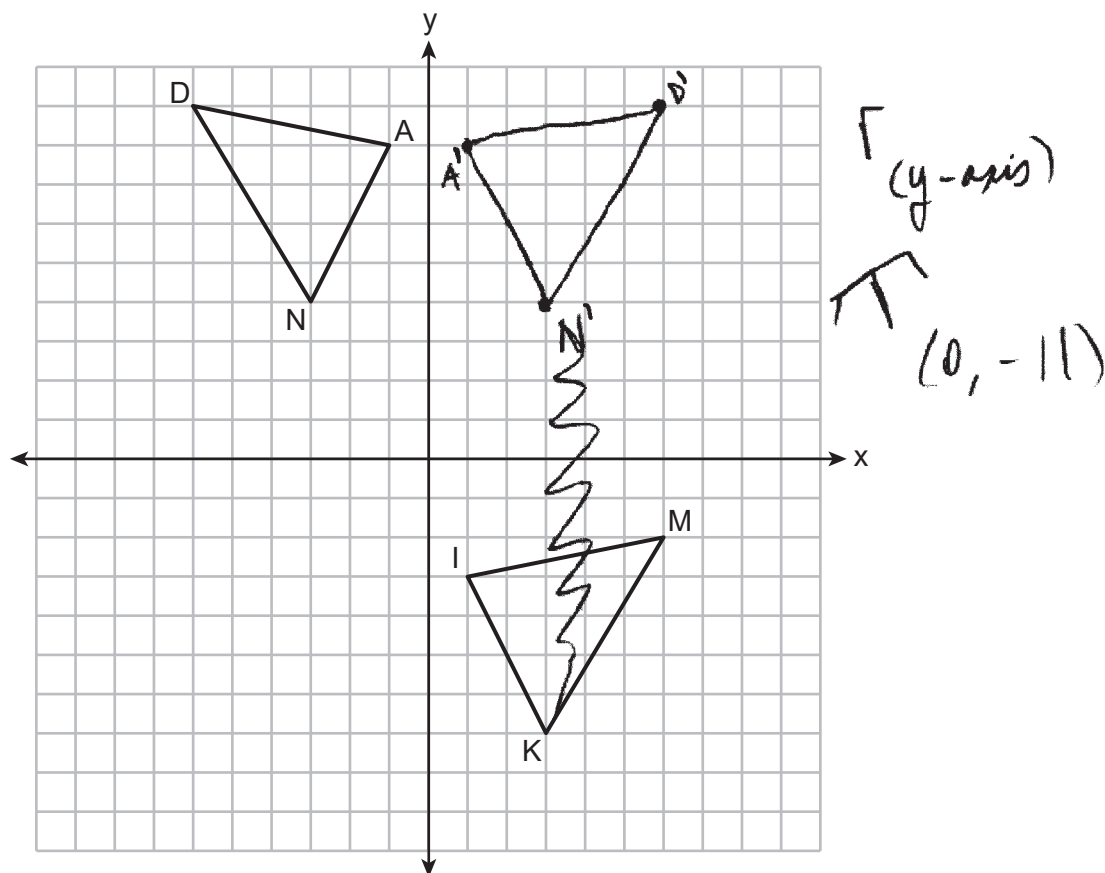
Describe a sequence of rigid motions that maps  $\triangle DAN$  onto  $\triangle MIK$ .

- a translation 11 units down
- a reflection over the y-axis

**Score 2:** The student gave a complete and correct response.

Question 25

25 On the set of axes below,  $\triangle DAN \cong \triangle MIK$ .



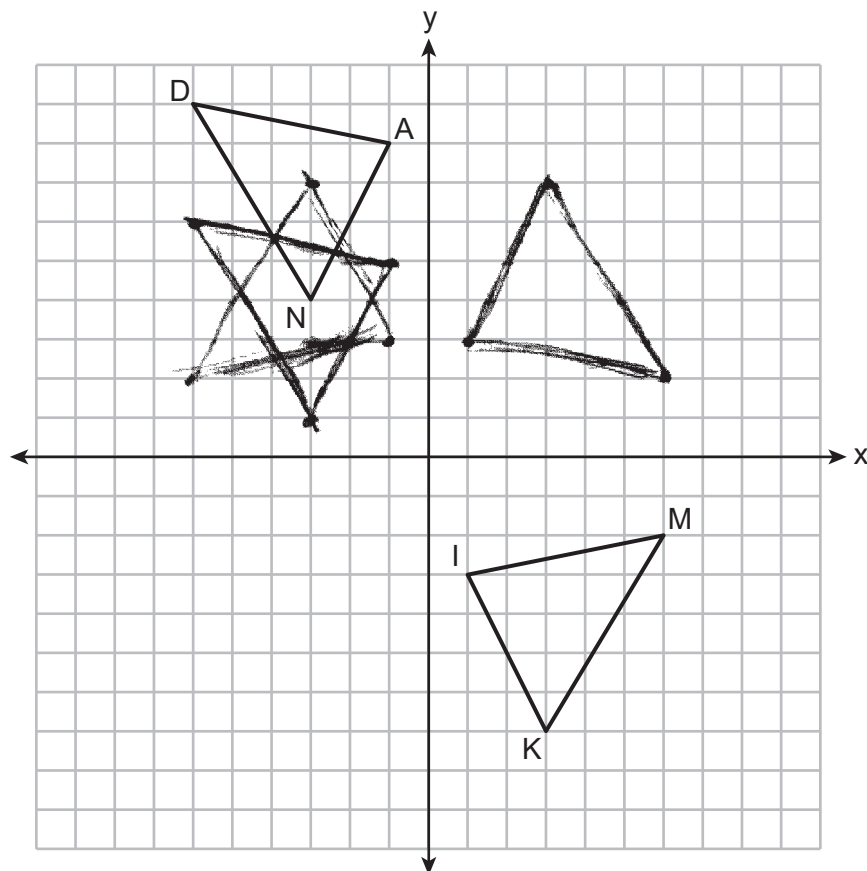
Describe a sequence of rigid motions that maps  $\triangle DAN$  onto  $\triangle MIK$ .

$$[T(0, -11) \circ \Gamma(y\text{-axis})]$$

**Score 2:** The student gave a complete and correct response.

Question 25

25 On the set of axes below,  $\triangle DAN \cong \triangle MIK$ .



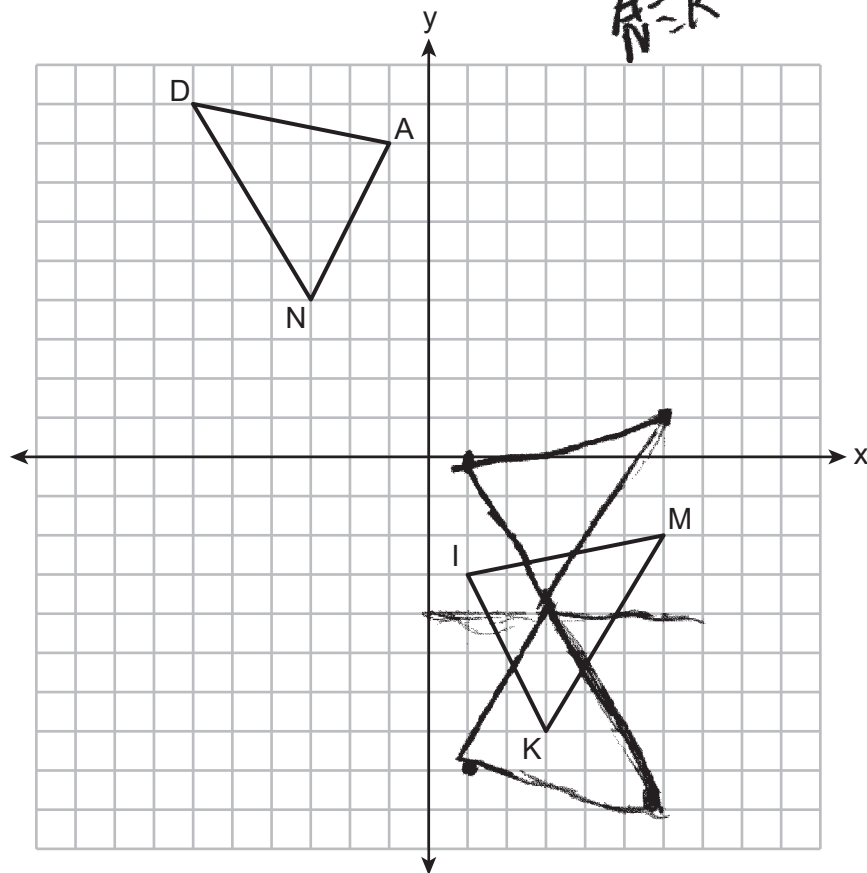
Describe a sequence of rigid motions that maps  $\triangle DAN$  onto  $\triangle MIK$ .

Reflection Across the x axis  
 Reflection Across the y axis  
 Reflection across  $y = 4$   
 Translation up 3 units

**Score 1:** The student mapped  $\triangle MIK$  onto  $\triangle DAN$ .

Question 25

25 On the set of axes below,  $\triangle DAN \cong \triangle MIK$ .



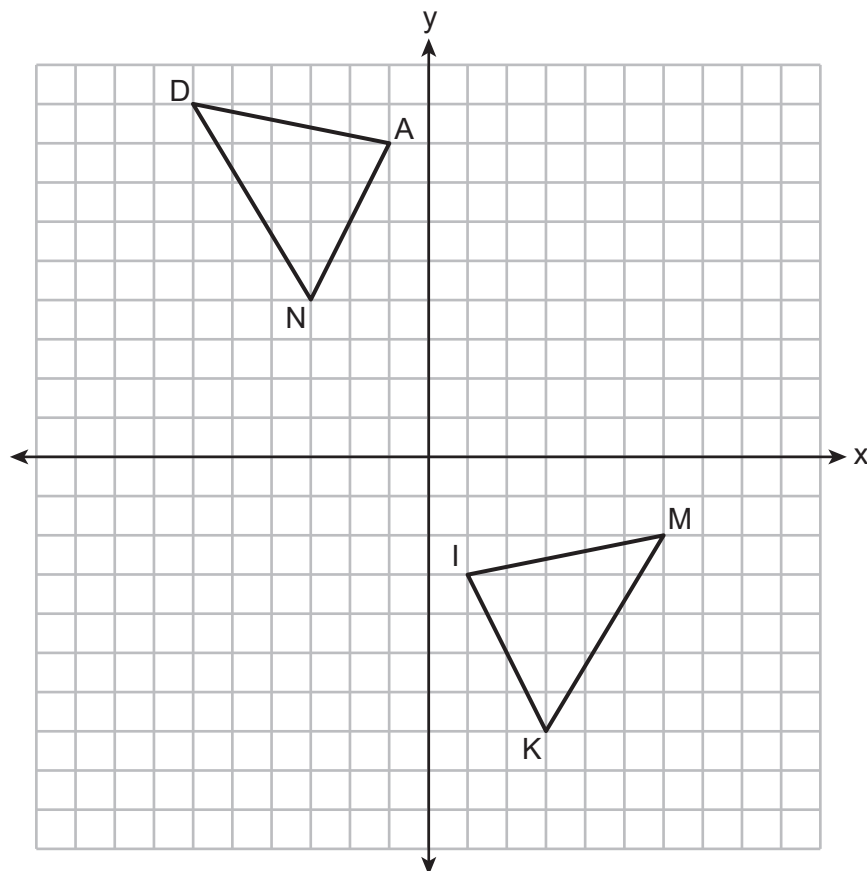
Describe a sequence of rigid motions that maps  $\triangle DAN$  onto  $\triangle MIK$ .

- rotation  $180^\circ$
- reflection over  $y = -4$
- Translation  $\langle 0, -3 \rangle$

**Score 1:** The student did not state the center of rotation.

Question 25

25 On the set of axes below,  $\triangle DAN \cong \triangle MIK$ .



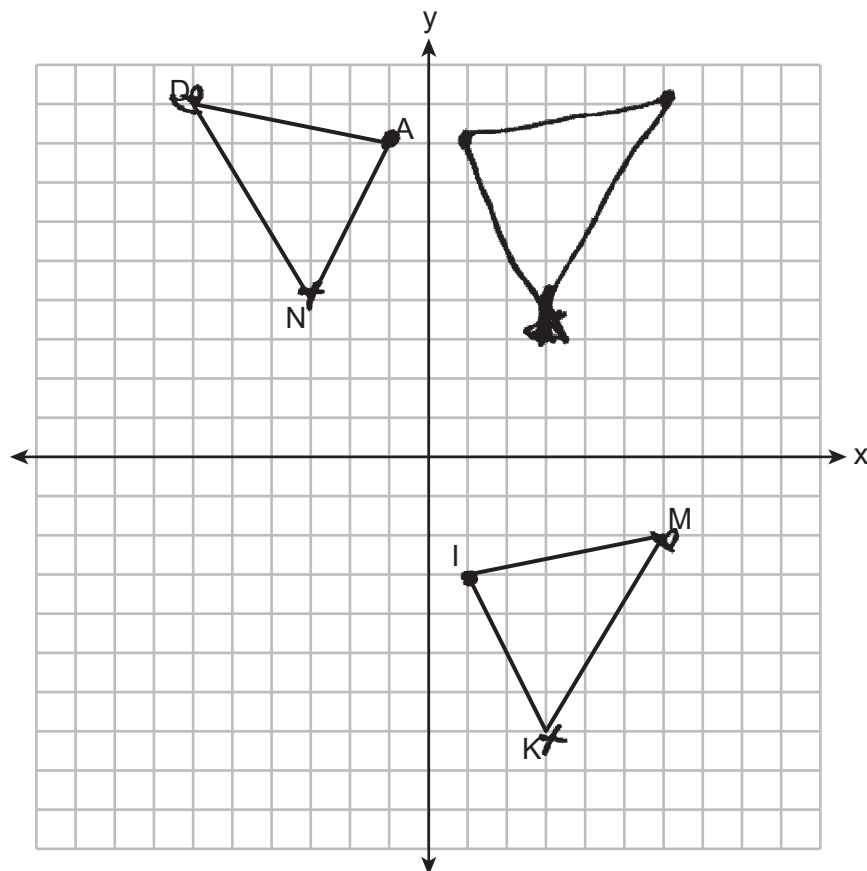
Describe a sequence of rigid motions that maps  $\triangle DAN$  onto  $\triangle MIK$ .

A reflection over the y-axis to bring  $\triangle DAN$  into quadrant I. A translation

**Score 1:** The student wrote an incomplete description of the translation.

Question 25

25 On the set of axes below,  $\triangle DAN \cong \triangle MIK$ .



Describe a sequence of rigid motions that maps  $\triangle DAN$  onto  $\triangle MIK$ .

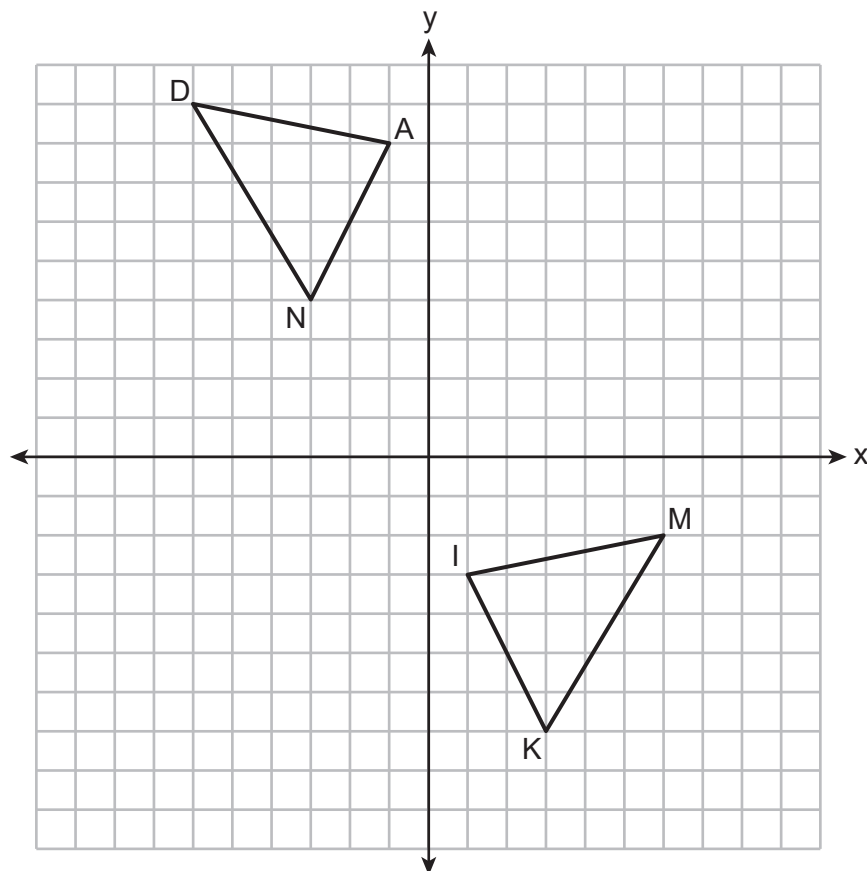
Reflect over  $x$

Down 10 units

**Score 0:** The student did not show enough relevant course-level work to receive any credit.

Question 25

25 On the set of axes below,  $\triangle DAN \cong \triangle MIK$ .



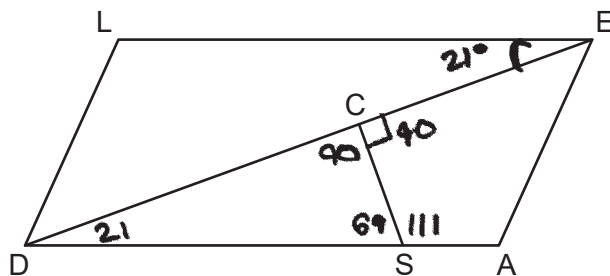
Describe a sequence of rigid motions that maps  $\triangle DAN$  onto  $\triangle MIK$ .

rotation by  $180^\circ$  around the origin

**Score 0:** The student did not show enough relevant course-level work to receive any credit.

Question 26

26 In parallelogram  $LEAD$  below,  $C$  is on  $\overline{DE}$  and  $S$  is on  $\overline{DA}$  such that  $\overline{SC} \perp \overline{DE}$ .



If  $m\angle LED = 21^\circ$ , determine and state the measure of  $\angle CSA$ .

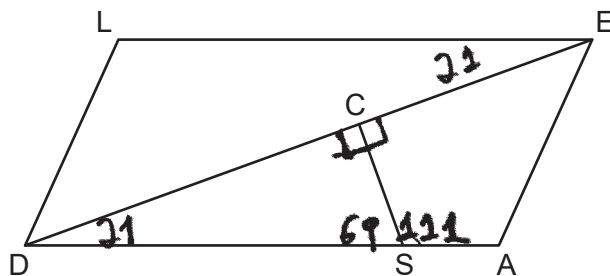


$$\begin{array}{r} 21 \\ + 90 \\ \hline 111 \end{array} \quad \begin{array}{r} 180 \\ - 111 \\ \hline 69 \end{array} \quad \begin{array}{r} 1260 \\ - 69 \\ \hline 111 \end{array}$$

**Score 2:** The student gave a complete and correct response.

Question 26

26 In parallelogram  $LEAD$  below,  $C$  is on  $\overline{DE}$  and  $S$  is on  $\overline{DA}$  such that  $\overline{SC} \perp \overline{DE}$ .



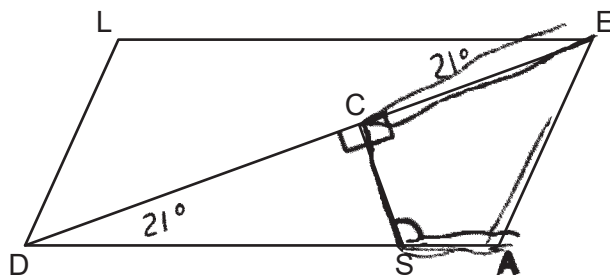
If  $m\angle LED = 21^\circ$ , determine and state the measure of  $\angle CSA$ .

$$m\angle CSA = 111^\circ$$

**Score 2:** The student gave a complete and correct response.

Question 26

26 In parallelogram  $LEAD$  below,  $C$  is on  $\overline{DE}$  and  $S$  is on  $\overline{DA}$  such that  $\overline{SC} \perp \overline{DE}$ .



If  $m\angle LED = 21^\circ$ , determine and state the measure of  $\angle CSA$ .

$$90 + 21 = 111$$

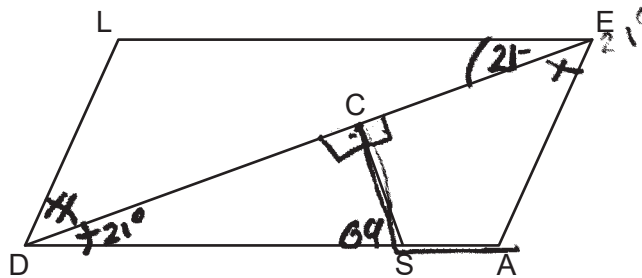
$$m\angle CSA = 111^\circ$$

**Score 2:** The student gave a complete and correct response.



Question 26

26 In parallelogram  $LEAD$  below,  $C$  is on  $\overline{DE}$  and  $S$  is on  $\overline{DA}$  such that  $\overline{SC} \perp \overline{DE}$ .



If  $m\angle LED = 21^\circ$ , determine and state the measure of  $\angle CSA$ .

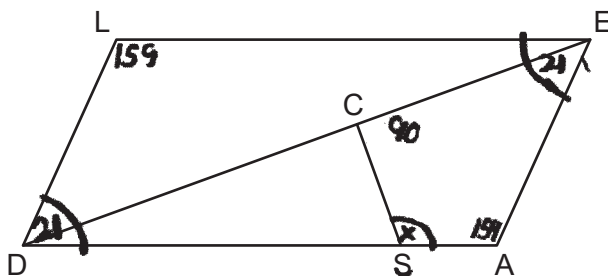
$$180 - 90 - 21 = 69^\circ \quad \angle DSC = 69^\circ$$

$\angle CDS = 21^\circ$  b/c of alternate interior angles are  $\cong$  in a parallelogram.

**Score 1:** The student showed correct work to determine  $m\angle CSD$ , but did not determine  $m\angle CSA$ .

## Question 26

26 In parallelogram  $LEAD$  below,  $C$  is on  $\overline{DE}$  and  $S$  is on  $\overline{DA}$  such that  $\overline{SC} \perp \overline{DE}$ .



If  $m\angle LED = 21^\circ$ , determine and state the measure of  $\angle CSA$ .

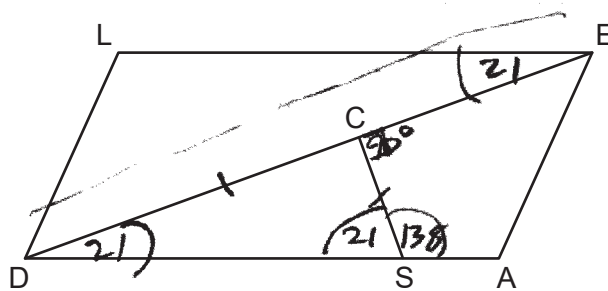
$$\begin{aligned}
 360 &= 159 + 90 + x + 10.5 \\
 360 &= 259.5 + x \\
 -259.5 &\quad -259.5 \\
 \hline
 100.5 &= x
 \end{aligned}$$

$$m\angle CSA = 100.5^\circ$$

**Score 0:** The student did not show enough correct work to receive any credit.

Question 26

26 In parallelogram  $LEAD$  below,  $C$  is on  $\overline{DE}$  and  $S$  is on  $\overline{DA}$  such that  $\overline{SC} \perp \overline{DE}$ .



If  $m\angle LED = 21^\circ$ , determine and state the measure of  $\angle CSA$ .

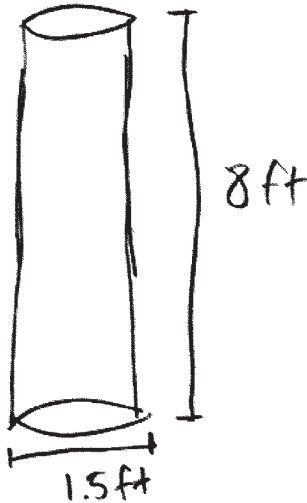
$$\angle CSA = 138^\circ$$

**Score 0:** The student did not show enough correct work to receive any credit.

**Question 27**

- 27** A section of tree trunk from a white pine tree can be modeled by a cylinder. The diameter of the trunk is 1.5 feet, and the height of this section is 8 feet.

If white pine weighs 25 pounds per cubic foot, determine and state the weight of this section of the white pine tree, to the *nearest pound*.



$$d = 2r$$

$$r = \frac{d}{2}$$

$$r = \frac{1.5 \text{ ft}}{2}$$

$$r = 0.75 \text{ ft}$$

$$V_{\text{cylinder}} = \pi r^2 h$$

$$V_{\text{cylinder}} = \pi (0.75)^2 (8)$$

$$\text{Weight} = \frac{25 \text{ lbs}}{1 \text{ ft}^3} \left( \pi (0.75)^2 (8) \right) \text{ ft}^3$$

$$\boxed{\text{Weight} = 353 \text{ lbs}}$$

**Score 2:** The student gave a complete and correct response.

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**Question 27**

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**27** A section of tree trunk from a white pine tree can be modeled by a cylinder. The diameter of the trunk is 1.5 feet, and the height of this section is 8 feet.

If white pine weighs 25 pounds per cubic foot, determine and state the weight of this section of the white pine tree, to the *nearest pound*.



$$\begin{aligned} A &= \pi r^2 \\ &= \pi \cdot 75^2 \\ &= 1.7671... \end{aligned}$$

$$\begin{aligned} V &= Bh \\ &= 1.76... \cdot 8 \\ &= 14.137... \end{aligned}$$

$$\frac{25 \text{ lbs}}{1 \text{ ft}^3} = \frac{x}{14.137 \text{ ft}^3}$$

$$x = 353.4...$$

353 lbs

**Score 2:** The student gave a complete and correct response.

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**Question 27**

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- 27** A section of tree trunk from a white pine tree can be modeled by a cylinder. The diameter of the trunk is 1.5 feet, and the height of this section is 8 feet.

If white pine weighs 25 pounds per cubic foot, determine and state the weight of this section of the white pine tree, to the *nearest pound*.

$$V = \pi r^2 h$$

$$V = \pi (0.75^2) 8$$

$$V = 4.5\pi$$

$$\frac{4.5\pi}{1} * \frac{25}{1} = \boxed{353.1\text{bs}}$$

**Score 2:** The student gave a complete and correct response.

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**Question 27**

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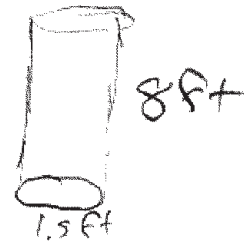
- 27** A section of tree trunk from a white pine tree can be modeled by a cylinder. The diameter of the trunk is 1.5 feet, and the height of this section is 8 feet.

If white pine weighs 25 pounds per cubic foot, determine and state the weight of this section of the white pine tree, to the *nearest pound*.

$$V = Bh$$

$$V = \pi r^2 (8)$$

$$V = \pi (2.25) (8)$$



$$V = \frac{56.549 \text{ ft}^3}{1} \times \frac{25 \text{ lbs}}{1 \text{ ft}^3} = 1413.7167165$$

$$\boxed{1414 \text{ lbs}}$$

**Score 1:** The student made an error using a radius of 1.5 when determining the volume.

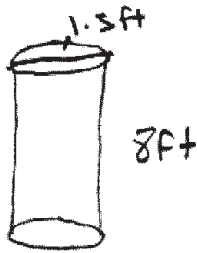
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**Question 27**

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- 27** A section of tree trunk from a white pine tree can be modeled by a cylinder. The diameter of the trunk is 1.5 feet, and the height of this section is 8 feet.

If white pine weighs 25 pounds per cubic foot, determine and state the weight of this section of the white pine tree, to the *nearest pound*.



$$\begin{aligned} V &= B \times h \\ V &= (\pi r^2) h \\ &= (\pi .5625) 8 \\ V &= 14 \end{aligned}$$

$$\begin{array}{r} 14 \\ \times 25 \\ \hline 350 \end{array}$$

the weight of this section is 350 pounds.

**Score 1:** The student made a rounding error when determining the volume of the section of white pine.

Question 27

27 A section of tree trunk from a white pine tree can be modeled by a cylinder. The diameter of the trunk is 1.5 feet, and the height of this section is 8 feet.

If white pine weighs 25 pounds per cubic foot, determine and state the weight of this section of the white pine tree, to the nearest pound.

$$D = \frac{1.5}{2}$$

$$r = 0.75$$

$$h = 8$$



$$D = \frac{W}{V} \quad W = D \cdot V$$

$$V = \frac{W}{D}$$



$$V = \pi (0.75)^2 \cdot 8$$

$$V = \pi 0.5625 \cdot 8$$

$$V = 4.5\pi$$

$$D = \frac{25}{4.5\pi}$$

$$D = 5.555555556\pi$$

$$D = 17.45329252$$

$$D = 17 \text{ pounds}$$

**Score 1:** The student correctly determined the volume.

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**Question 27**

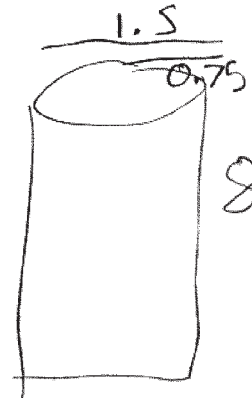
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**27** A section of tree trunk from a white pine tree can be modeled by a cylinder. The diameter of the trunk is 1.5 feet, and the height of this section is 8 feet.

If white pine weighs 25 pounds per cubic foot, determine and state the weight of this section of the white pine tree, to the *nearest pound*.

$$(0.75^2 \pi) \cdot 8 = 5.964117303$$
$$5.96 \dots \cdot 25 = 149.1029$$

149 pounds



**Score 1:** The student made an error when determining the volume, but found an appropriate weight.

### Question 27

**27** A section of tree trunk from a white pine tree can be modeled by a cylinder. The diameter of the trunk is 1.5 feet, and the height of this section is 8 feet.

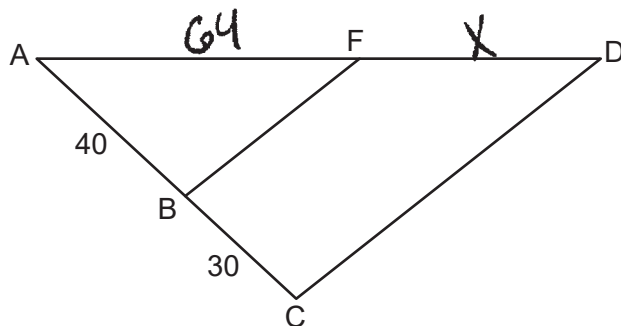
If white pine weighs 25 pounds per cubic foot, determine and state the weight of this section of the white pine tree, to the *nearest pound*.


$$V = 7.068$$
$$\begin{array}{r} 25 \\ \times 7.068 \\ \hline \end{array}$$
$$W = 176.7 \text{ lbs}$$

**Score 0:** The student made an error when determining the volume of the section of tree trunk and made a rounding error.

# Question 28

28 In  $\triangle ADC$  below, points  $B$  and  $F$  are on  $\overline{AC}$  and  $\overline{AD}$ , respectively, such that  $AB = 40$ ,  $BC = 30$ , and  $\overline{BF}$  is drawn.



If  $AF = 64$ , determine and state the length of  $\overline{FD}$  that would prove  $\overline{BF} \parallel \overline{CD}$ .

~~$$\frac{64}{40} = \frac{x}{30}$$~~

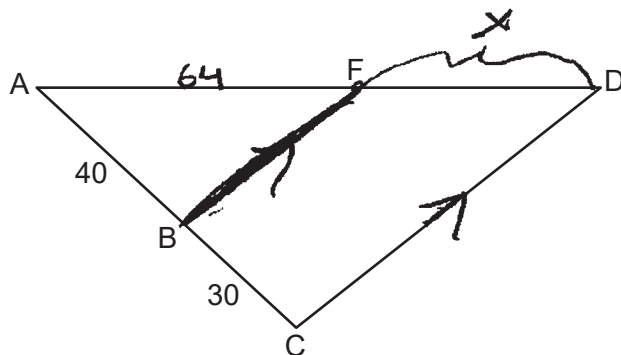
$$\frac{40x}{40} = \frac{1920}{40}$$

$$x = 48$$

**Score 2:** The student gave a complete and correct response.

Question 28

28 In  $\triangle ADC$  below, points  $B$  and  $F$  are on  $\overline{AC}$  and  $\overline{AD}$ , respectively, such that  $\underline{AB = 40}$ ,  $\underline{BC = 30}$ , and  $\overline{BF}$  is drawn.



If  $\underline{AF = 64}$ , determine and state the length of  $\underline{FD}$  that would prove  $\underline{BF \parallel CD}$ .

$$\frac{64}{x} = \frac{40}{30}$$

$$\frac{40x}{40} = \frac{1920}{40}$$

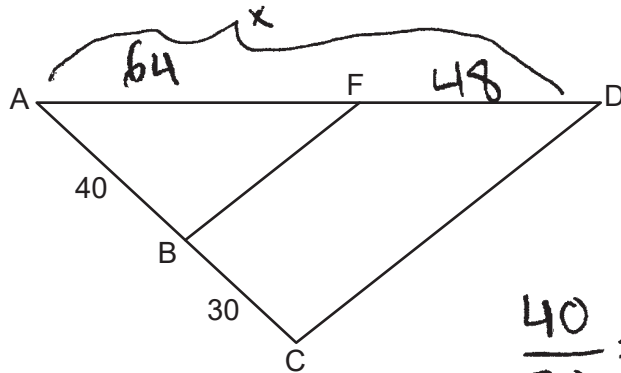
$$x = 48$$

$$\boxed{FD = 48}$$

**Score 2:** The student gave a complete and correct response.

Question 28

28 In  $\triangle ADC$  below, points  $B$  and  $F$  are on  $\overline{AC}$  and  $\overline{AD}$ , respectively, such that  $AB = 40$ ,  $BC = 30$ , and  $\overline{BF}$  is drawn.



$$\frac{40}{70} = 57.14285\%$$

If  $AF = 64$ , determine and state the length of  $\overline{FD}$  that would prove  $\overline{BF} \parallel \overline{CD}$ .

$$\frac{57.14285}{100} = \frac{64}{x}$$

$$6400 = 57.14285x$$

$$112 = x$$

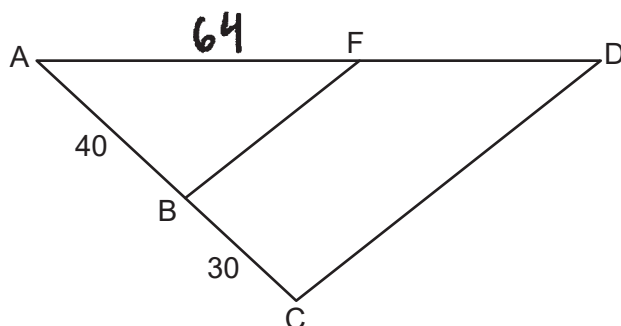
$$112 - 64 =$$

48

**Score 2:** The student gave a complete and correct response.

# Question 28

28 In  $\triangle ADC$  below, points  $B$  and  $F$  are on  $\overline{AC}$  and  $\overline{AD}$ , respectively, such that  $AB = 40$ ,  $BC = 30$ , and  $\overline{BF}$  is drawn.



If  $AF = 64$ , determine and state the length of  $\overline{FD}$  that would prove  $\overline{BF} \parallel \overline{CD}$ .

$$\frac{40}{70} = \frac{64}{x}$$

$$\frac{40x}{40} = \frac{4480}{40}$$

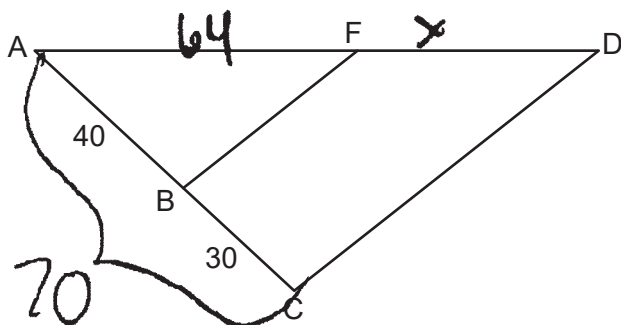
$$x = 112$$

$$112$$

**Score 1:** The student correctly determined the length of  $\overline{AD}$ .

Question 28

28 In  $\triangle ADC$  below, points  $B$  and  $F$  are on  $\overline{AC}$  and  $\overline{AD}$ , respectively, such that  $AB = 40$ ,  $BC = 30$ , and  $\overline{BF}$  is drawn.



If  $AF = 64$ , determine and state the length of  $\overline{FD}$  that would prove  $\overline{BF} \parallel \overline{CD}$ .

$$\frac{64}{40} = \frac{64+x}{70}$$

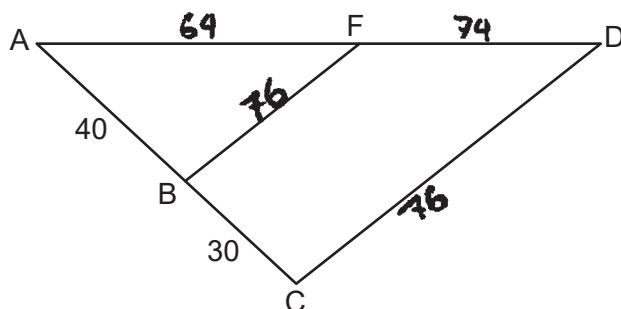
$$\frac{2560x = 1120}{2560} \quad \frac{2560}{2560}$$

$$x = 21.75$$

**Score 1:** The student wrote a correct proportion, but made an error when cross-multiplying.

Question 28

28 In  $\triangle ADC$  below, points  $B$  and  $F$  are on  $\overline{AC}$  and  $\overline{AD}$ , respectively, such that  $AB = 40$ ,  $BC = 30$ , and  $\overline{BF}$  is drawn.



If  $AF = 64$ , determine and state the length of  $\overline{FD}$  that would prove  $\overline{BF} \parallel \overline{CD}$ .

$$AB + AF \quad (40 + 64) - 180 = 76$$

$$BC + CD \quad (30 + 76) - 180 = 74$$

$$[\overline{BF} \parallel \overline{CD}]$$

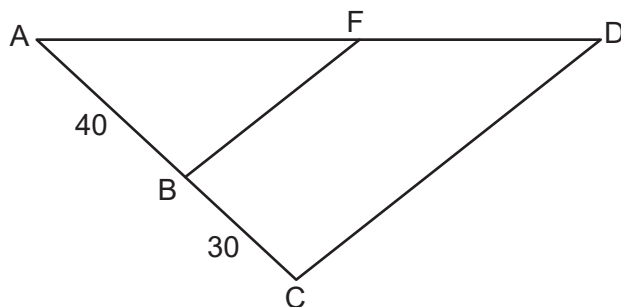
**Score 0:** The student did not show enough correct relevant work to receive any credit.

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**Question 28**

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**28** In  $\triangle ADC$  below, points  $B$  and  $F$  are on  $\overline{AC}$  and  $\overline{AD}$ , respectively, such that  $AB = 40$ ,  $BC = 30$ , and  $\overline{BF}$  is drawn.



If  $AF = 64$ , determine and state the length of  $\overline{FD}$  that would prove  $\overline{BF} \parallel \overline{CD}$ .

$$\begin{array}{r} 40 + 30 = 70 \\ + 64 \\ \hline 134 \end{array}$$

~~134~~

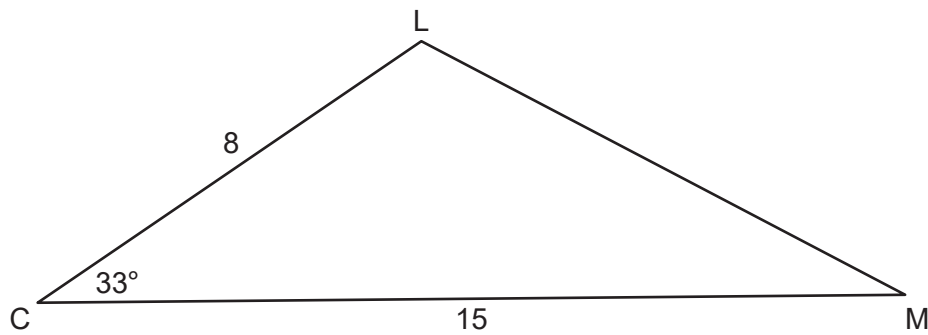
**Score 0:** The student did not show enough correct relevant work to receive any credit.

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**Question 29**

---

**29** In  $\triangle CLM$  below,  $m\angle C = 33^\circ$ ,  $CL = 8$ , and  $CM = 15$ .



Determine and state, to the *nearest tenth*, the area of  $\triangle CLM$ .

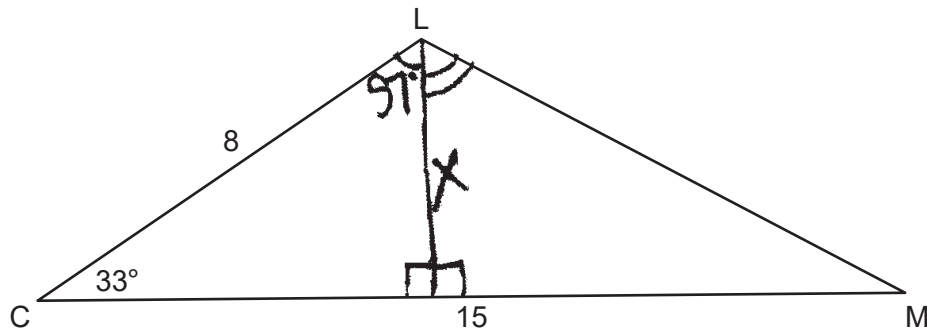
$$\frac{1}{2}(8)(15)\sin 33$$
$$A = 32.7$$

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**Score 2:** The student gave a complete and correct response.

Question 29

29 In  $\triangle CLM$  below,  $m\angle C = 33^\circ$ ,  $CL = 8$ , and  $CM = 15$ .



Determine and state, to the *nearest tenth*, the area of  $\triangle CLM$ .

$$\frac{\cos(57)}{1} = \frac{x}{8} \quad x \approx 4.357$$

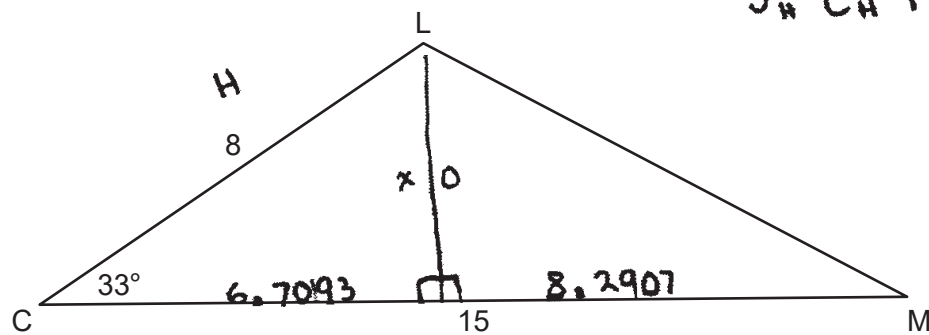
$$\frac{15 \cdot 4.357}{2} \approx 32.7$$

$$\boxed{\text{area of } \triangle CLM \approx 32.7}$$

**Score 2:** The student gave a complete and correct response.

Question 29

29 In  $\triangle CLM$  below,  $m\angle C = 33^\circ$ ,  $CL = 8$ , and  $CM = 15$ .



Determine and state, to the nearest tenth, the area of  $\triangle CLM$ .

$$15 - 6.7093 = 8.2907$$

$$\frac{\sin 33}{1} = \frac{x}{8}$$

$$x = \sin 33 (8)$$

$$x = 4.3571$$

$$(4.3571)^2 + b^2 = (8)^2$$

$$18.9843 + b^2 = 64$$

$$-18.9843 \quad -18.9843$$

$$\sqrt{b^2} = \sqrt{45.0157}$$

$$b = 6.7093$$

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(6.7093)(4.3571)$$

$$A = 14.6165$$

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(8.2907)(4.3571)$$

$$A = 18.0617$$

$$18.0617 + 14.6165 = 32.7 \text{ units}^2$$

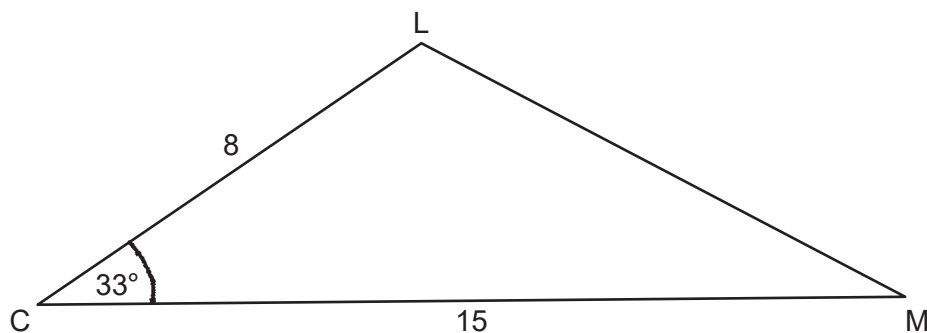
**Score 2:** The student gave a complete and correct response.

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**Question 29**

---

**29** In  $\triangle CLM$  below,  $m\angle C = 33^\circ$ ,  $CL = 8$ , and  $CM = 15$ .



Determine and state, to the nearest tenth, the area of  $\triangle CLM$ .

$$A_d = ab \sin C$$

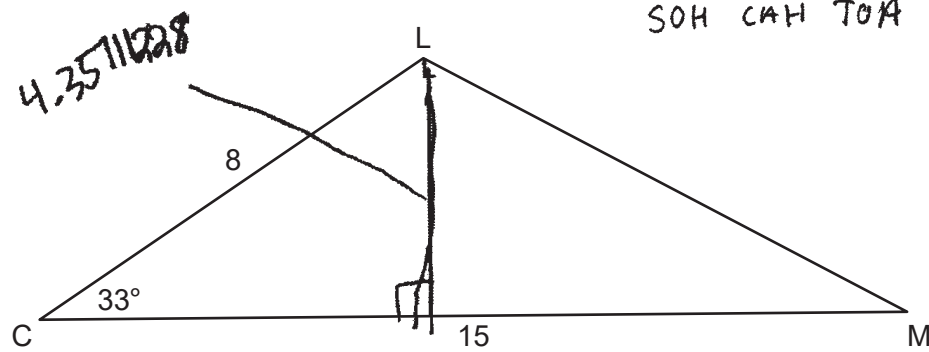
$$A_d = 8 \cdot 15 \cdot \sin 33$$

$$A_d \approx 65.4$$

**Score 1:** The student made an error by not using  $\frac{1}{2}$  in the formula, but found an appropriate answer.

Question 29

29 In  $\triangle CLM$  below,  $m\angle C = 33^\circ$ ,  $CL = 8$ , and  $CM = 15$ .



Determine and state, to the nearest tenth, the area of  $\triangle CLM$ .

$$A = \frac{1}{2}(b)(h)$$

$$\frac{\sin(33)}{1} = \frac{x}{8}$$

$$x = 4.3571228$$

$$A = \frac{1}{2}(15)(4.3571228)$$

$$A =$$

$$A \approx 32.68 \text{ units}^2$$

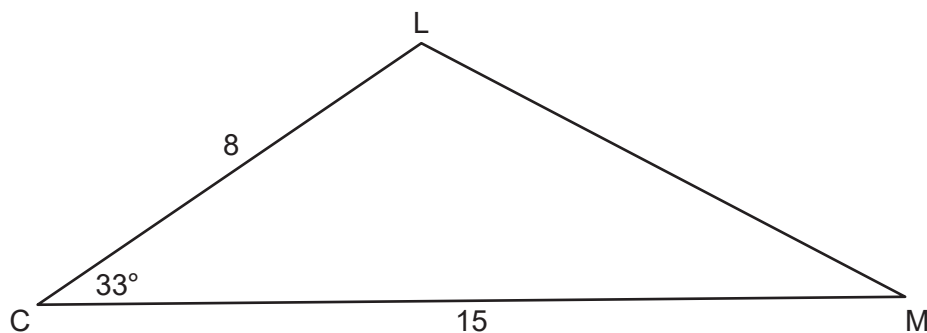
**Score 1:** The student made a rounding error.

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**Question 29**

---

**29** In  $\triangle CLM$  below,  $m\angle C = 33^\circ$ ,  $CL = 8$ , and  $CM = 15$ .



Determine and state, to the *nearest tenth*, the area of  $\triangle CLM$ .

$$A = \frac{1}{2} b \cdot h$$

$$A = \left(\frac{1}{2}\right)(15)(8)$$

$$A = 60.0$$

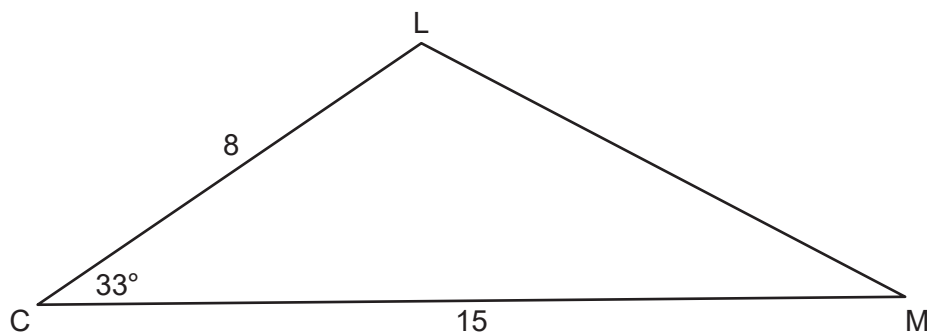
**Score 0:** The student did not show enough correct relevant course-level work to receive any credit.

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**Question 29**

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**29** In  $\triangle CLM$  below,  $m\angle C = 33^\circ$ ,  $CL = 8$ , and  $CM = 15$ .



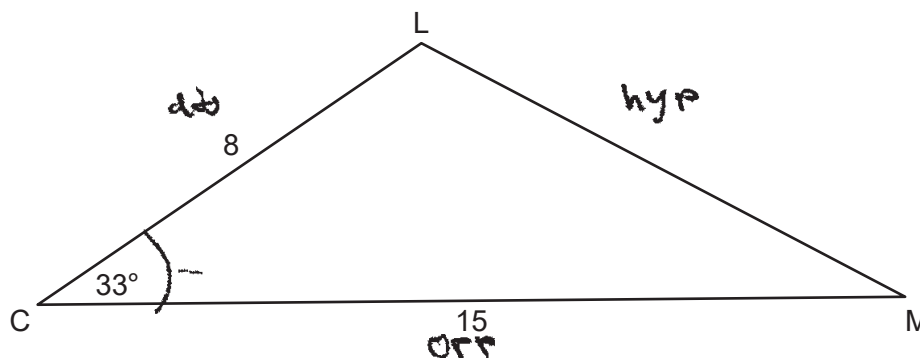
Determine and state, to the *nearest tenth*, the area of  $\triangle CLM$ .

$$A = \frac{1}{2}(15)(33)(\tan 33) = 8.6$$

**Score 0:** The student did not show enough correct relevant course-level work to receive any credit.

Question 29

29 In  $\triangle CLM$  below,  $m\angle C = 33^\circ$ ,  $CL = 8$ , and  $CM = 15$ .



Determine and state, to the *nearest tenth*, the area of  $\triangle CLM$ .

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 8^2 + 15^2 &= c^2 \\ \downarrow \\ \frac{289}{33} &= c \quad \text{or} \quad \sqrt{289} = 17 \\ c &= 8.8 \end{aligned}$$

**Score 0:** The student did not show enough correct relevant course-level work to receive any credit.

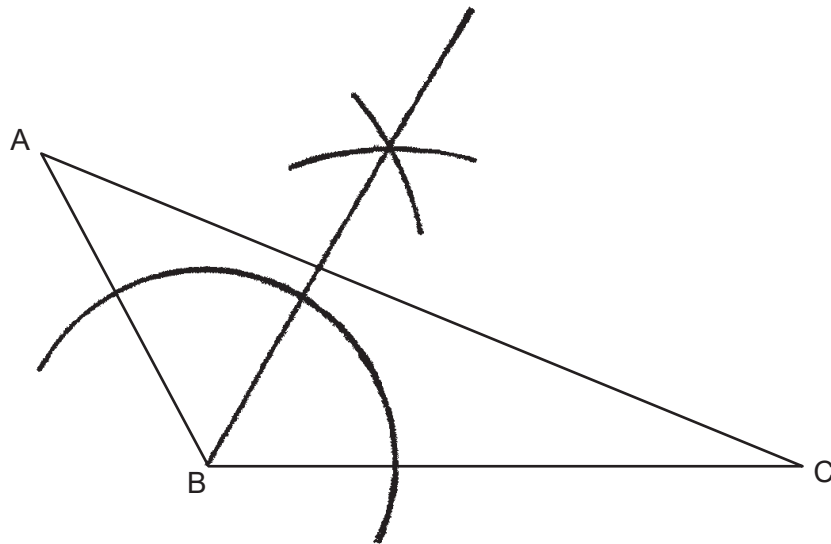
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**Question 30**

---

**30** In the diagram of  $\triangle ABC$  below, use a compass and straightedge to construct the angle bisector of  $\angle ABC$ .

[Leave all construction marks.]



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**Score 2:** The student gave a complete and correct response.

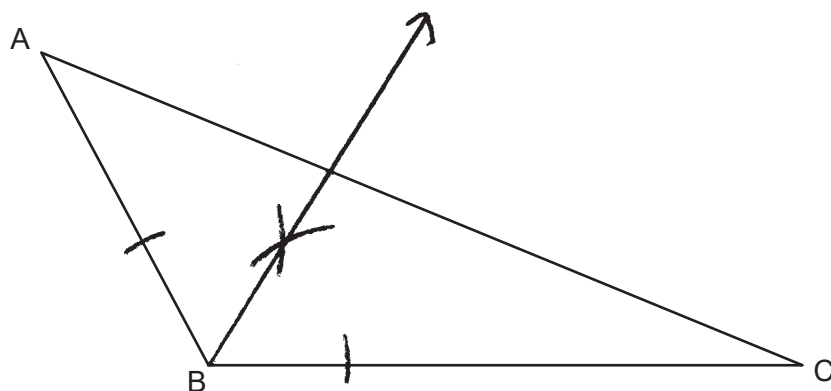
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**Question 30**

---

**30** In the diagram of  $\triangle ABC$  below, use a compass and straightedge to construct the angle bisector of  $\angle ABC$ .

[Leave all construction marks.]



**Score 2:** The student gave a complete and correct response.

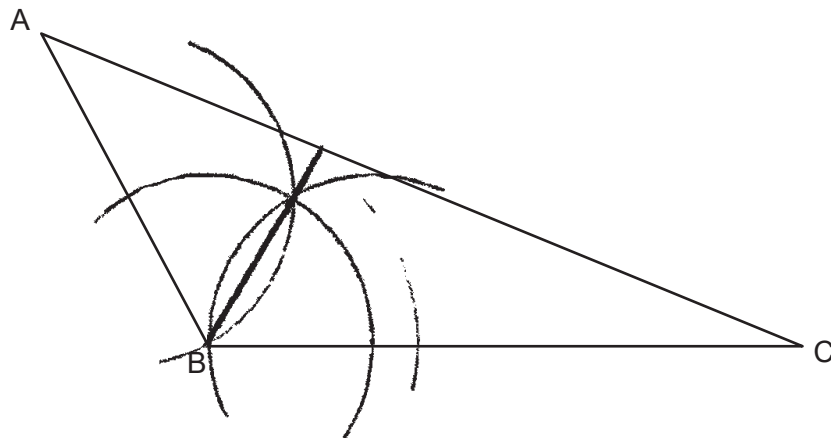
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**Question 30**

---

**30** In the diagram of  $\triangle ABC$  below, use a compass and straightedge to construct the angle bisector of  $\angle ABC$ .

[Leave all construction marks.]



**Score 2:** The student gave a complete and correct response.

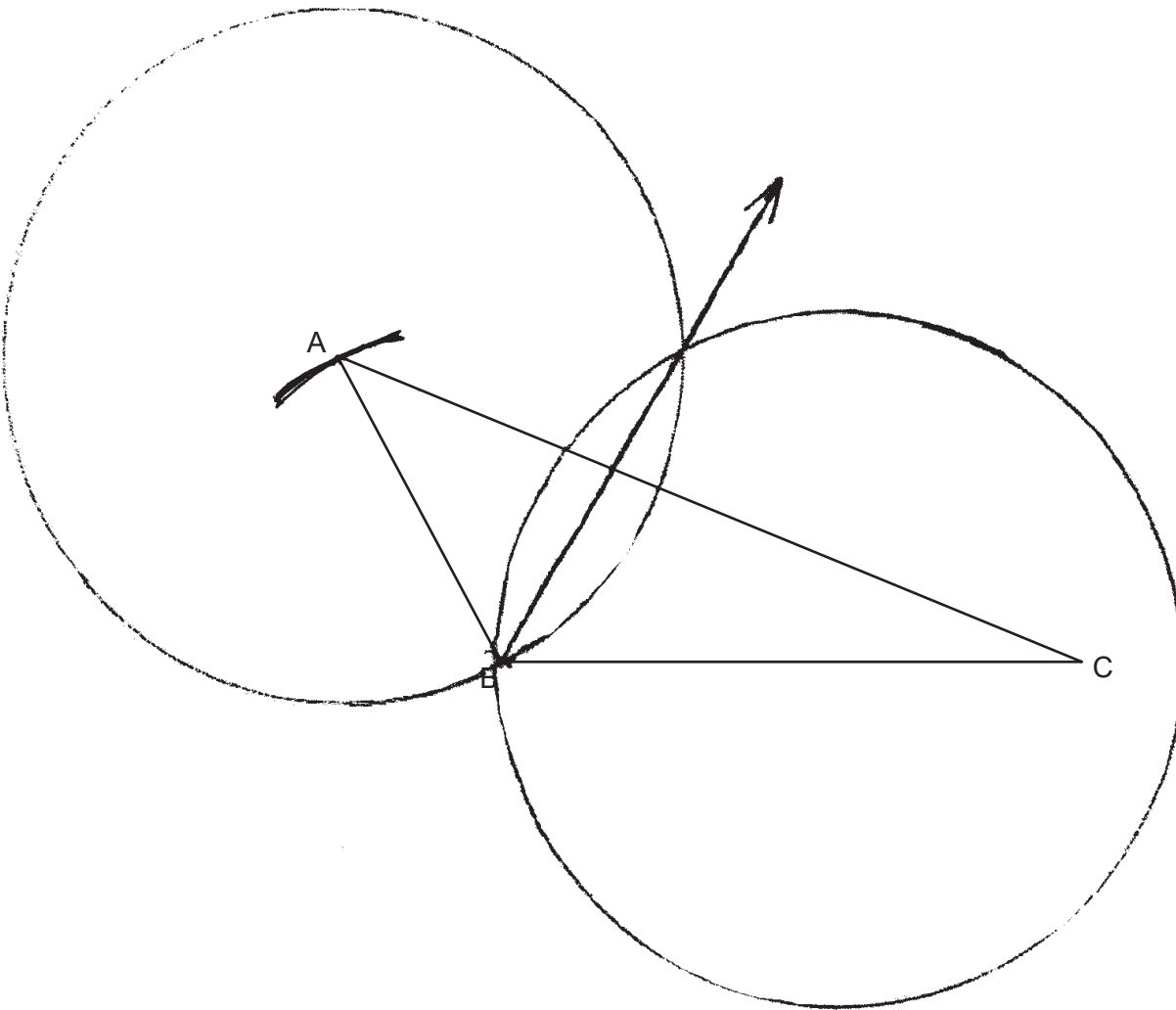
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**Question 30**

---

**30** In the diagram of  $\triangle ABC$  below, use a compass and straightedge to construct the angle bisector of  $\angle ABC$ .

[Leave all construction marks.]



**Score 1:** The student did not construct the arc on  $\overline{BC}$  that is the center of the second circle.

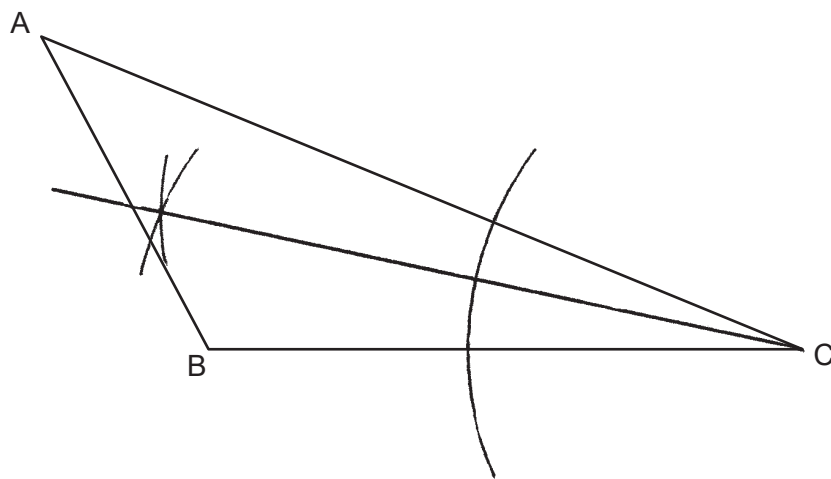
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**Question 30**

---

**30** In the diagram of  $\triangle ABC$  below, use a compass and straightedge to construct the angle bisector of  $\angle ABC$ .

[Leave all construction marks.]



**Score 1:** The student constructed the bisector of  $\angle C$ .

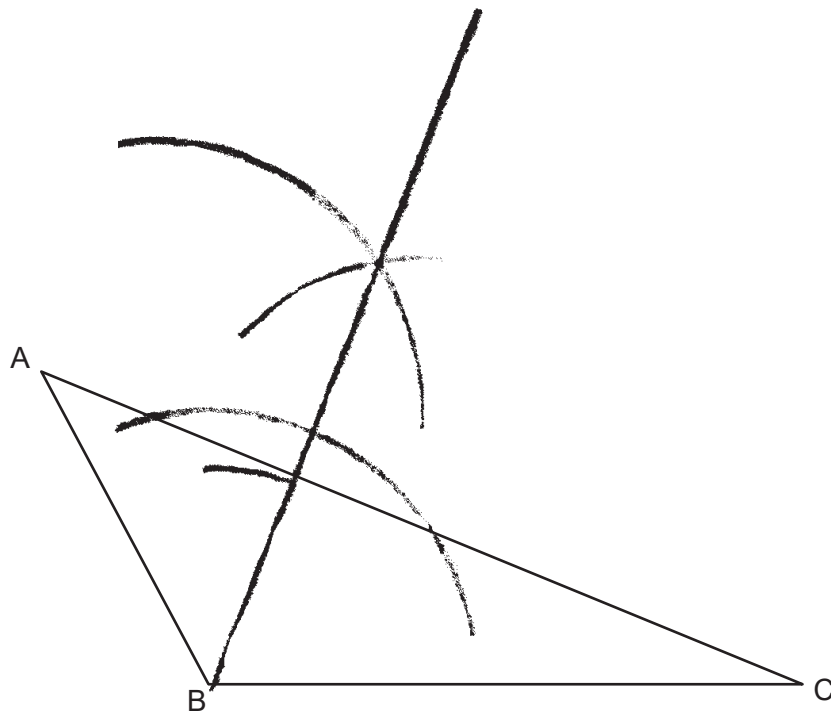
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**Question 30**

---

**30** In the diagram of  $\triangle ABC$  below, use a compass and straightedge to construct the angle bisector of  $\angle ABC$ .

[Leave all construction marks.]



**Score 0:** The student constructed an altitude from vertex  $B$ .

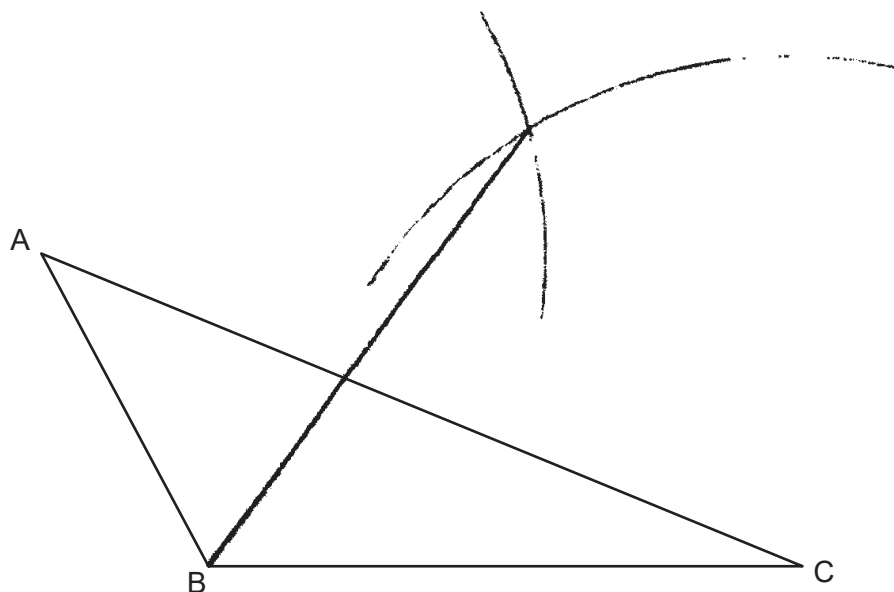
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**Question 30**

---

**30** In the diagram of  $\triangle ABC$  below, use a compass and straightedge to construct the angle bisector of  $\angle ABC$ .

[Leave all construction marks.]



**Score 0:** The student did not show enough correct relevant course-level work to receive any credit.

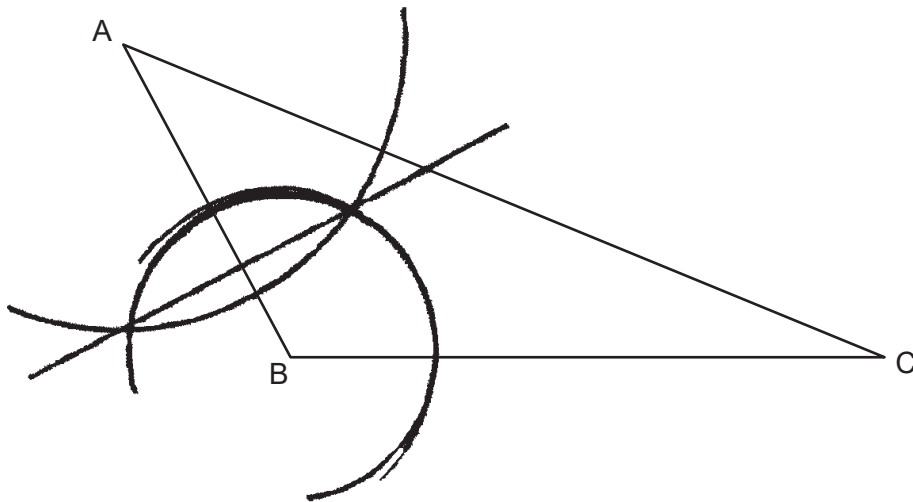
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**Question 30**

---

**30** In the diagram of  $\triangle ABC$  below, use a compass and straightedge to construct the angle bisector of  $\angle ABC$ .

[Leave all construction marks.]

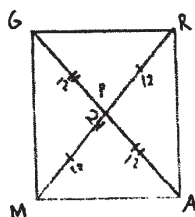


**Score 0:** The student did not show enough correct relevant course-level work to receive any credit.

### Question 31

31 The diagonals of parallelogram  $GRAM$  intersect at  $P$ .

If  $RP = 12$  and  $GA = 24$ , explain why  $GRAM$  is a rectangle.



$GRAM$  is  $\square$ ,  $RP = 12$ ,  $GA = 24$

$\overline{MP} \cong \overline{PR}$  and  $\overline{GP} \cong \overline{PA}$  because diagonals bisect each other in parallelograms.  
 $GP = 12$ ,  $PA = 12$  since segment bisectors divide a segment into two equal halves.  
 $24/2 = 12$

Since  $\overline{MP} \cong \overline{PR}$ ,  $MP = 12$

$GP + PA = MP + PR$ , so diagonals  $\overline{GA} \cong \overline{MR}$ .  
 $12 + 12 = 12 + 12$

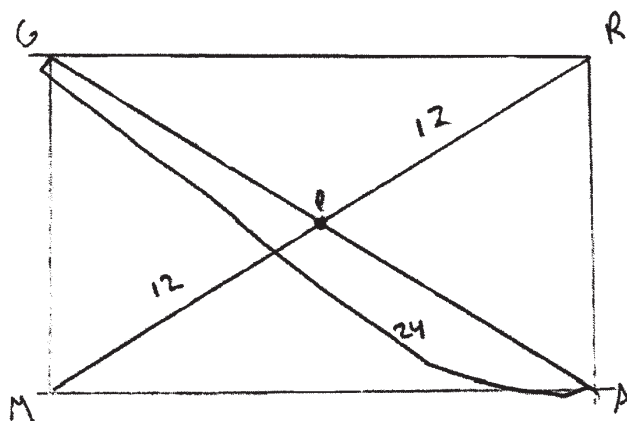
$GRAM$  is a rectangle since rectangles are parallelograms with  $\cong$  diagonals.

**Score 2:** The student gave a complete and correct response.

### Question 31

31 The diagonals of parallelogram  $GRAM$  intersect at  $P$ .

If  $RP = 12$  and  $GA = 24$ , explain why  $GRAM$  is a rectangle.



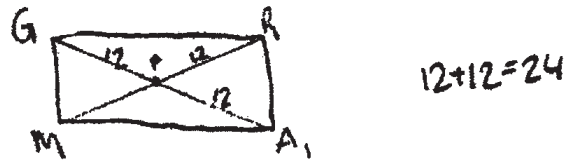
| Statement                                           | Reason                                                                                     |
|-----------------------------------------------------|--------------------------------------------------------------------------------------------|
| 1) $GRAM$ is a parallelogram<br>$RP = 12$ $GA = 24$ | 1) Given                                                                                   |
| 2) $\overline{RP} \cong \overline{MP}$              | 2) Parallelograms diagonals bisect each other, bisector cuts segment into 2 $\cong$ parts. |
| 3) $\overline{RM} \cong \overline{GA}$              | 3) $24 = 24$                                                                               |
| 4) $GRAM$ is a rectangle                            | 4) $PGRAM$ w/ $\cong$ diagonals is a rectangle                                             |

**Score 2:** The student gave a complete and correct response.

### Question 31

31 The diagonals of parallelogram  $GRAM$  intersect at  $P$ .

If  $RP = 12$  and  $GA = 24$ , explain why  $GRAM$  is a rectangle.



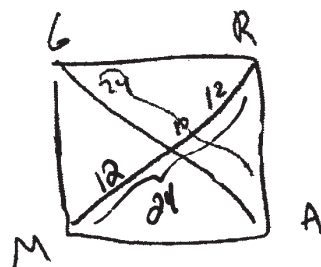
We know that  $GRAM$  is a rectangle because the diagonals of a parallelogram bisect each other, so if diagonal  $GA = 24$  and the diagonals intersect at  $P$   $GP = 12$   $PA = 12$  since  $24 \div 2 = 12$ . And since  $RP = 12$  we know  $RM = 24$  for the same reason diagonals of a parallelogram bisect each other so if half the diagonal is 12 the whole diagonal is 24. Now that I've proven  $RM = 24$  and  $GA = 24$   $\overline{RM} \cong \overline{GA}$  therefore parallelogram  $GRAM$  is a rectangle since the diagonals are congruent.

**Score 2:** The student gave a complete and correct response.

### Question 31

31 The diagonals of parallelogram  $GRAM$  intersect at  $P$ .

If  $RP = 12$  and  $GA = 24$ , explain why  $GRAM$  is a rectangle.



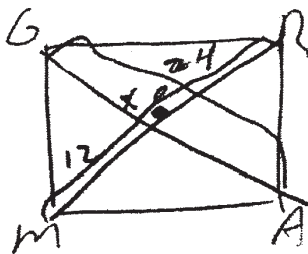
$\overline{RM} \cong \overline{GA}$ ,  $24 = 24$  Parallelogram  
 $GRAM$  is a rectangle  
because diagonals are  $\cong$

**Score 2:** The student gave a complete and correct response.

### Question 31

31 The diagonals of parallelogram  $GRAM$  intersect at  $P$ .

If  $RP = 12$  and  $GA = 24$ , explain why  $GRAM$  is a rectangle.



$$\begin{array}{r} 24 \\ -12 \\ \hline 12 \end{array}$$

$$\begin{array}{r} 12 \\ +12 \\ \hline 24 \end{array}$$

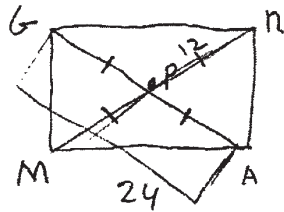
$$\therefore \overline{GA} \cong \overline{RM}$$

**Score 1:** The student wrote a partially correct explanation.

### Question 31

31 The diagonals of parallelogram  $GRAM$  intersect at  $P$ .

If  $RP = 12$  and  $GA = 24$ , explain why  $GRAM$  is a rectangle.



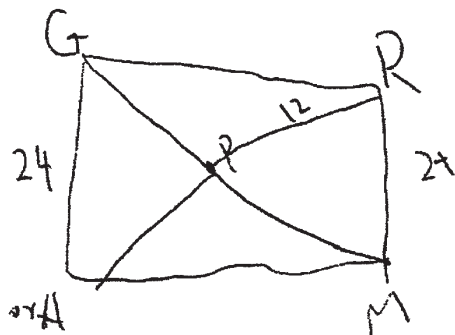
for parallelogram  
 $GRAM$  to be a  
rectangle the  
diagonals of the  
shape have to  
intersect at the  
midpoints which they  
do at point  $P$ .  
also bc all of  
the diagonals are  
 $\cong$  this means the  
opposite sides are  
 $\cong$  making  $GRAM$   
a rectangle

**Score 1:** The student wrote a partially correct explanation.

### Question 31

31 The diagonals of parallelogram  $GRAM$  intersect at  $P$ .

If  $RP = 12$  and  $GA = 24$ , explain why  $GRAM$  is a rectangle.



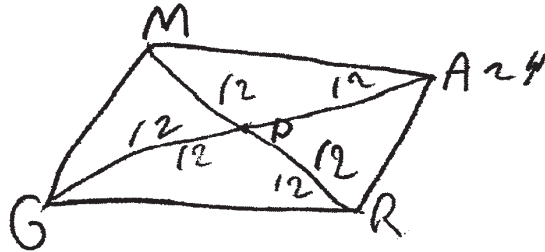
$GRAM$  is a rectangle  
because  $GA$  and  $RM$  will have the  
same side length and they  
all have intersect at  $P$

**Score 0:** The student did not show enough correct relevant course-level work to receive any credit.

### Question 31

31 The diagonals of parallelogram  $GRAM$  intersect at  $P$ .

If  $RP = 12$  and  $GA = 24$ , explain why  $GRAM$  is a rectangle.

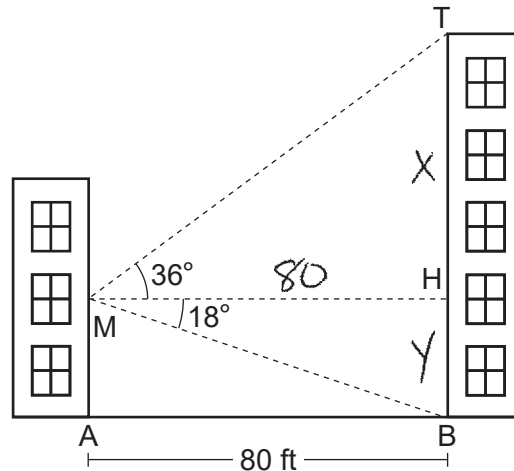


GRAM is a rectangle because  
it has equal sides like a  
square does, but it doesn't have  
4  $90^\circ$  angles so instead of a  
square it's a rectangle

**Score 0:** The student did not show enough correct relevant course-level work to receive any credit.

**Question 32**

- 32** As modeled below, Maria wants to determine the height of the building across the street from her position,  $M$ . The angle of elevation from  $M$  to the top of the building,  $T$ , is  $36^\circ$ . From  $M$ , the angle of depression to the base of the building,  $B$ , is  $18^\circ$ . The buildings are 80 feet apart.



Determine and state, to the *nearest foot*, the height of the building,  $TB$ , across the street from Maria.

$$\tan 36 = \frac{x}{80}$$

$$\tan 18 = \frac{y}{80}$$

$$x = 80 (\tan 36)$$

$$y = 80 (\tan 18)$$

$$x = 58.12340224$$

$$y = 25.9935757$$

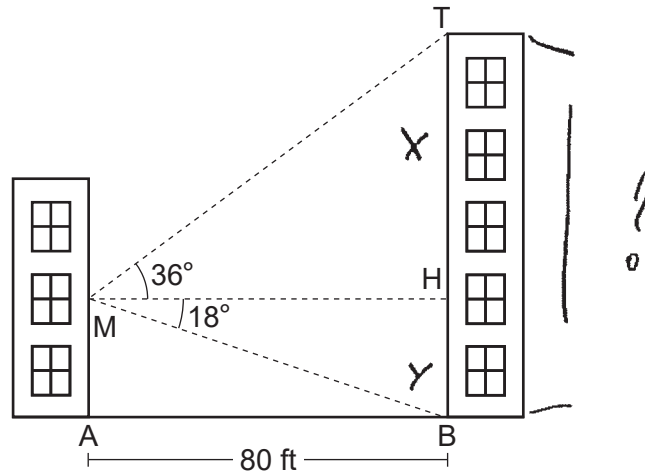
$$x + y = 84.11697794$$

$$\textcircled{84}$$

**Score 4:** The student gave a complete and correct response.

### Question 32

- 32** As modeled below, Maria wants to determine the height of the building across the street from her position,  $M$ . The angle of elevation from  $M$  to the top of the building,  $T$ , is  $36^\circ$ . From  $M$ , the angle of depression to the base of the building,  $B$ , is  $18^\circ$ . The buildings are 80 feet apart.



Determine and state, to the nearest foot, the height of the building,  $TB$ , across the street from Maria.

Schonheit

1 0 9  
n s n

hyp

$36^\circ$

80 adj

opp

$\tan 36^\circ = \frac{X}{80}$

$X = 58$

hyp

$18^\circ$

80 adj

opp

$\tan 18^\circ = \frac{Y}{80}$

$Y = 26$

$58$   
 $+ 26$   

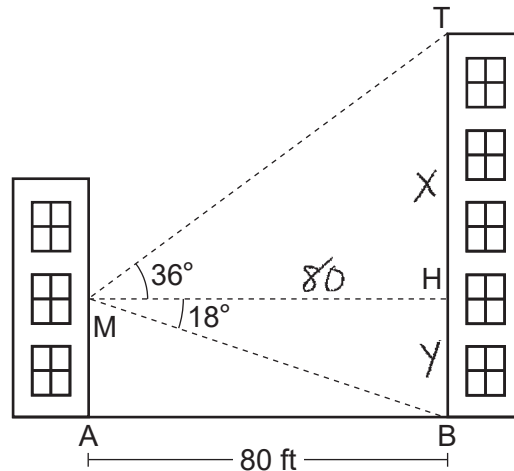
---

 $84$  is the height of the building

**Score 4:** The student gave a complete and correct response.

Question 32

- 32 As modeled below, Maria wants to determine the height of the building across the street from her position,  $M$ . The angle of elevation from  $M$  to the top of the building,  $T$ , is  $36^\circ$ . From  $M$ , the angle of depression to the base of the building,  $B$ , is  $18^\circ$ . The buildings are 80 feet apart.



Determine and state, to the *nearest foot*, the height of the building,  $TB$ , across the street from Maria.

$$\tan 36 = \frac{x}{80}$$

$$\sin 18 = \frac{y}{80}$$

$$x = (\tan 36)(80)$$

$$y = (\sin 18)(80)$$

$$x = 58.12340224$$

$$y = 24.72135955$$

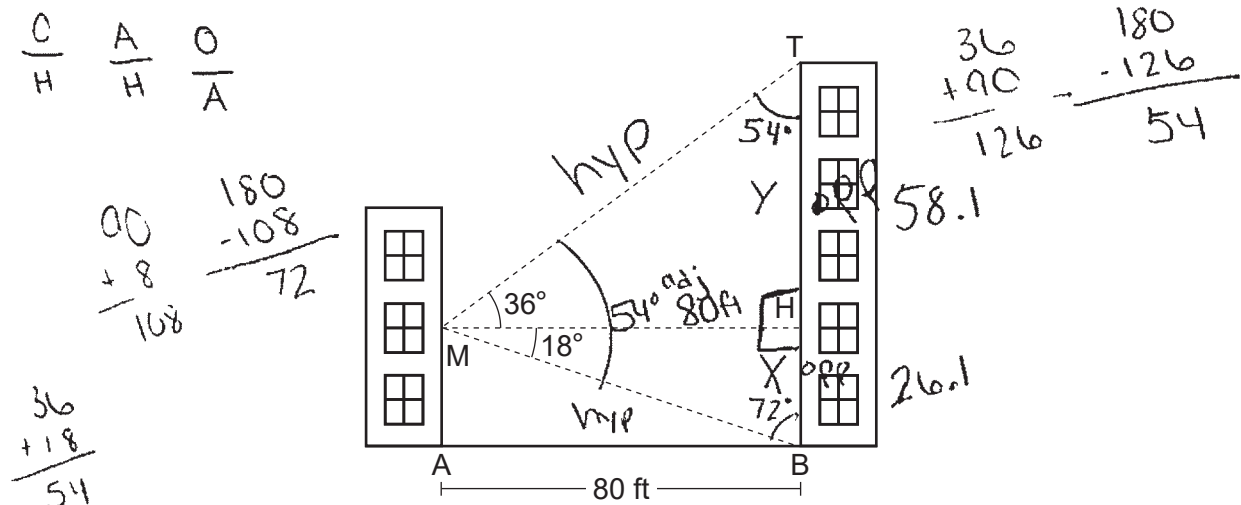
$$x + y = 82.84476179$$

$$\textcircled{83}$$

**Score 3:** The student made an error when determining the length of  $\overline{HB}$ , but found an appropriate length for  $\overline{TB}$ .

Question 32

- 32 As modeled below, Maria wants to determine the height of the building across the street from her position,  $M$ . The angle of elevation from  $M$  to the top of the building,  $T$ , is  $36^\circ$ . From  $M$ , the angle of depression to the base of the building,  $B$ , is  $18^\circ$ . The buildings are 80 feet apart.



Determine and state, to the nearest foot, the height of the building,  $TB$ , across the street from Maria.

$$\frac{\tan 18^\circ}{1} = \frac{x}{80}$$

$$\frac{\tan 36^\circ}{1} = \frac{y}{80}$$

$$x = 80 \tan 18^\circ$$

$$y = 80 \tan 36^\circ$$

$$x = 26.1$$

$$y = 58.1$$

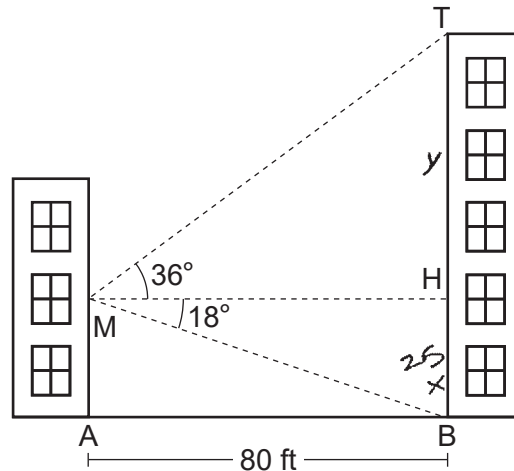
$$\begin{array}{r} 58.1 \\ + 26.1 \\ \hline 84.2 \end{array}$$

height of building  $TB = 84$  ft

**Score 3:** The student made a computational error when determining the length of  $\overline{HB}$ .

### Question 32

- 32** As modeled below, Maria wants to determine the height of the building across the street from her position,  $M$ . The angle of elevation from  $M$  to the top of the building,  $T$ , is  $36^\circ$ . From  $M$ , the angle of depression to the base of the building,  $B$ , is  $18^\circ$ . The buildings are 80 feet apart.



Determine and state, to the *nearest foot*, the height of the building,  $TB$ , across the street from Maria.

SOH ~~CAH~~ <sup>CAH</sup> TOA

$$\frac{\sin 18}{1} = \frac{x}{80}$$

$$\frac{\tan 36}{1} = \frac{y}{80}$$

$$80 \sin 18 = x$$

$$x \approx 25$$

$$80 \tan 36 = y$$

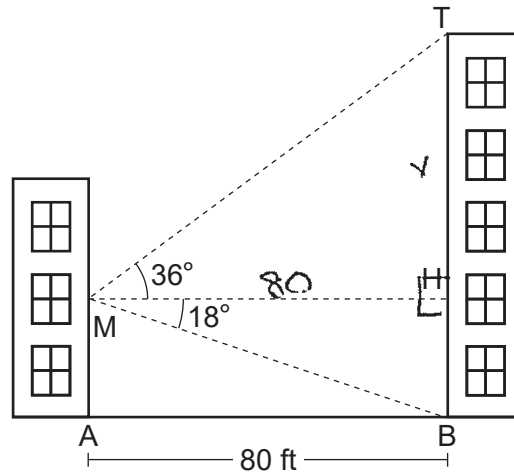
$$y \approx 58 \text{ ft}$$

approx  
Building  $TB$  is  $\hat{84}$  ft

**Score 2:** The student made errors when determining the lengths of  $\overline{HB}$  and  $\overline{TB}$ .

**Question 32**

- 32** As modeled below, Maria wants to determine the height of the building across the street from her position,  $M$ . The angle of elevation from  $M$  to the top of the building,  $T$ , is  $36^\circ$ . From  $M$ , the angle of depression to the base of the building,  $B$ , is  $18^\circ$ . The buildings are 80 feet apart.



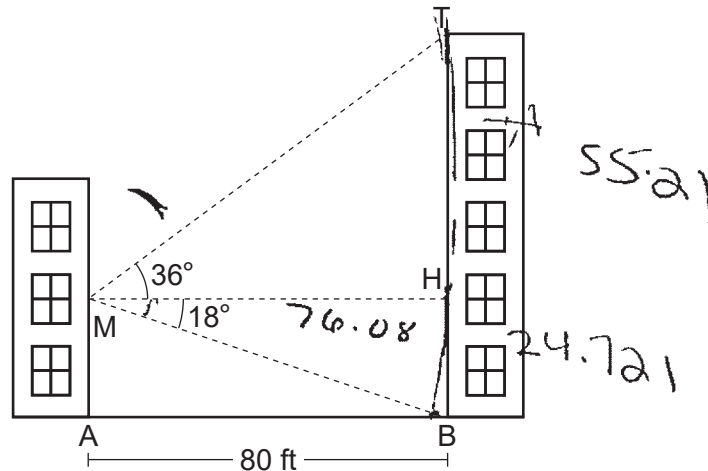
Determine and state, to the *nearest foot*, the height of the building,  $TB$ , across the street from Maria.

$$\begin{aligned}\tan(36) &= \frac{x}{80} \\ \tan(36) \cdot 80 &= x \\ 58.123 &= x\end{aligned}$$

**Score 2:** The student correctly determined the length of  $\overline{TH}$ .

### Question 32

- 32** As modeled below, Maria wants to determine the height of the building across the street from her position,  $M$ . The angle of elevation from  $M$  to the top of the building,  $T$ , is  $36^\circ$ . From  $M$ , the angle of depression to the base of the building,  $B$ , is  $18^\circ$ . The buildings are 80 feet apart.



Determine and state, to the *nearest foot*, the height of the building,  $TB$ , across the street from Maria.

$$\frac{\sin(18)}{1} = \frac{HB}{80}$$

$$HB = \sin(18)(80)$$

$$HB = 24.721$$

$$\frac{\cos(18)}{1} = \frac{MH}{80}$$

$$MH = \cos(18)(80)$$

$$MH = 76.08$$

$$\frac{\tan(36)}{1} = \frac{TH}{76}$$

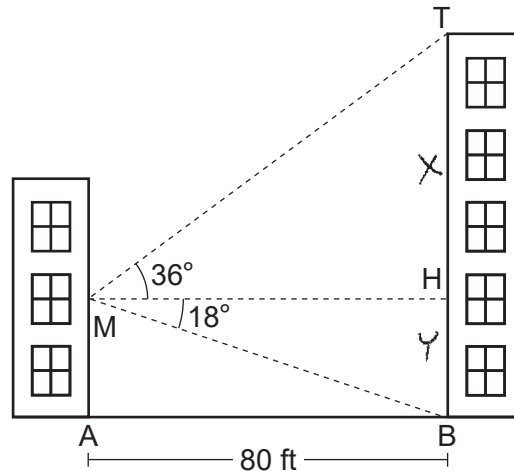
$$TH = \tan(36)(76)$$

$$TB = 79.938$$

**Score 1:** The student made errors when determining the lengths of  $\overline{HB}$  and  $\overline{MH}$  and made a rounding error when determining the length of  $\overline{TB}$ .

Question 32

- 32 As modeled below, Maria wants to determine the height of the building across the street from her position,  $M$ . The angle of elevation from  $M$  to the top of the building,  $T$ , is  $36^\circ$ . From  $M$ , the angle of depression to the base of the building,  $B$ , is  $18^\circ$ . The buildings are 80 feet apart.



Determine and state, to the *nearest foot*, the height of the building,  $TB$ , across the street from Maria.

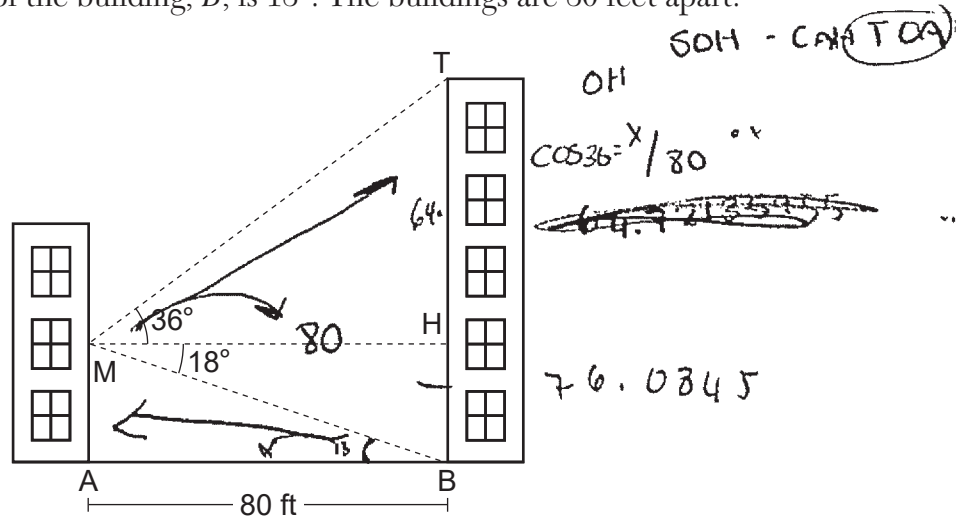
$$\tan 36^\circ = \frac{x}{80} \quad \left| \quad \sin 18^\circ = \frac{y}{80}$$

$$\begin{array}{l|l} x = .36 \times 80 & y = .18 \times 80 \\ x = 28.8 & y = 14.4 \\ x + y = \cancel{50} & \begin{array}{r} 28.8 \\ + 14.4 \\ \hline 43.2 \end{array} \end{array}$$

**Score 1:** The student wrote one correct relevant trigonometric equation.

### Question 32

- 32** As modeled below, Maria wants to determine the height of the building across the street from her position,  $M$ . The angle of elevation from  $M$  to the top of the building,  $T$ , is  $36^\circ$ . From  $M$ , the angle of depression to the base of the building,  $B$ , is  $18^\circ$ . The buildings are 80 feet apart.



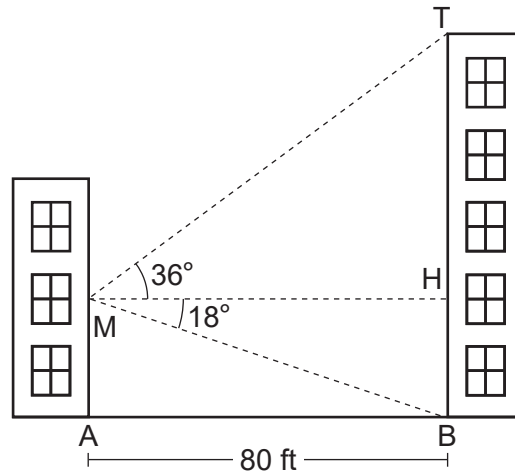
Determine and state, to the *nearest foot*, the height of the building,  $TB$ , across the street from Maria.

58.1234042...

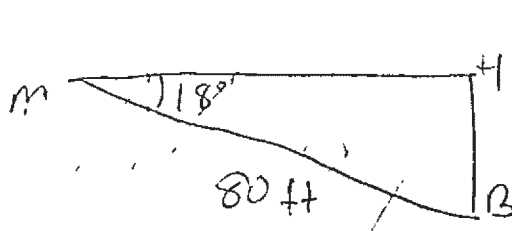
**Score 0:** The student did not show enough correct relevant course-level work to receive any credit.

### Question 32

- 32** As modeled below, Maria wants to determine the height of the building across the street from her position,  $M$ . The angle of elevation from  $M$  to the top of the building,  $T$ , is  $36^\circ$ . From  $M$ , the angle of depression to the base of the building,  $B$ , is  $18^\circ$ . The buildings are 80 feet apart.

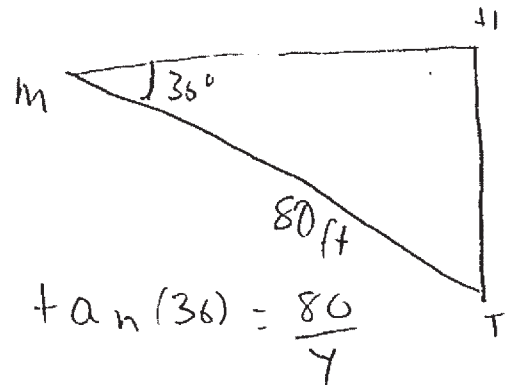


Determine and state, to the *nearest foot*, the height of the building,  $TB$ , across the street from Maria.



$$\tan(18) = \frac{80}{X}$$

$$80 \tan(18) = X$$



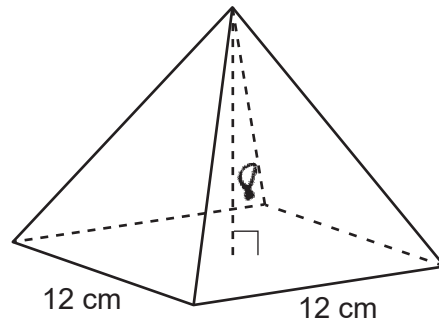
$$\tan(36) = \frac{80}{Y}$$

$$80 \tan(36) = Y$$

**Score 0:** The student did not show enough correct relevant course-level work to receive any credit.

### Question 33

- 33** An artist uses clay to make solid pyramids with a square base whose sides measure 12 cm, as modeled below.



The height of each pyramid is 8 cm, and the density of the clay is  $1.25 \text{ g/cm}^3$ . If the artist has 15 kilograms of clay, determine and state the maximum number of pyramids that can be made.

$$12 \cdot 12 = 144$$

$$m = d \cdot V$$

$$\frac{1}{3} \cdot 144 \cdot 8 = 384$$

$$384 \cdot 1.25 = 480 \text{ g} \quad 0.48 \text{ kg}$$

$$\frac{15}{0.48} = 31.25 \approx 31 \text{ pyramids}$$

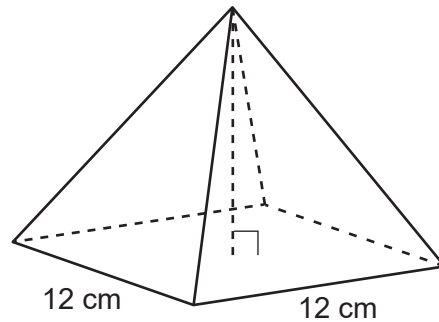
**Score 4:** The student gave a complete and correct response.

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**Question 33**

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- 33** An artist uses clay to make solid pyramids with a square base whose sides measure 12 cm, as modeled below.



The height of each pyramid is 8 cm, and the density of the clay is  $1.25 \text{ g/cm}^3$ . If the artist has 15 kilograms of clay, determine and state the maximum number of pyramids that can be made.

$$\frac{1}{3} 12^2 (8) = 384$$

$$\frac{1.25}{1} = \frac{m}{384 \text{ cm}^3}$$

$$\frac{15}{.48} = 31.25$$

$$m = 480 \text{ grams}$$

$$\frac{480 \text{ g}}{1000} = .48 \text{ kilograms}$$

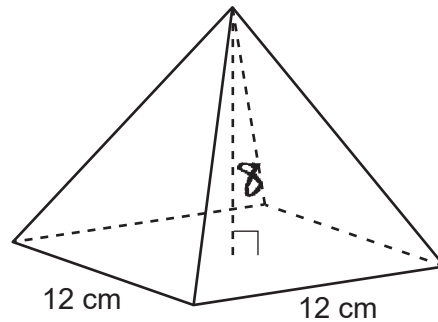
31 pyramids

---

**Score 4:** The student gave a complete and correct response.

### Question 33

**33** An artist uses clay to make solid pyramids with a square base whose sides measure 12 cm, as modeled below.



The height of each pyramid is 8 cm, and the density of the clay is  $1.25 \text{ g/cm}^3$ . If the artist has 15 kilograms of clay, determine and state the maximum number of pyramids that can be made.

$$V = \frac{1}{3}Bh$$

$$V = \frac{1}{3}(12 \cdot 12)(8)$$

$$V = \frac{1}{3}(144)(8)$$

$$V = \frac{1}{3}(1152)$$

$$V = 384$$

$$\begin{aligned} & \rightarrow 15000 \text{ g} \\ & D \cdot V \\ & (1.25)(384) \end{aligned}$$

$$480$$

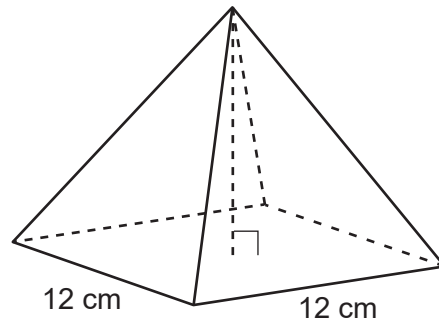
$$\frac{15000}{480} = 31.25$$

max 31 pyramids can be made

**Score 4:** The student gave a complete and correct response.

### Question 33

- 33 An artist uses clay to make solid pyramids with a square base whose sides measure 12 cm, as modeled below.



The height of each pyramid is 8 cm, and the density of the clay is  $1.25 \text{ g/cm}^3$ . If the artist has 15 kilograms of clay, determine and state the maximum number of pyramids that can be made.

$$15 \text{ kg} = 15,000 \text{ g}$$

$$D = \frac{M}{V}$$

$$1.25 = \frac{15,000}{x}$$

$$\frac{1.25x}{1.25} = \frac{15,000}{1.25}$$

$$x = 12,000 \text{ cm}^3$$

$$V = \frac{1}{3} B \cdot h$$

$$= \frac{1}{3} (12 \cdot 12) (8)$$

$$= \frac{1}{3} (1152)$$

$$V = 384 \text{ cm}^3$$

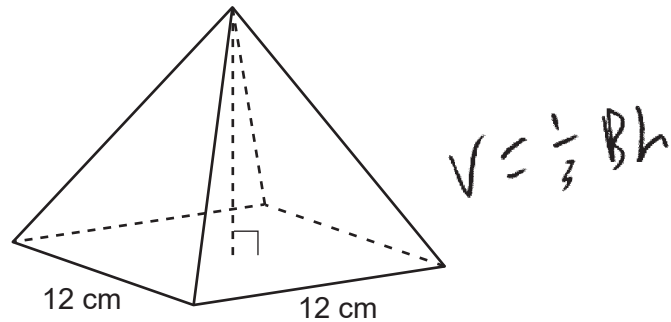
$$\frac{12,000}{384} = 31.25$$

31

**Score 4:** The student gave a complete and correct response.

### Question 33

- 33** An artist uses clay to make solid pyramids with a square base whose sides measure 12 cm, as modeled below.



The height of each pyramid is 8 cm, and the density of the clay is  $1.25 \text{ g/cm}^3$ . If the artist has 15 kilograms of clay, determine and state the maximum number of pyramids that can be made.

$$\frac{1}{3}(12 \times 8) = 32$$
$$32 \times 1.25 = 40g$$

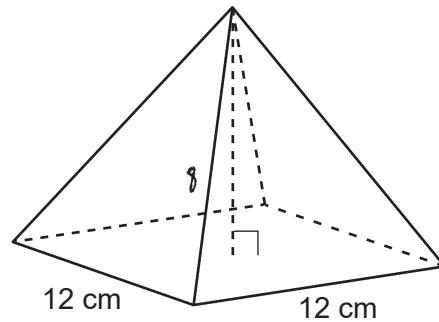
$$\frac{15000}{40} = 375$$

375 pyramids could  
be made

**Score 3:** The student made an error when determining the volume of one pyramid.

### Question 33

- 33** An artist uses clay to make solid pyramids with a square base whose sides measure 12 cm, as modeled below.



The height of each pyramid is 8 cm, and the density of the clay is  $1.25 \text{ g/cm}^3$ . If the artist has 15 kilograms of clay, determine and state the maximum number of pyramids that can be made.

$$V_{\text{of Pyramid}} = (8 \cdot 12 \cdot 12) \frac{1}{3}$$

$$V = 384 \text{ cm}^3$$

$$1.25 \times 384 = g$$

$$480 = g$$

$$480 / 100 = \text{kg}$$

$$4.8 = \text{kg}$$

$$15 / 4.8 = \text{max \# of pyramids}$$

$$3.125$$

↓

3 pyramids

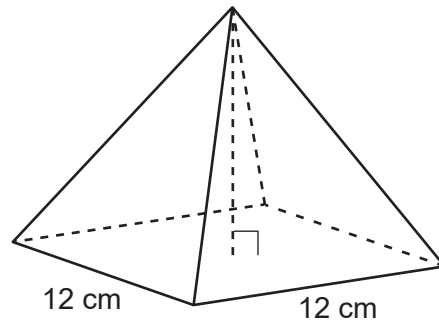
**Score 3:** The student made an error converting grams to kilograms.

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**Question 33**

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- 33** An artist uses clay to make solid pyramids with a square base whose sides measure 12 cm, as modeled below.



The height of each pyramid is 8 cm, and the density of the clay is  $1.25 \text{ g/cm}^3$ . If the artist has 15 kilograms of clay, determine and state the maximum number of pyramids that can be made.

$$V = \frac{1}{3} B h$$

$$V = \frac{1}{3} (12 \cdot 12) (8)$$

$$V = \frac{1}{3} (144) (8)$$

$$V = 384 \text{ cm}^3$$

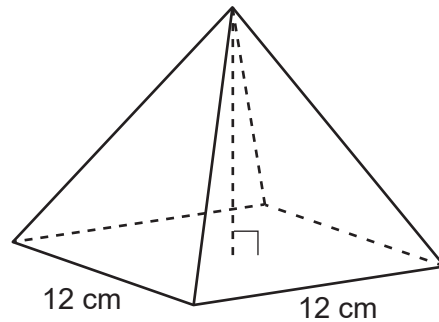
$$M = 384 (1.25)$$

$$M = 480$$

**Score 2:** The student correctly determined the mass of one pyramid.

### Question 33

- 33** An artist uses clay to make solid pyramids with a square base whose sides measure 12 cm, as modeled below.



The height of each pyramid is 8 cm, and the density of the clay is  $1.25 \text{ g/cm}^3$ . If the artist has 15 kilograms of clay, determine and state the maximum number of pyramids that can be made.

pyramid

$$V = \frac{1}{3}bh$$

$$V = \frac{1}{3}(12 \cdot 12) 8$$

$$V = 384 \text{ cm}^3$$

32 pyramids

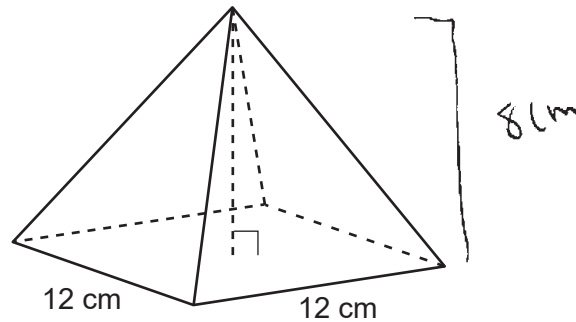
$$384 \cdot 1.25 = 480$$

$$480/15 = 32$$

**Score 2:** The student correctly determined the mass of one pyramid.

### Question 33

- 33 An artist uses clay to make solid pyramids with a square base whose sides measure 12 cm, as modeled below.



The height of each pyramid is 8 cm, and the density of the clay is  $1.25 \text{ g/cm}^3$ . If the artist has 15 kilograms of clay, determine and state the maximum number of pyramids that can be made.

*of base*  
 $A = 12 \cdot 12 = 144$

$$V = \frac{1}{3} (144) (8)$$

$$V = 384$$

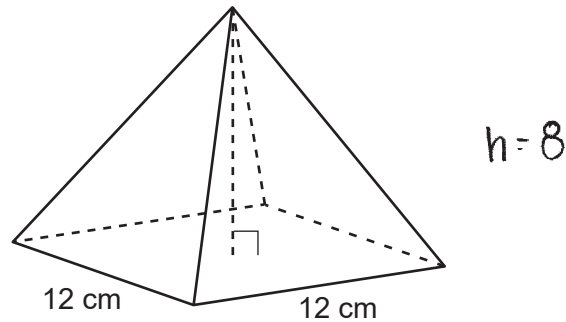
$$\frac{384}{1.25} = 307.2$$

artist  
can make  
25 pyramids

**Score 1:** The student correctly determined the volume of one pyramid.

### Question 33

- 33 An artist uses clay to make solid pyramids with a square base whose sides measure 12 cm, as modeled below.



The height of each pyramid is 8 cm, and the density of the clay is  $1.25 \text{ g/cm}^3$ . If the artist has 15 kilograms of clay, determine and state the maximum number of pyramids that can be made.

$$A_{\text{base}} = 12(12) \\ = 144$$

$$384 / 1.25$$

$$V = \frac{1}{3}(144)(8)$$

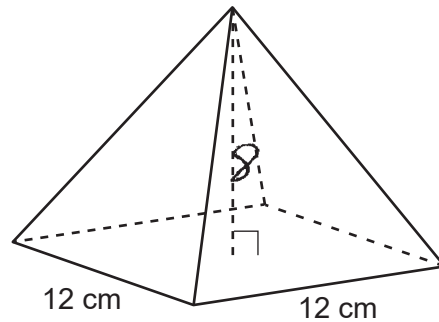
$$= 387.2$$

$$V = 384$$

**Score 1:** The student correctly determined the volume of one pyramid.

**Question 33**

- 33** An artist uses clay to make solid pyramids with a square base whose sides measure 12 cm, as modeled below.



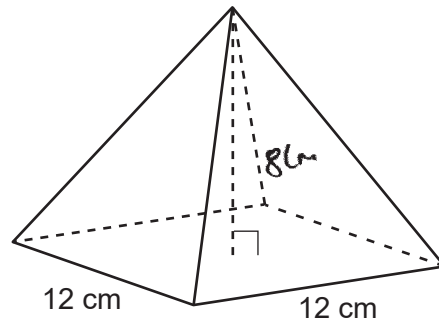
The height of each pyramid is 8 cm, and the density of the clay is  $1.25 \text{ g/cm}^3$ . If the artist has 15 kilograms of clay, determine and state the maximum number of pyramids that can be made.

$$\begin{aligned} d &= \frac{m}{V} & 1.25 &= \frac{m}{256} & \frac{1}{3}Bh \\ & & & & \frac{1}{3}(12)(12)(8) \\ & & 320 &= m & 256 \\ & & 320/15 &= 21.3333333 & \\ & & 21 & \text{pyramids} & \end{aligned}$$

**Score 1:** The student made a computational error when determining the volume of one pyramid, but found an appropriate mass.

### Question 33

- 33** An artist uses clay to make solid pyramids with a square base whose sides measure 12 cm, as modeled below.



The height of each pyramid is 8 cm, and the density of the clay is  $1.25 \text{ g/cm}^3$ . If the artist has 15 kilograms of clay, determine and state the maximum number of pyramids that can be made.

$$\begin{array}{l}
 12 \times 12 = 144 \\
 12 \times 8 = 96 \\
 \begin{array}{r}
 \times 4 \\
 \hline
 384
 \end{array} \\
 \begin{array}{r}
 384 \\
 + 144 \\
 \hline
 528
 \end{array} \\
 \boxed{44}
 \end{array}$$

$$\boxed{44} = \frac{660}{15} = \frac{15}{15}$$

$$\frac{15}{528} = \frac{125}{1}$$

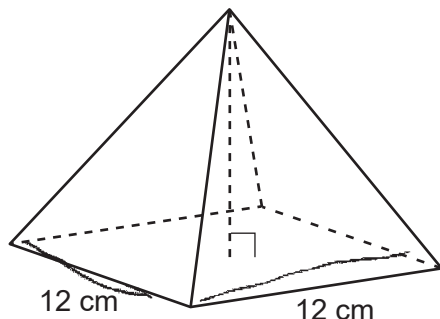
**Score 0:** The student did not show enough correct relevant work to receive any credit.

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**Question 33**

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- 33** An artist uses clay to make solid pyramids with a square base whose sides measure 12 cm, as modeled below.



The height of each pyramid is 8 cm, and the density of the clay is  $1.25 \text{ g/cm}^3$ . If the artist has 15 kilograms of clay, determine and state the maximum number of pyramids that can be made.

$$12 \cdot 12 \cdot 8 = 1152$$

$$1152 / 1.25 \text{ g/cm}^3$$

$$= 921.6^3 / 15$$

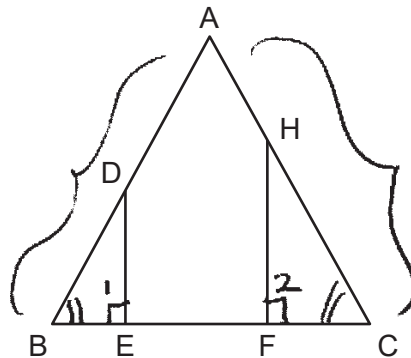
$$= 61.44$$

$$\approx 61 \text{ pyramids.}$$

**Score 0:** The student did not show enough correct relevant work to receive any credit.

Question 34

34 Given: Triangle  $ABC$ ,  $\overline{AB} \cong \overline{AC}$ ,  $\overline{DE} \perp \overline{BC}$ , and  $\overline{HF} \perp \overline{BC}$



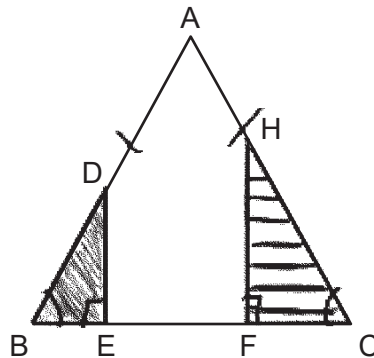
Prove:  $\frac{DE}{BE} = \frac{HF}{CF}$

| $\triangle ABC$ Statements                                                                                        | Reasons                                                     |
|-------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------|
| ① $\overline{AB} \cong \overline{AC}$ , $\overline{DE} \perp \overline{BC}$ , $\overline{HF} \perp \overline{BC}$ | ① Given                                                     |
| ② $\triangle ABC$ is isos.                                                                                        | ② $2 \cong$ legs $\rightarrow$ isos. $\triangle$            |
| ③ $\angle B \cong \angle C$                                                                                       | ③ isos $\triangle \rightarrow 2 \cong$ base $\angle$ 's     |
| ④ $\angle 1, \angle 2$ are right                                                                                  | ④ $\perp$ lines $\rightarrow$ right $\angle$ 's             |
| ⑤ $\angle 1 \cong \angle 2$                                                                                       | ⑤ Right $\angle$ 's $\rightarrow \cong \angle$ 's           |
| ⑥ $\triangle DBE \sim \triangle HCF$                                                                              | ⑥ AA $\sim$                                                 |
| ⑦ $\frac{DE}{BE} = \frac{HF}{CF}$                                                                                 | ⑦ $\sim \triangle$ s $\rightarrow$ proportional corr. sides |

**Score 4:** The student gave a complete and correct response.

Question 34

34 Given: Triangle  $ABC$ ,  $\overline{AB} \cong \overline{AC}$ ,  $\overline{DE} \perp \overline{BC}$ , and  $\overline{HF} \perp \overline{BC}$



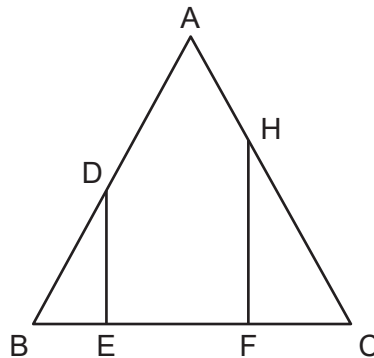
Prove:  $\frac{DE}{BE} = \frac{HF}{CF}$

| Statements                                                                                                                             | Reasons                                                           |
|----------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------|
| ① $\triangle ABC$ , $\overline{AB} \cong \overline{AC}$ , $\overline{DE} \perp \overline{BC}$ ,<br>$\overline{HF} \perp \overline{BC}$ | ① Given                                                           |
| ② $\angle B \cong \angle C$                                                                                                            | ② In a $\triangle$ , angles opposite<br>$\cong$ sides are $\cong$ |
| ③ $\angle DEB \cong \angle HFC$                                                                                                        | ③ $\perp$ lines form $\cong$ right $\angle$ 's                    |
| ④ $\triangle DEB \sim \triangle HFC$                                                                                                   | ④ AA $\sim$                                                       |
| ⑤ $\frac{DE}{BE} = \frac{HF}{CF}$                                                                                                      | ⑤ Corresponding sides of<br>$\sim \triangle$ 's are in proportion |

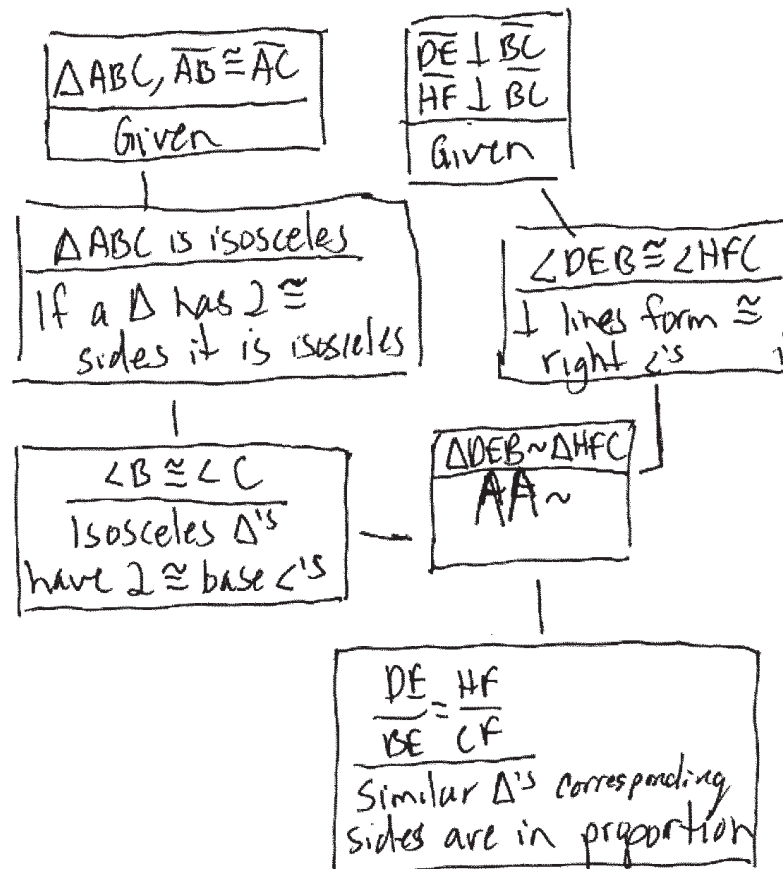
**Score 4:** The student gave a complete and correct response.

### Question 34

34 Given: Triangle  $ABC$ ,  $\overline{AB} \cong \overline{AC}$ ,  $\overline{DE} \perp \overline{BC}$ , and  $\overline{HF} \perp \overline{BC}$



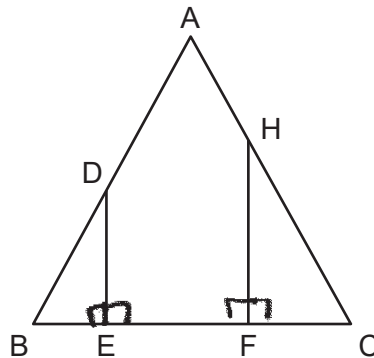
Prove:  $\frac{DE}{BE} = \frac{HF}{CF}$



**Score 4:** The student gave a complete and correct response.

Question 34

34 Given: Triangle  $ABC$ ,  $\overline{AB} \cong \overline{AC}$ ,  $\overline{DE} \perp \overline{BC}$ , and  $\overline{HF} \perp \overline{BC}$



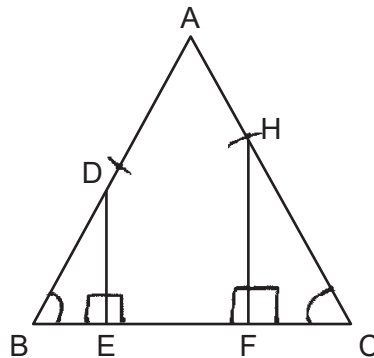
Prove:  $\frac{DE}{BE} = \frac{HF}{CF}$

| Statement                                                                                                                            | Reason                                                     |
|--------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------|
| 1. $\triangle ABC$ , $\overline{AB} \cong \overline{AC}$ , $\overline{DE} \perp \overline{BC}$ , $\overline{HF} \perp \overline{BC}$ | 1. Given                                                   |
| 2. $\angle DEB$ and $\angle HFC$ are right $\angle$ 's                                                                               | 2. $\perp$ segments form right $\angle$ 's                 |
| 3. $\angle DEB \cong \angle HFC$                                                                                                     | 3. All right $\angle$ 's are $\cong$                       |
| 4. $\triangle ABC$ is isosceles                                                                                                      | 4. A $\triangle$ with 2 $\cong$ sides is isosceles         |
| 5. $\angle B \cong \angle C$                                                                                                         | 5. Base $\angle$ 's of a $\cong \triangle$ are $\cong$     |
| 6. $\triangle DBE \sim \triangle HCF$                                                                                                | 6. AA $\cong$ AA                                           |
| 7. $\frac{DE}{BE} = \frac{HF}{CF}$                                                                                                   | 7. Corr. sides of similar $\triangle$ 's are in proportion |

**Score 3:** The student wrote an incorrect reason in step 5.

# Question 34

34 Given: Triangle  $ABC$ ,  $\overline{AB} \cong \overline{AC}$ ,  $\overline{DE} \perp \overline{BC}$ , and  $\overline{HF} \perp \overline{BC}$



Prove:  $\frac{DE}{BE} = \frac{HF}{CF}$

product of means equals product of extremes

corresponding parts of corresponding triangles are congruent

①  $\overline{AB} \cong \overline{AC}$   
 $\overline{DE} \perp \overline{BC}$   
 $\overline{HF} \perp \overline{BC}$   
 $\triangle ABC$

① Given

②  $\angle ABC \cong \angle ACB$

② if 2 sides of a triangle are  $\cong$  then the  $\angle$ 's opposite those sides are  $\cong$

③  $\angle DEB$  and  $\angle HFC$  are right angles  
 ③  $\perp$  lines form right angles

④  $\angle DEB \cong \angle HFC$

③ All right angles are congruent

⑤  $\triangle DBE \sim \triangle HCF$

⑤ AA~

⑥  $\overline{DE} \cong \overline{CF}$   $\overline{HF} \cong \overline{BE}$

⑥ corresponding parts of corresponding triangles are congruent

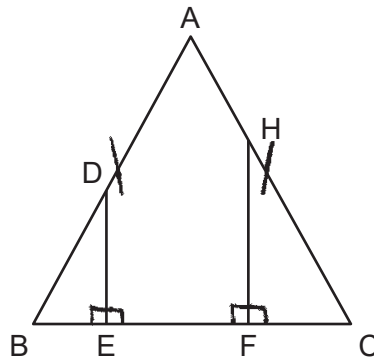
⑦  $\frac{DE}{BE} = \frac{HF}{CF}$

⑦ Products of means is equal to the product of the extremes

**Score 3:** The student correctly proved  $\triangle DBE \sim \triangle HCF$ .

### Question 34

34 Given: Triangle  $ABC$ ,  $\overline{AB} \cong \overline{AC}$ ,  $\overline{DE} \perp \overline{BC}$ , and  $\overline{HF} \perp \overline{BC}$



$$DE \cdot CF = BE \cdot HF$$

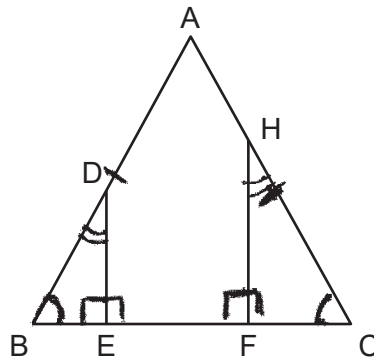
Prove:  $\frac{DE}{BE} = \frac{HF}{CF}$

| Statement                                                                                                                               | Reason                                                      |
|-----------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------|
| 1 $\overline{AB} \cong \overline{AC}$ , $\overline{DE} \perp \overline{BC}$ , $\overline{HF} \perp \overline{BC}$ , $ABC$ is a triangle | 1 given                                                     |
| 2 $\angle ABE \cong \angle ACF$                                                                                                         | 2 all base $\angle$ 's of an isosceles $\Delta$ are $\cong$ |
| 3 $\angle DEB$ , $\angle HFC$ are right $\angle$ 's                                                                                     | 3 perpendicular lines form right $\angle$ 's                |
| 4 $\angle DEB \cong \angle HFC$                                                                                                         | 4 all right $\angle$ 's are $\cong$                         |
| 5 $\triangle BDE \sim \triangle CHF$                                                                                                    | 5 AA~                                                       |
| 6 $\frac{DE}{BE} = \frac{HF}{CF}$                                                                                                       | 6 product of the means equals the product of the extremes   |

**Score 2:** The student was missing a statement and reason to prove step 2. The student wrote an incorrect reason in step 6.

Question 34

34 Given: Triangle  $ABC$ ,  $\overline{AB} \cong \overline{AC}$ ,  $\overline{DE} \perp \overline{BC}$ , and  $\overline{HF} \perp \overline{BC}$



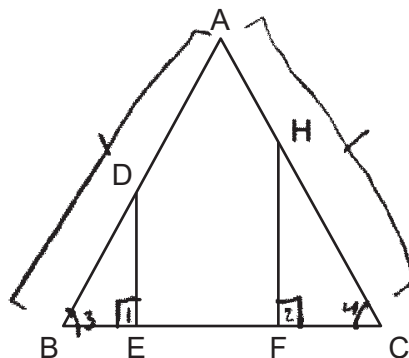
Prove:  $\frac{DE}{BE} = \frac{HF}{CF}$

- |                                                                                                                                  |                                                                                             |
|----------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------|
| 1. $\triangle ABC, \overline{AB} \cong \overline{AC},$<br>$\overline{DE} \perp \overline{BC}, \overline{HF} \perp \overline{BC}$ | 1. Given                                                                                    |
| 2. $\angle ABE \cong \angle ACF$                                                                                                 | 2. Definition of isosceles                                                                  |
| 3. $\angle DEB$ and $\angle HFC$ are $\cong$ right<br>$\angle$ 's                                                                | 3. Perpendicular l's<br>form $\cong$ right angles                                           |
| 4. $\triangle DBE \sim \triangle HCF$                                                                                            | 4. AA~                                                                                      |
| 5. $\frac{DE}{BE} = \frac{HF}{CF}$                                                                                               | 5. Due to similarity<br>since the angles are the<br>same they are similar<br>which prove it |

**Score 2:** The student wrote correct relevant statements and reasons in steps 3 and 4.

# Question 34

34 Given: Triangle  $ABC$ ,  $\overline{AB} \cong \overline{AC}$ ,  $\overline{DE} \perp \overline{BC}$ , and  $\overline{HF} \perp \overline{BC}$



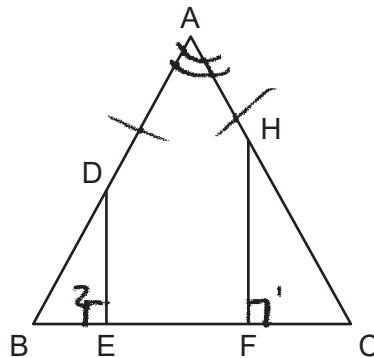
Prove:  $\frac{DE}{BE} = \frac{HF}{CF}$

| S                                                                                                                                 | R                                                    |
|-----------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------|
| 1. $\triangle ABC$ $\overline{AB} \cong \overline{AC}$<br>$\overline{DE} \perp \overline{BC}$ $\overline{HF} \perp \overline{BC}$ | 1. Given                                             |
| 2. $\angle 1 \cong \angle 2$                                                                                                      | 2. $\perp$ lines form $\cong$ rt $\angle$ 's         |
| 3. $\angle 3 \cong \angle 4$                                                                                                      | 3. Congruent sides form $\cong$ opposite $\angle$ 's |
| 4. $\triangle DEB \sim \triangle HFC$                                                                                             | 4. AA $\sim$                                         |
| 5. $\frac{DE}{BE} = \frac{HF}{CF}$                                                                                                | 5. CPSTS                                             |

**Score 2:** The student wrote an incomplete reason in step 3 and an incorrect reason in step 5.

Question 34

34 Given: Triangle  $ABC$ ,  $\overline{AB} \cong \overline{AC}$ ,  $\overline{DE} \perp \overline{BC}$ , and  $\overline{HF} \perp \overline{BC}$



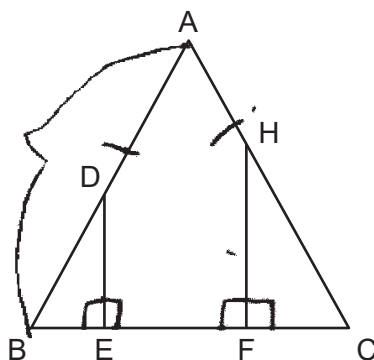
Prove:  $\frac{DE}{BE} = \frac{HF}{CF}$

| Statements                                                                                                        | Reason                                                                    |
|-------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------|
| ① $\overline{AB} \cong \overline{AC}$ , $\overline{DE} \perp \overline{BC}$ , $\overline{HF} \perp \overline{BC}$ | ① Given                                                                   |
| ② $\angle 1$ and $\angle 2$ are right $\angle$ 's                                                                 | ② All $\perp$ lines form right $\angle$ 's                                |
| ③ $\angle 2 \cong \angle 1$                                                                                       | ③ All right $\angle$ 's are $\cong$                                       |
| ④ $\angle A \cong \angle A$                                                                                       | ④ Reflexive property                                                      |
| ⑤ $\triangle DBE \sim \triangle HCF$                                                                              | ⑤ AA~                                                                     |
| ⑥ $\frac{DE}{BE} = \frac{HF}{CF}$                                                                                 | ⑥ 2 $\Delta$ 's similar, $\frac{2}{2}$ $\angle$ 's are crossed multiplied |

**Score 1:** The student correctly proved  $\angle 2 \cong \angle 1$ .

### Question 34

34 Given: Triangle  $ABC$ ,  $\overline{AB} \cong \overline{AC}$ ,  $\overline{DE} \perp \overline{BC}$ , and  $\overline{HF} \perp \overline{BC}$



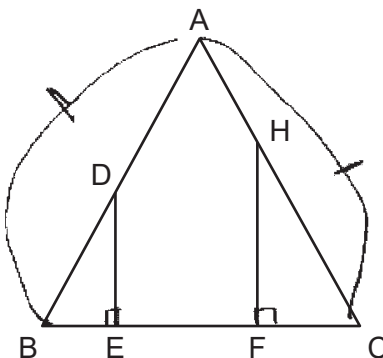
Prove:  $\frac{DE}{BE} = \frac{HF}{CF}$

we are given  $\overline{AB} \cong \overline{AC}$   $\overline{DE} \perp \overline{BC}$  and  $\overline{HF} \perp \overline{BC}$ .  
 $\triangle ABC$  is isosceles b/c it has  
 2 congruent sides  $\overline{AB}, \overline{AC}$ , so  $\angle B \cong \angle C$ .

**Score 1:** The student wrote one correct relevant statement and reason.

# Question 34

34 Given: Triangle  $ABC$ ,  $\overline{AB} \cong \overline{AC}$ ,  $\overline{DE} \perp \overline{BC}$ , and  $\overline{HF} \perp \overline{BC}$



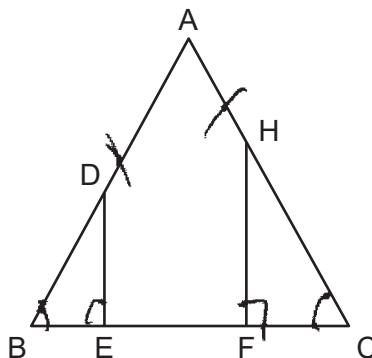
Prove:  $\frac{DE}{BE} = \frac{HF}{CF}$

| <u>I know</u>                          | <u>b/c</u>                        |
|----------------------------------------|-----------------------------------|
| 1. $\overline{AB} \cong \overline{AC}$ | 1. given                          |
| 2. $\overline{DE} \perp \overline{BC}$ | 2. given                          |
| 3. $\angle DEB$ is a rt $\angle$       | 3. $\perp$ lines form rt $\angle$ |
| 4. $\overline{HF} \perp \overline{BC}$ | 4. given                          |
| 5. $\angle HFC$ is a rt $\angle$       | 5. $\perp$ lines form rt $\angle$ |
| 6. $\angle DEB \cong \angle HFC$       | 6. All rt $\angle$ s are $\cong$  |
| 7. $\triangle DEB \sim \triangle HFC$  | 7. AA                             |
| 8.                                     |                                   |

**Score 1:** The student correctly proved  $\angle DEB \cong \angle HFC$ .

### Question 34

34 Given: Triangle  $ABC$ ,  $\overline{AB} \cong \overline{AC}$ ,  $\overline{DE} \perp \overline{BC}$ , and  $\overline{HF} \perp \overline{BC}$



Prove:  $\frac{DE}{BE} = \frac{HF}{CF}$

$$\begin{aligned} 1) \angle B &\cong \angle C \\ 2) \angle E &\cong \angle F \end{aligned}$$

A.A

$$3.) \frac{DE}{BE} = \frac{HF}{CF}$$

1) base  $\angle$ s are  $\cong$

2)  $\perp$   $\angle$ s are  $\cong$

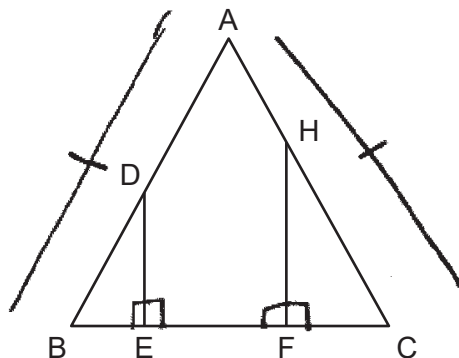
2.)

3.) A.A

**Score 0:** The student did not show enough correct work to receive any credit.

### Question 34

34 Given: Triangle  $ABC$ ,  $\overline{AB} \cong \overline{AC}$ ,  $\overline{DE} \perp \overline{BC}$ , and  $\overline{HF} \perp \overline{BC}$



Prove:  $\frac{DE}{BE} = \frac{HF}{CF}$

| Statement                                                                                                                              | Reasons                           |
|----------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------|
| 1. $\triangle ABC$ , $\overline{AB} \cong \overline{AC}$ , $\overline{DE} \perp \overline{BC}$ and $\overline{HF} \perp \overline{BC}$ | 1. Given                          |
| 2.                                                                                                                                     |                                   |
| 3. $\triangle DEB \sim \triangle HFC$                                                                                                  | AA similarity                     |
| 4. $\frac{DE}{BE} = \frac{HF}{CF}$                                                                                                     | Prod of means = prod of extremes. |

**Score 0:** The student did not show enough correct work to receive any credit.

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**Question 35**

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**35** Triangle  $ABC$  has vertices whose coordinates are  $A(-3, -1)$ ,  $B(-5, 2)$ , and  $C(-1, 8)$ .

State the coordinates of point  $D$  such that quadrilateral  $ABCD$  is a parallelogram.

[The use of the set of axes on the next page is optional.]

$$D(1, 5)$$

Prove  $ABCD$  is a parallelogram.

[The use of the set of axes on the next page is optional.]

$$\begin{array}{lll} m_{\overline{AB}} = -\frac{3}{2} & m_{\overline{BC}} = \frac{6}{4} & \therefore ABCD \text{ is a parallelogram} \\ & & \text{because both sets} \\ m_{\overline{CD}} = \frac{-3}{2} & m_{\overline{AD}} = \frac{6}{4} & \text{of opposite sides are} \\ \text{same slope} & \text{same slope} & \text{parallel} \\ \overline{AB} \parallel \overline{CD} & \overline{BC} \parallel \overline{AD} & \end{array}$$

State the coordinates of point  $E$ , the midpoint of  $\overline{BC}$ .

[The use of the set of axes on the next page is optional.]

$$E(-3, 5)$$

**Question 35 is continued on the next page.**

**Score 6:** The student gave a complete and correct response.

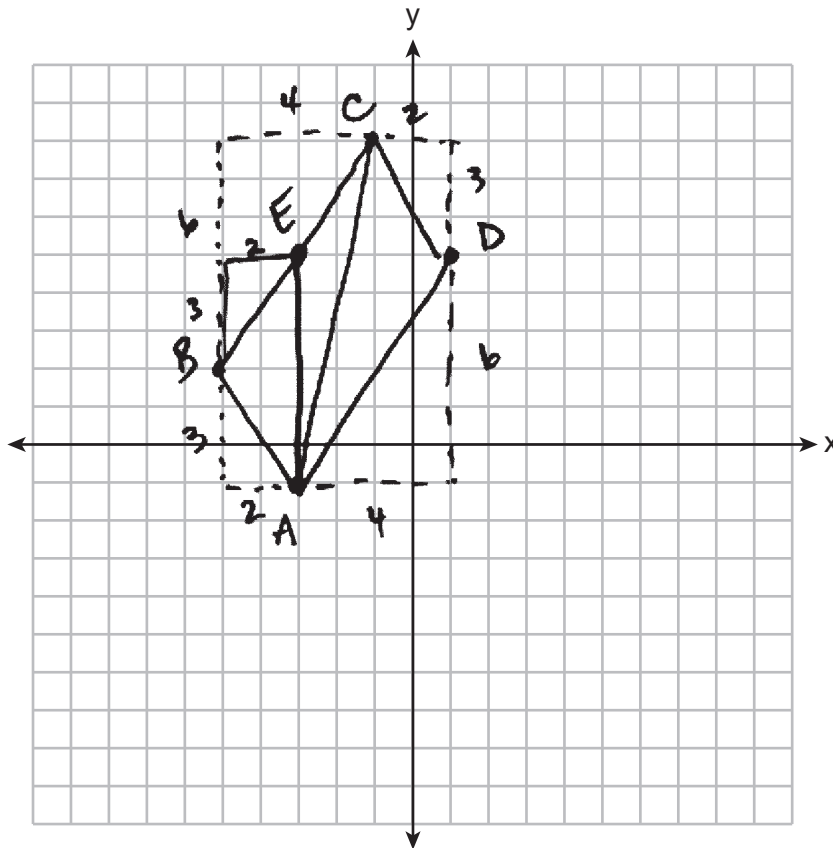
Question 35 continued.

Prove  $\triangle ABE$  is an isosceles triangle.

[The use of the set of axes below is optional.]

$$\begin{aligned} d_{BE} &= \sqrt{3^2 + 2^2} = \sqrt{13} \\ d_{AB} &= \sqrt{(-3)^2 + (2)^2} = \sqrt{13} \end{aligned} \quad \left. \vphantom{\begin{aligned} d_{BE} &= \sqrt{3^2 + 2^2} = \sqrt{13} \\ d_{AB} &= \sqrt{(-3)^2 + (2)^2} = \sqrt{13} \end{aligned}} \right\} \overline{BE} \cong \overline{AB}$$

$\therefore \triangle ABE$  is an isosceles triangle because 2 sides are congruent.



**Question 35**

**35** Triangle  $ABC$  has vertices whose coordinates are  $A(-3, -1)$ ,  $B(-5, 2)$ , and  $C(-1, 8)$ .

State the coordinates of point  $D$  such that quadrilateral  $ABCD$  is a parallelogram.

[The use of the set of axes on the next page is optional.]

$$D(1, 5)$$

Prove  $ABCD$  is a parallelogram.

[The use of the set of axes on the next page is optional.]

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$d_{AB} = \sqrt{(-3 + 5)^2 + (-1 - 2)^2} = \sqrt{4 + 9} = \sqrt{13}$$

$$d_{CD} = \sqrt{(1 + 1)^2 + (5 - 8)^2} = \sqrt{4 + 9} = \sqrt{13}$$

$$d_{BC} = \sqrt{(-5 + 1)^2 + (2 - 8)^2} = \sqrt{16 + 36} = \sqrt{52}$$

$$d_{AD} = \sqrt{(1 + 3)^2 + (5 + 1)^2} = \sqrt{16 + 36} = \sqrt{52}$$

$ABCD$  is a p-gram b/c both pairs of opposite sides are congruent.

State the coordinates of point  $E$ , the midpoint of  $\overline{BC}$ .

[The use of the set of axes on the next page is optional.]

$$E(-3, 5)$$

**Question 35 is continued on the next page.**

**Score 6:** The student gave a complete and correct response.

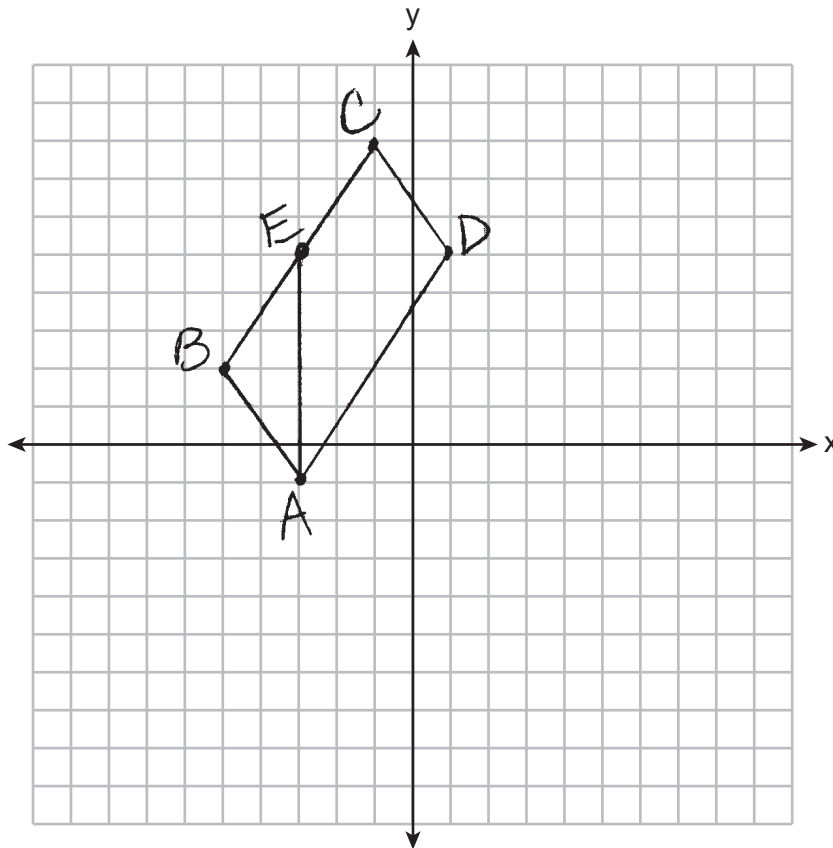
Question 35 continued.

Prove  $\triangle ABE$  is an isosceles triangle.

[The use of the set of axes below is optional.]

$$\begin{aligned} AB &= \sqrt{13} & d_{BE} &= \sqrt{(-5+3)^2 + (2-5)^2} \\ & & &= \sqrt{4+9} \\ & & &= \sqrt{13} \end{aligned}$$

$\triangle ABE$  is isosceles because it has 2  $\cong$  sides.  
( $\overline{AB} \cong \overline{BE}$ ).



**Question 35**

**35** Triangle  $ABC$  has vertices whose coordinates are  $A(-3, -1)$ ,  $B(-5, 2)$ , and  $C(-1, 8)$ .

State the coordinates of point  $D$  such that quadrilateral  $ABCD$  is a parallelogram.

[The use of the set of axes on the next page is optional.]

$$D \begin{matrix} 3 \\ \downarrow \end{matrix}, \begin{matrix} 2 \\ \rightarrow \end{matrix} \quad D(1, 5)$$

Prove  $ABCD$  is a parallelogram.

[The use of the set of axes on the next page is optional.]

$$\begin{aligned} \text{Midpoint } \overline{AC} &= \left( \frac{-3+(-1)}{2}, \frac{-1+8}{2} \right) & \text{Midpoint } \overline{BD} &= \left( \frac{-5+1}{2}, \frac{2+5}{2} \right) \\ &= \left( -\frac{4}{2}, \frac{7}{2} \right) & &= \left( -\frac{4}{2}, \frac{7}{2} \right) \\ &= (-2, 3.5) & &= (-2, 3.5) \end{aligned}$$

Since diagonals  $\overline{AC}$  and  $\overline{BD}$  have the same midpoint, they bisect each other.

Since the diagonals bisect each other,  $ABCD$  is a parallelogram.

State the coordinates of point  $E$ , the midpoint of  $\overline{BC}$ .

[The use of the set of axes on the next page is optional.]

$$\begin{aligned} \text{Midpoint of } \overline{BC} &= \left( \frac{-5+(-1)}{2}, \frac{2+8}{2} \right) \\ &= \left( -\frac{6}{2}, \frac{10}{2} \right) \\ E &(-3, 5) \end{aligned}$$

**Question 35 is continued on the next page.**

**Score 6:** The student gave a complete and correct response.

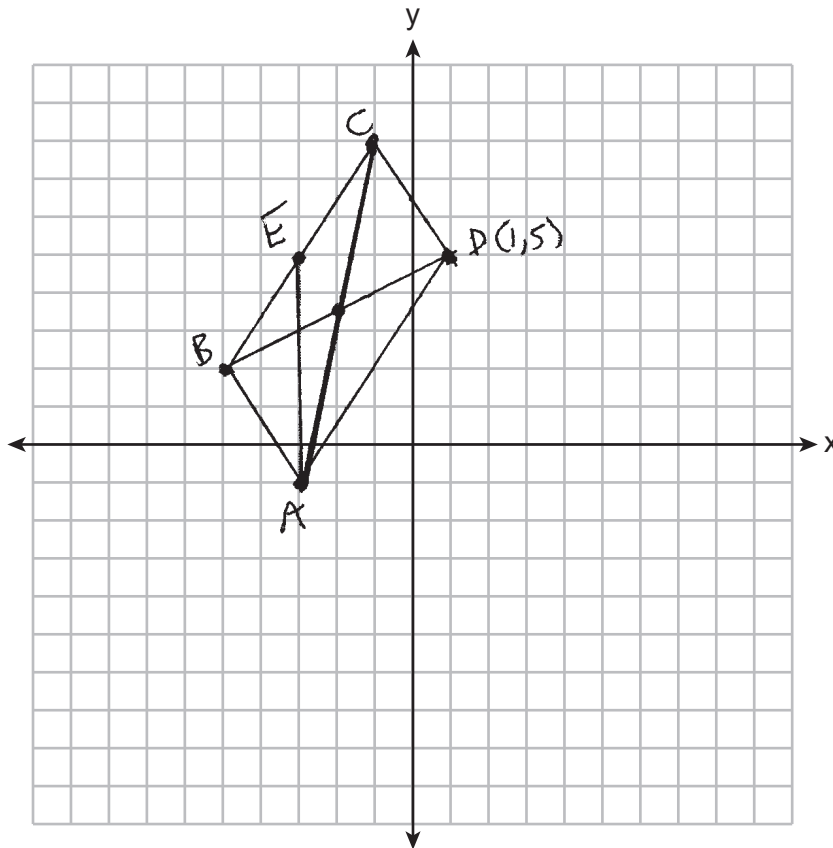
Question 35 continued.

Prove  $\triangle ABE$  is an isosceles triangle.

[The use of the set of axes below is optional.]

$$\begin{array}{l|l}
 AB = \sqrt{(-5 - (-3))^2 + (2 - (-1))^2} & BE = \sqrt{(-3 - (-5))^2 + (5 - 2)^2} \\
 = \sqrt{(-2)^2 + 3^2} & = \sqrt{2^2 + 3^2} \\
 = \sqrt{4 + 9} & = \sqrt{4 + 9} \\
 AB = \sqrt{13} & BE = \sqrt{13} \\
 \hline
 \overline{AB} \cong \overline{BE}
 \end{array}$$

Since 2 sides of  $\triangle ABE$  are  $\cong$ , it is isosceles.



### Question 35

**35** Triangle  $ABC$  has vertices whose coordinates are  $A(-3, -1)$ ,  $B(-5, 2)$ , and  $C(-1, 8)$ .

State the coordinates of point  $D$  such that quadrilateral  $ABCD$  is a parallelogram.

[The use of the set of axes on the next page is optional.]

$$D(1, 5)$$

Prove  $ABCD$  is a parallelogram.

[The use of the set of axes on the next page is optional.]

OPP sides parallel

$$m_{\overline{BA}} = \frac{-1-2}{-3+5} = \frac{-3}{2}$$

$$m_{\overline{CD}} = \frac{5-8}{1+1} = \frac{-3}{2}$$

$$m_{\overline{CB}} = \frac{2-8}{-5+1} = \frac{-6}{-4} = \frac{3}{2}$$

$$m_{\overline{DA}} = \frac{-1+5}{-3-1} = \frac{4}{-4} = -1$$

$\overline{BA} \parallel \overline{CD}$

$\overline{CB} \parallel \overline{DA}$

Since both pairs of opp sides  $\parallel$ ,  
 $ABCD$  is a parallelogram

State the coordinates of point  $E$ , the midpoint of  $\overline{BC}$ .

[The use of the set of axes on the next page is optional.]

$$\frac{-5+1}{2} = -2$$

$$\frac{8+2}{2} = 5$$

$$(-2, 5)$$

Question 35 is continued on the next page.

**Score 6:** The student gave a complete and correct response.

Question 35 continued.

Prove  $\triangle ABE$  is an isosceles triangle.

[The use of the set of axes below is optional.]

$$A(-3, -1) \quad B(-5, 2) \quad E(-3, 5)$$

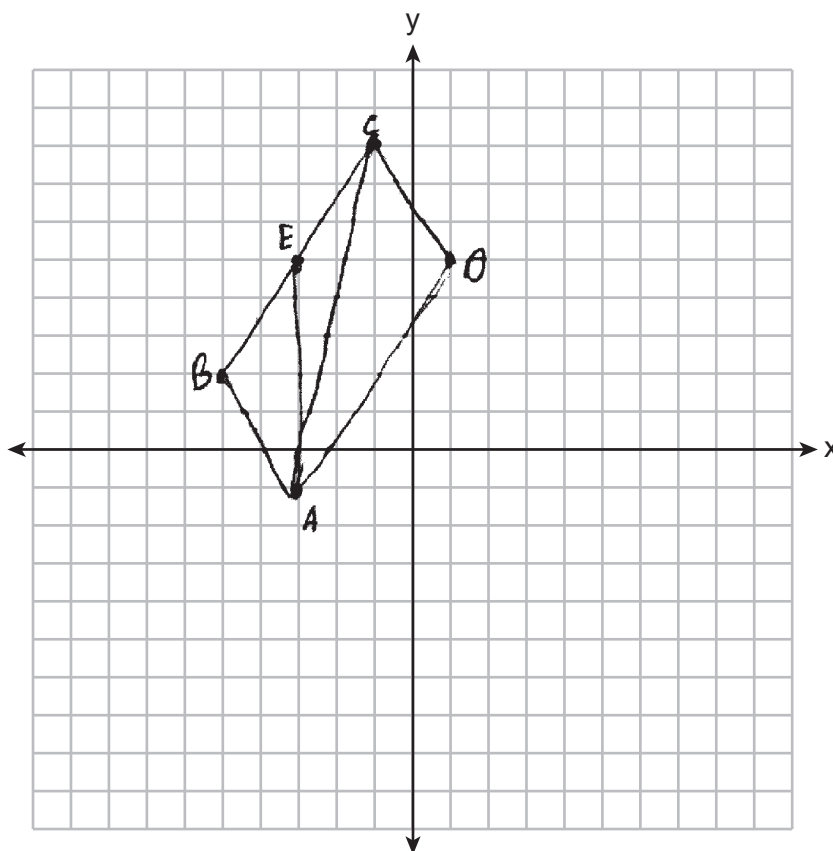
$$d_{AB} = \frac{\sqrt{(2+1)^2 + (-5+3)^2}}{\sqrt{13}}$$

$$d_{BE} = \frac{\sqrt{(5-2)^2 + (-3+5)^2}}{\sqrt{13}}$$

$$\overline{AB} \cong \overline{BE}$$

Since 2 Sides

$\cong$ ,  $\triangle ABE$  is an  
isosceles triangle



### Question 35

**35** Triangle  $ABC$  has vertices whose coordinates are  $A(-3, -1)$ ,  $B(-5, 2)$ , and  $C(-1, 8)$ .

State the coordinates of point  $D$  such that quadrilateral  $ABCD$  is a parallelogram.

[The use of the set of axes on the next page is optional.]

$$D: (1, 5)$$

Prove  $ABCD$  is a parallelogram.

[The use of the set of axes on the next page is optional.]

$$\text{slope: } \overline{AD}: \frac{3}{2} \quad \overline{BC}: \frac{3}{2} \quad \overline{CD}: \frac{-3}{2} \quad \overline{AB}: \frac{-3}{2}$$

same so //

if 2 sets opposite parallel lines  
then it is a parallelogram.

State the coordinates of point  $E$ , the midpoint of  $\overline{BC}$ .

[The use of the set of axes on the next page is optional.]

$$\begin{array}{ccc} & \boxed{(-3, 5)} & \\ B & & C \\ -5, 2 & & -1, 8 \end{array}$$

$$\left( \frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2} \right)$$

$$\left( \frac{-5 + -1}{2}, \frac{2 + 8}{2} \right)$$

$$(-3, 5)$$

Question 35 is continued on the next page.

**Score 5:** The student did not show work when finding the slopes of  $\overline{CD}$  and  $\overline{DA}$ .

Question 35 continued.

Prove  $\triangle ABE$  is an isosceles triangle.

[The use of the set of axes below is optional.]

distance :  $AB : \sqrt{13}$   $BE : \sqrt{13}$

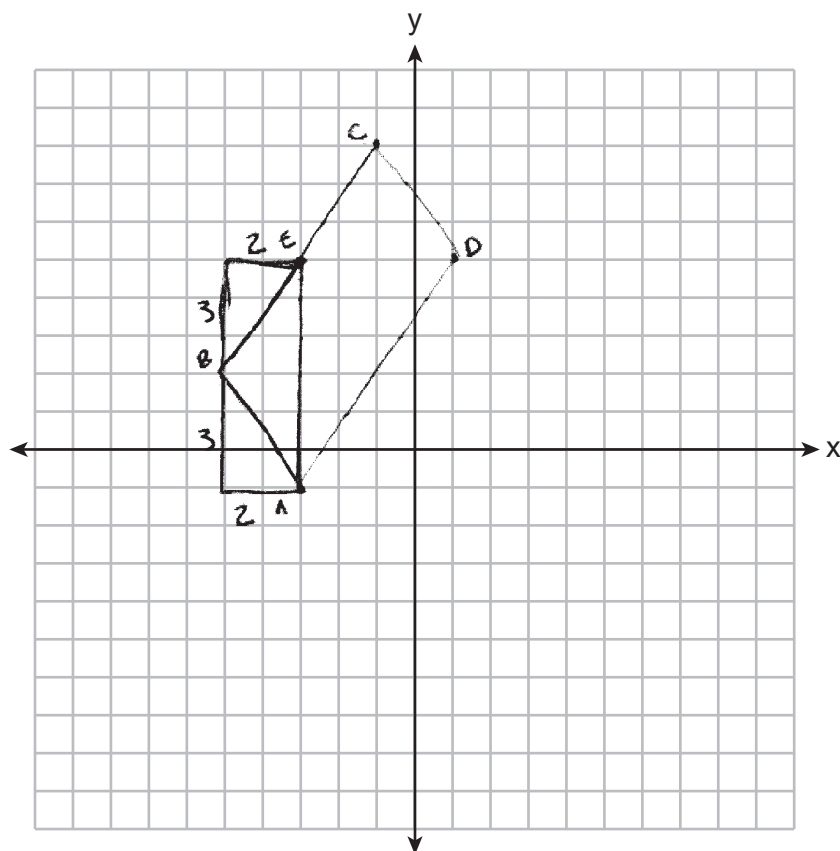
$\underbrace{\hspace{1.5cm}}$   
 Same so  
 $\cong$

$$2^2 + 3^2 = c^2$$

$$\sqrt{4 + 9} = c$$

$$\sqrt{13}$$

$\triangle ABE$  is an isosceles triangle because  
it has 2 congruent sides.



Question 35

35 Triangle  $ABC$  has vertices whose coordinates are  $A(-3, -1)$ ,  $B(-5, 2)$ , and  $C(-1, 8)$ .

State the coordinates of point  $D$  such that quadrilateral  $ABCD$  is a parallelogram.

[The use of the set of axes on the next page is optional.]

$$D(1, 5)$$

Prove  $ABCD$  is a parallelogram.

[The use of the set of axes on the next page is optional.]

$$\begin{aligned} D_{BC} &= \sqrt{4^2 + 6^2} = \sqrt{16 + 36} = \sqrt{52} \\ D_{CD} &= \sqrt{(-3)^2 + 2^2} = \sqrt{9 + 4} = \sqrt{13} \\ D_{DA} &= \sqrt{(-4)^2 + 6^2} = \sqrt{16 + 36} = \sqrt{52} \\ D_{AB} &= \sqrt{(2)^2 + (-3)^2} = \sqrt{4 + 9} = \sqrt{13} \end{aligned} \quad \left. \begin{array}{l} \text{distances are =} \\ \text{so sides are } \cong \end{array} \right\}$$

State the coordinates of point  $E$ , the midpoint of  $\overline{BC}$ .

[The use of the set of axes on the next page is optional.]

$$\begin{aligned} &\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &\left( \frac{-5 + (-1)}{2}, \frac{2 + 8}{2} \right) \end{aligned} \quad (-3, 5)$$

Question 35 is continued on the next page.

**Score 5:** The student wrote an incomplete conclusion when proving  $ABCD$  is a parallelogram.

Question 35 continued.

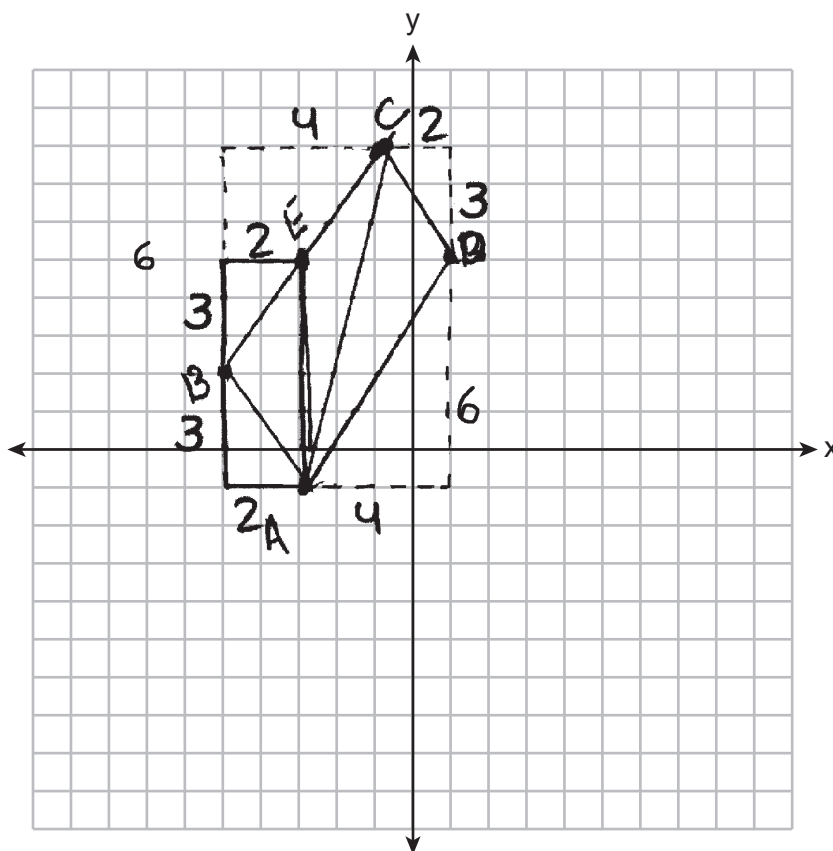
Prove  $\triangle ABE$  is an isosceles triangle.

[The use of the set of axes below is optional.]

$$D_{\overline{BE}} = \sqrt{3^2 + 2^2} = \sqrt{9+4} = \sqrt{13}$$

$$D_{\overline{BA}} = \sqrt{(-3)^2 + 2^2} = \sqrt{9+4} = \sqrt{13}$$

Two sides are  $\cong$  so the  $\triangle$  is isosceles



### Question 35

**35** Triangle  $ABC$  has vertices whose coordinates are  $A(-3, -1)$ ,  $B(-5, 2)$ , and  $C(-1, 8)$ .

State the coordinates of point  $D$  such that quadrilateral  $ABCD$  is a parallelogram.

[The use of the set of axes on the next page is optional.]

$$m_{\overline{AB}} = \frac{2 - (-1)}{-5 - (-3)} = \frac{3}{-2}$$

$$m_{\overline{CD}} = \frac{8 - y}{-1 - x} = \frac{3}{-2}$$

$$D = (1, 5)$$

Prove  $ABCD$  is a parallelogram.

[The use of the set of axes on the next page is optional.]

$$m_{\overline{AB}} = \frac{3}{-2}$$

$$m_{\overline{CD}} = \frac{8 - 5}{-1 - 1} = \frac{3}{-2}$$

same slope  
so  $\overline{AB} \parallel \overline{CD}$

One pair of opp  
sides are  $\parallel + \cong$   
so it's a parallelogram

$$\begin{aligned} d_{\overline{AB}} &= 3.6055... \\ \text{distance}_{\overline{CD}} &= 3.6055... \end{aligned} \quad \left. \begin{array}{l} \text{same distance} \\ \text{so } \overline{AB} \cong \overline{CD} \end{array} \right\}$$

State the coordinates of point  $E$ , the midpoint of  $\overline{BC}$ .

[The use of the set of axes on the next page is optional.]

$$E(-3, 5)$$

**Question 35 is continued on the next page.**

**Score 5:** The student did not show work when determining the lengths of  $\overline{AB}$ ,  $\overline{CD}$ , and  $\overline{EB}$ .

Question 35 continued.

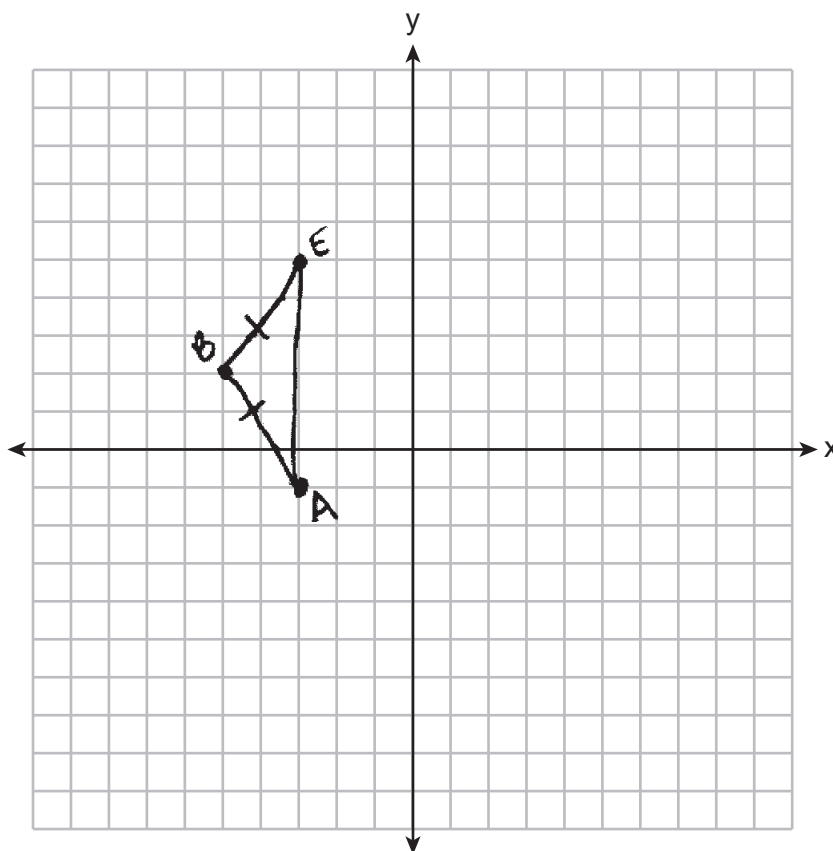
Prove  $\triangle ABE$  is an isosceles triangle.

[The use of the set of axes below is optional.]

$$d_{\overline{AB}} = 3.6055... \quad \left. \begin{array}{l} \text{same distance} \\ \text{So } \overline{AB} \cong \overline{BE} \end{array} \right\}$$

$$d_{\overline{BE}} = 3.6055...$$

$\triangle ABE$  is an isos triangle  
because 2 sides are  $\cong$



### Question 35

**35** Triangle  $ABC$  has vertices whose coordinates are  $A(-3, -1)$ ,  $B(-5, 2)$ , and  $C(-1, 8)$ .

State the coordinates of point  $D$  such that quadrilateral  $ABCD$  is a parallelogram.

[The use of the set of axes on the next page is optional.]

$$m = \frac{\Delta y}{\Delta x} \quad \frac{6}{4} = \frac{3}{2}$$

$$D = (1, 5)$$

Prove  $ABCD$  is a parallelogram.

[The use of the set of axes on the next page is optional.]

$$c = \sqrt{a^2 + b^2}$$

$$AB = \sqrt{2^2 + 3^2} = \sqrt{13}$$

$$BC = \sqrt{4^2 + 6^2} = \sqrt{52}$$

$$CD = \sqrt{2^2 + 3^2} = \sqrt{13}$$

$$DA = \sqrt{4^2 + 6^2} = \sqrt{52}$$

$$\overline{AB} \cong \overline{CD}$$

$$\overline{BC} \cong \overline{DA}$$

(2 pairs of opp sides  $\cong$ )  
 $\therefore \square ABCD$

State the coordinates of point  $E$ , the midpoint of  $\overline{BC}$ .

[The use of the set of axes on the next page is optional.]

$$E = (-3, 5)$$

**Question 35 is continued on the next page.**

**Score 4:** The student showed no correct work when proving  $\triangle ABE$  is isosceles.

**Question 35 continued.**

Prove  $\triangle ABE$  is an isosceles triangle.

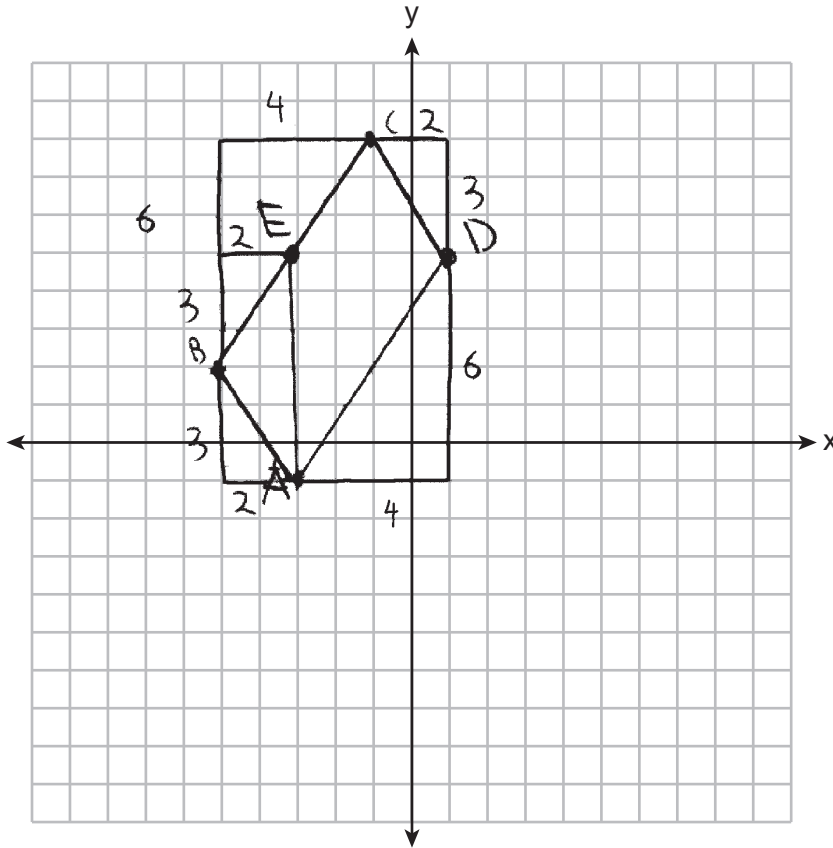
[The use of the set of axes below is optional.]

$$M = \frac{\Delta y}{\Delta x}$$

$\triangle ABE$  is isosceles (A isosceles  $\triangle$  has 2  $\cong$  sides)

$$EB = \frac{3}{2}$$

$$AB = \frac{-3}{2}$$



### Question 35

**35** Triangle  $ABC$  has vertices whose coordinates are  $A(-3, -1)$ ,  $B(-5, 2)$ , and  $C(-1, 8)$ .

State the coordinates of point  $D$  such that quadrilateral  $ABCD$  is a parallelogram.

[The use of the set of axes on the next page is optional.]

$$\text{Slope } \overline{AB}: \frac{2 - (-1)}{-5 - (-3)} = \frac{3}{-2} \rightarrow$$

$$D(1, 5)$$

Prove  $ABCD$  is a parallelogram.

[The use of the set of axes on the next page is optional.]

$$\begin{aligned} \text{Slope } \overline{BC}: \frac{8 - 2}{-1 - (-5)} &= \frac{6}{-4} = -\frac{3}{2} \\ \text{Slope } \overline{AD}: \frac{-1 - 5}{-3 - 1} &= \frac{-6}{-4} = \frac{3}{2} \end{aligned} \quad \leftarrow \text{parallel line}$$

$$\begin{aligned} \text{Slope } \overline{BA}: \frac{-1 - 2}{-3 - (-5)} &= \frac{-3}{2} \\ \text{Slope } \overline{CD}: \frac{5 - 8}{1 - (-1)} &= \frac{-3}{2} \end{aligned} \quad \leftarrow \text{parallel line}$$

State the coordinates of point  $E$ , the midpoint of  $\overline{BC}$ .

[The use of the set of axes on the next page is optional.]

$$\text{Slope } \overline{BC}: \frac{8 - 2}{-1 - (-5)} = \frac{6}{-4} = -\frac{3}{2} \rightarrow \quad E(-3, 5)$$

**Question 35 is continued on the next page.**

**Score 4:** The student wrote an incomplete conclusion when proving  $ABCD$  is a parallelogram. The student wrote a partially correct concluding statement when proving  $\triangle ABE$  is isosceles.

Question 35 continued.

Prove  $\triangle ABE$  is an isosceles triangle.

[The use of the set of axes below is optional.]

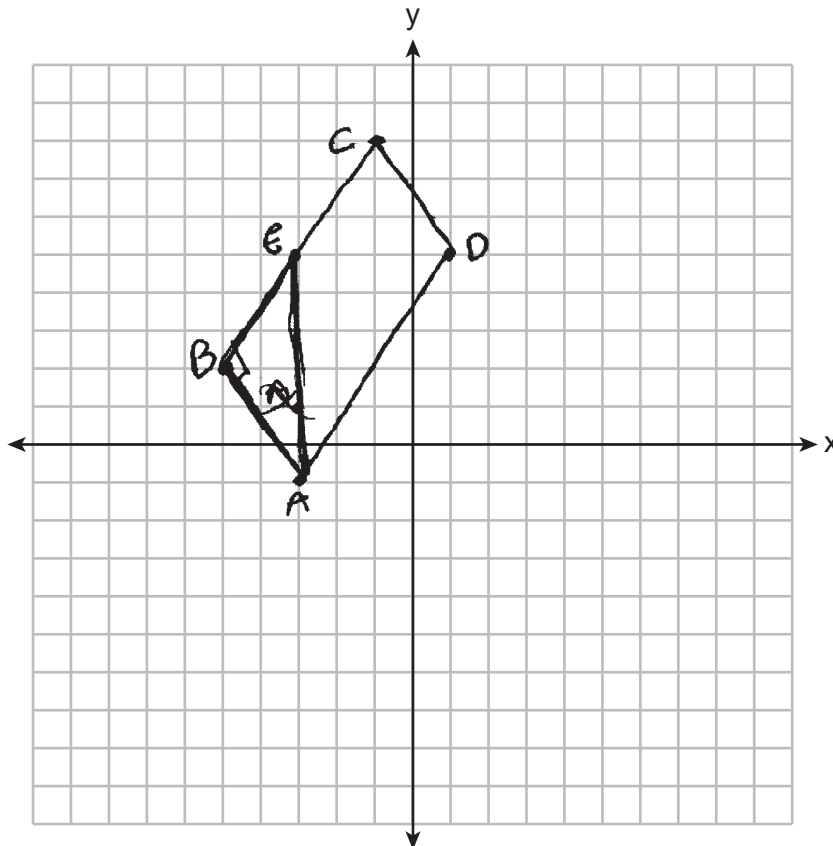
$$\begin{aligned} D_{AE} &: \sqrt{(-3+5)^2 + (-1-5)^2} \\ &: \sqrt{36} \quad D_{AE} = 6 \end{aligned}$$

$$\begin{aligned} \text{slope}_{BE} &: \frac{5-2}{-3+5} = \frac{3}{2} \\ \text{slope}_{BA} &: \frac{-1-2}{-3+5} = \frac{-3}{2} \end{aligned} \quad \left. \begin{array}{l} \\ \end{array} \right\} \perp$$

$$\begin{aligned} D_{EB} &: \sqrt{(-3+5)^2 + (5-2)^2} \\ &: \sqrt{13} \quad D_{EB} = \sqrt{13} \end{aligned}$$

$$\begin{aligned} D_{AB} &: \sqrt{(-3+5)^2 + (-1-2)^2} \\ &: \sqrt{13} \quad D_{AB} = \sqrt{13} \end{aligned} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{Equal}$$

$\therefore \triangle ABE$  is an isosceles triangle b/c it has 2 equal sides and  $90^\circ \angle$ .  $\perp$  lines form rt.  $\angle$ 's.



---

**Question 35**

---

**35** Triangle  $ABC$  has vertices whose coordinates are  $A(-3, -1)$ ,  $B(-5, 2)$ , and  $C(-1, 8)$ .

State the coordinates of point  $D$  such that quadrilateral  $ABCD$  is a parallelogram.

[The use of the set of axes on the next page is optional.]

Prove  $ABCD$  is a parallelogram.

[The use of the set of axes on the next page is optional.]

$$m\overline{BA} = \frac{2+1}{-5+3} = \boxed{\frac{3}{2}}$$

$$\boxed{D(x,y) = (1,5)}$$

State the coordinates of point  $E$ , the midpoint of  $\overline{BC}$ .

[The use of the set of axes on the next page is optional.]

$$E(x,y) = (-3,5)$$

**Question 35 is continued on the next page.**

**Score 3:** The student correctly determined the coordinates of points  $D$  and  $E$  and correctly determined the lengths of  $\overline{AB}$  and  $\overline{EB}$ .

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**Question 35 continued.**

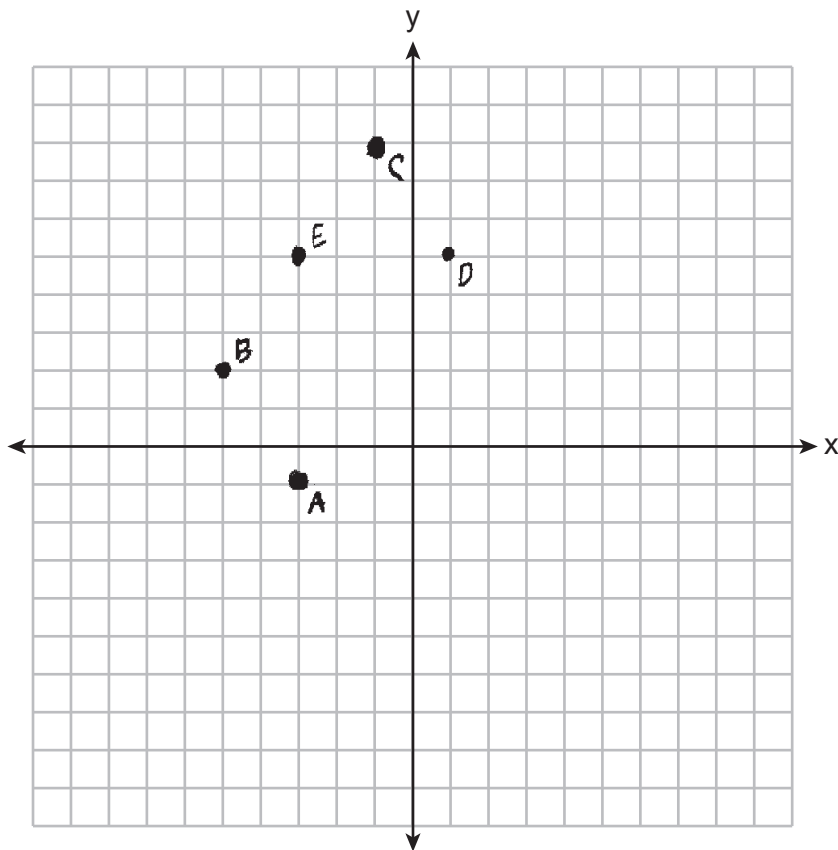
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Prove  $\triangle ABE$  is an isosceles triangle.

[The use of the set of axes below is optional.]

$$dBA = \sqrt{(-5+3)^2 + (2+1)^2} \rightarrow \sqrt{(-2)^2 + 3^2} \rightarrow \sqrt{4+9} = \boxed{\sqrt{13}}$$

$$dBE = \sqrt{(-5+3)^2 + (2-5)^2} \rightarrow \sqrt{(-2)^2 + (-3)^2} \rightarrow \sqrt{4+9} = \boxed{\sqrt{13}}$$



---

**Question 35**

---

**35** Triangle  $ABC$  has vertices whose coordinates are  $A(-3, -1)$ ,  $B(-5, 2)$ , and  $C(-1, 8)$ .

State the coordinates of point  $D$  such that quadrilateral  $ABCD$  is a parallelogram.

[The use of the set of axes on the next page is optional.]

$(1, 5)$

Prove  $ABCD$  is a parallelogram.

[The use of the set of axes on the next page is optional.]

$$\begin{aligned} m\overline{BA} &= \frac{-2}{3} \\ m\overline{CD} &= \frac{-2}{3} \\ m\overline{CB} &= \frac{4}{6} \\ m\overline{AD} &= \frac{4}{6} \end{aligned}$$

$ABCD$  is a parallelogram

State the coordinates of point  $E$ , the midpoint of  $\overline{BC}$ .

[The use of the set of axes on the next page is optional.]

$(-3, 5)$

**Question 35 is continued on the next page.**

**Score 3:** The student made an error when determining the slopes of the four sides of  $ABCD$ . The student wrote an incomplete conclusion when proving  $ABCD$  is a parallelogram. The student did not show enough work when determining the lengths of  $\overline{AB}$  and  $\overline{EB}$ .

Question 35 continued.

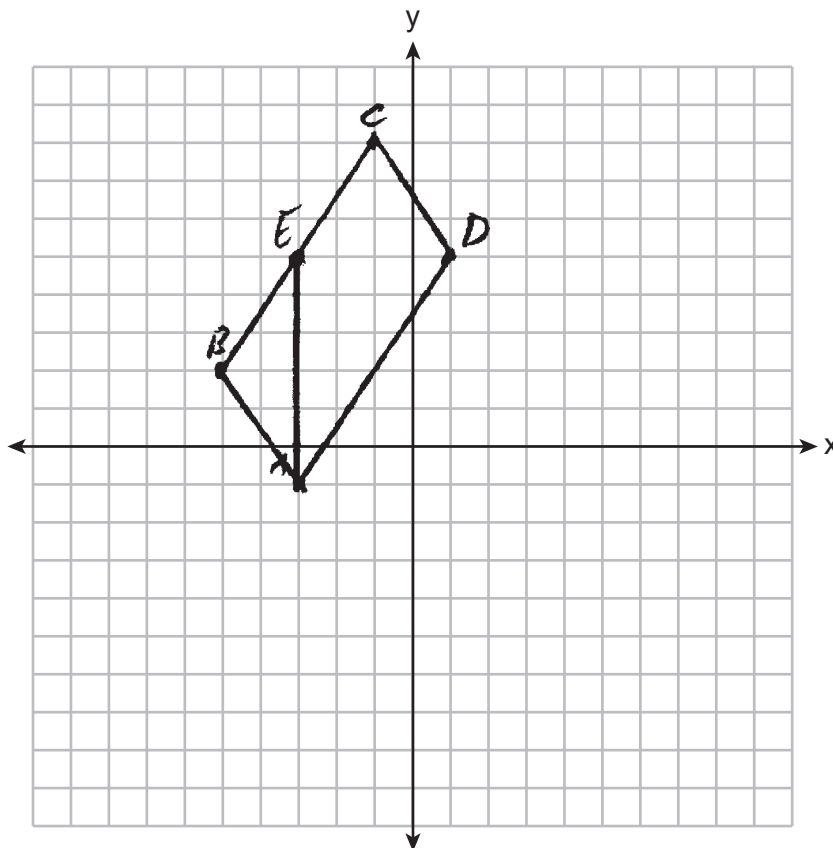
Prove  $\triangle ABE$  is an isosceles triangle.

[The use of the set of axes below is optional.]

$$BA = \sqrt{(2)^2 + (-3)^2} = \sqrt{13}$$

$$EB = \sqrt{(2)^2 + (-3)^2} = \sqrt{13}$$

$\triangle ABE$  is an isosceles triangle  
because it has 2 congruent sides



### Question 35

**35** Triangle  $ABC$  has vertices whose coordinates are  $A(-3, -1)$ ,  $B(-5, 2)$ , and  $C(-1, 8)$ .

State the coordinates of point  $D$  such that quadrilateral  $ABCD$  is a parallelogram.

[The use of the set of axes on the next page is optional.]

$$D(1, 5)$$

$$B \rightarrow C$$

$$A \rightarrow D$$

$$\uparrow 6 \rightarrow 4$$

$$\uparrow 6 \rightarrow 4$$

Prove  $ABCD$  is a parallelogram.

[The use of the set of axes on the next page is optional.]

$BC + AD$  have the same slope  $6/4$

def of Parallel lines

// lines make a Parallelogram

State the coordinates of point  $E$ , the midpoint of  $\overline{BC}$ .

[The use of the set of axes on the next page is optional.]

$$B(-5, 2) \quad C(-1, 8)$$

$$\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$\left( \frac{-6}{2}, \frac{10}{2} \right)$$

$$\left( \frac{-5 + (-1)}{2}, \frac{2 + 8}{2} \right)$$

$$E(-3, 5)$$

Question 35 is continued on the next page.

**Score 2:** The student correctly determined the coordinates of points  $D$  and  $E$ .

Question 35 continued.

Prove  $\triangle ABE$  is an isosceles triangle.

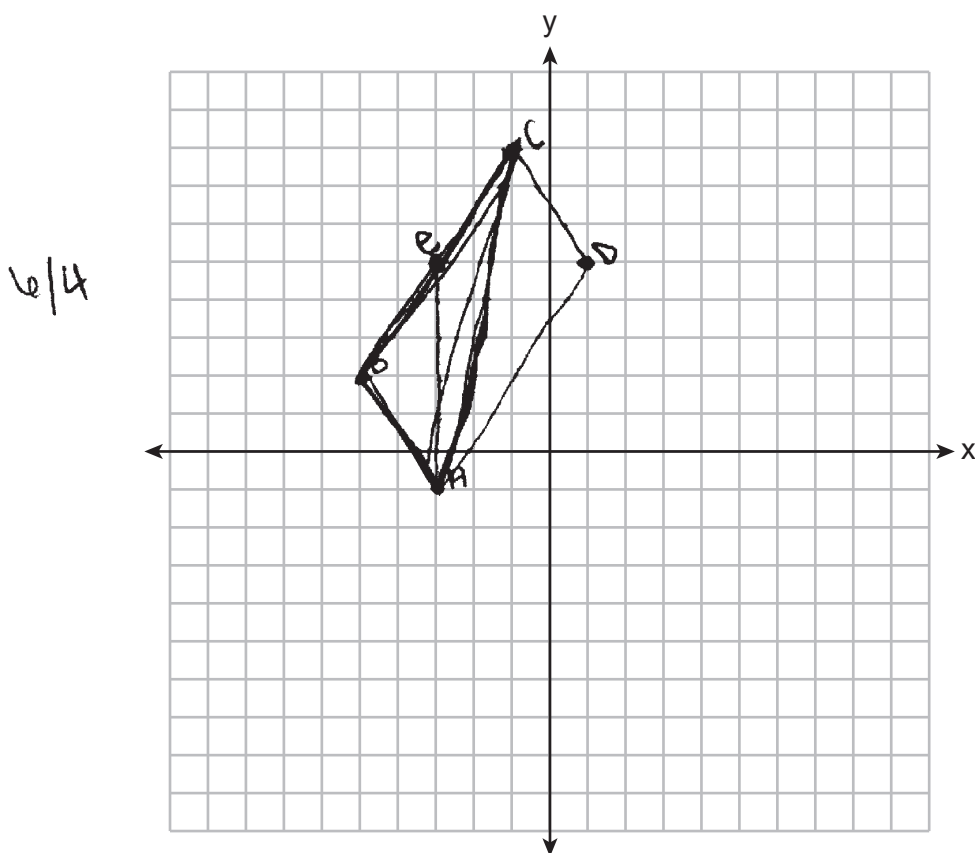
[The use of the set of axes below is optional.]

$\angle eba$  is a right angle  $90^\circ$

making  $\angle e + \angle a = 45^\circ$

$$90^\circ + 45^\circ + 45^\circ = 180^\circ \checkmark$$

$\angle e + \angle a$  are  $\cong$  def of ~~an~~ isosceles  $\triangle$



Question 35

35 Triangle  $ABC$  has vertices whose coordinates are  $A(-3, -1)$ ,  $B(-5, 2)$ , and  $C(-1, 8)$ .

State the coordinates of point  $D$  such that quadrilateral  $ABCD$  is a parallelogram.

[The use of the set of axes on the next page is optional.]

(1, 5)

Prove  $ABCD$  is a parallelogram.

[The use of the set of axes on the next page is optional.]

$$\begin{array}{l}
 \overline{BA} \\
 3^2 + 2^2 = c^2 \\
 9 + 4 \\
 \sqrt{13} = c \\
 \sqrt{13} = c
 \end{array}
 \left\{
 \begin{array}{l}
 \overline{DA} \\
 6^2 + 4^2 = c^2 \\
 36 + 16 \\
 \sqrt{52} = \sqrt{c^2} \\
 \sqrt{52} = c
 \end{array}
 \right\}
 \left\{
 \begin{array}{l}
 \overline{CD} \\
 3^2 + 2^2 = c^2 \\
 9 + 4 \\
 \sqrt{13} = \sqrt{c^2} \\
 \sqrt{13} = c
 \end{array}
 \right\}
 \left\{
 \begin{array}{l}
 \overline{BC} \\
 6^2 + 4^2 = c^2 \\
 36 + 16 = c^2 \\
 \sqrt{52} = \sqrt{c^2} \\
 \sqrt{52} = c
 \end{array}
 \right\}
 \begin{array}{l}
 a^2 + b^2 = c^2
 \end{array}$$

State the coordinates of point  $E$ , the midpoint of  $\overline{BC}$ .

[The use of the set of axes on the next page is optional.]

$$\begin{array}{l}
 M = \left( \frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2} \right) \\
 \left( \frac{-1 + 5}{2}, \frac{8 + 2}{2} \right) \\
 (-2, 5)
 \end{array}
 \quad
 \begin{array}{l}
 B(-5, 2) C(-1, 8) \\
 x \quad y \quad x \quad y
 \end{array}$$

Question 35 is continued on the next page.

**Score 2:** The student correctly determined the coordinates of point  $D$  and correctly determined the lengths of the four sides of  $ABCD$ .

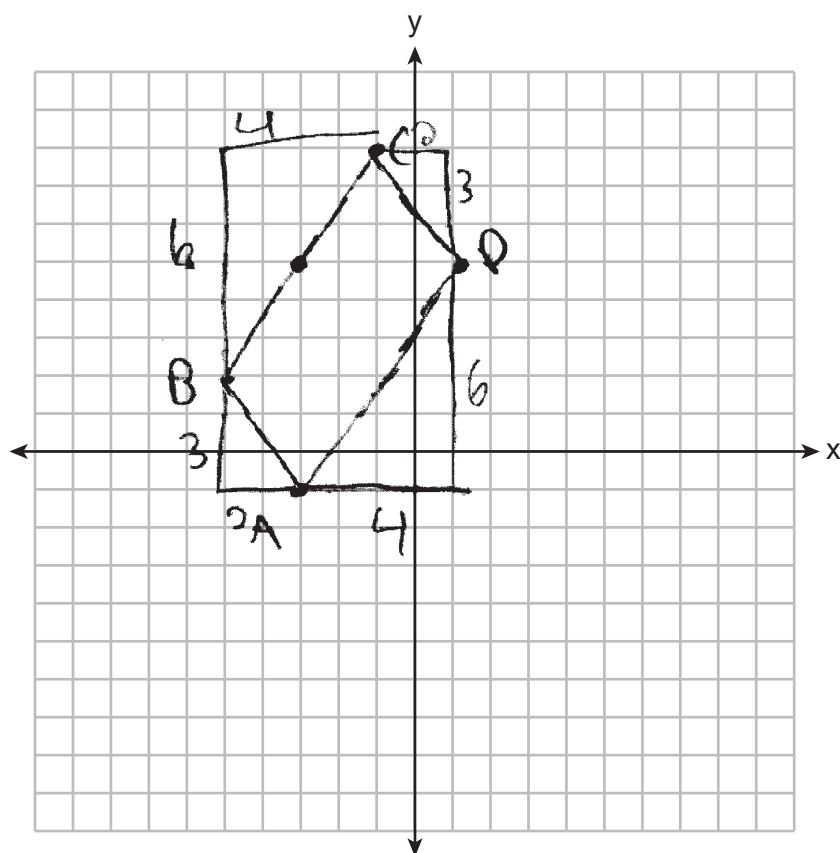
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**Question 35 continued.**

---

Prove  $\triangle ABE$  is an isosceles triangle.

[The use of the set of axes below is optional.]



### Question 35

**35** Triangle  $ABC$  has vertices whose coordinates are  $A(-3, -1)$ ,  $B(-5, 2)$ , and  $C(-1, 8)$ .

State the coordinates of point  $D$  such that quadrilateral  $ABCD$  is a parallelogram.

[The use of the set of axes on the next page is optional.]

$(1, 5)$

Prove  $ABCD$  is a parallelogram.

[The use of the set of axes on the next page is optional.]

$$D_{BC} = \sqrt{4^2 + 6^2} = \sqrt{16 + 36} = \sqrt{52}$$

$\overline{BC} \parallel \overline{AD}$

$$D_{AD} = \sqrt{4^2 + 6^2} = \sqrt{16 + 36} = \sqrt{52}$$

$\overline{BA} \parallel \overline{CD}$

$$D_{CD} = \sqrt{2^2 + 3^2} = \sqrt{4 + 9} = \sqrt{13}$$

$$D_{BA} = \sqrt{2^2 + 3^2} = \sqrt{4 + 9} = \sqrt{13}$$

All sides are parallel so it makes a parallelogram

State the coordinates of point  $E$ , the midpoint of  $\overline{BC}$ .

[The use of the set of axes on the next page is optional.]

$$\frac{-5 + -1}{2}, \frac{2 + 8}{2}$$

$$-3, 5$$

Question 35 is continued on the next page.

**Score 2:** The student wrote an incorrect concluding statement when proving  $ABCD$  is a parallelogram. The student did not state point  $E$  as a coordinate. The student did not prove  $\triangle ABE$  is isosceles.

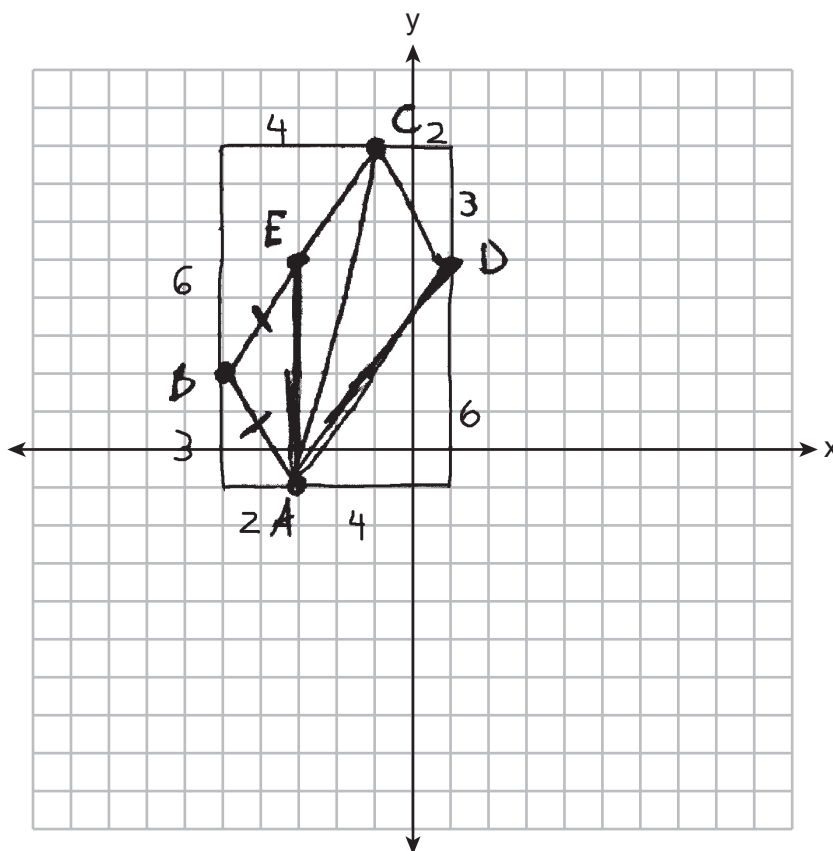
Question 35 continued.

Prove  $\triangle ABE$  is an isosceles triangle.

[The use of the set of axes below is optional.]

$$\overline{BE} \cong \overline{BA}$$

The base sides are congruent  
which makes the triangle isosceles



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**Question 35**

---

**35** Triangle  $ABC$  has vertices whose coordinates are  $A(-3, -1)$ ,  $B(-5, 2)$ , and  $C(-1, 8)$ .

State the coordinates of point  $D$  such that quadrilateral  $ABCD$  is a parallelogram.

[The use of the set of axes on the next page is optional.]

$(1, 5)$

Prove  $ABCD$  is a parallelogram.

[The use of the set of axes on the next page is optional.]

$$\begin{aligned}\frac{2 - (-1)}{-5 - (-3)} &= \boxed{-\frac{3}{2}} & \frac{8 - 2}{-1 - (-5)} &= \boxed{-\frac{3}{2}} \\ \frac{5 - 8}{1 - (-1)} &= \boxed{-\frac{3}{2}} & \frac{-1 - 5}{1 - (-3)} &= \boxed{-\frac{3}{2}}\end{aligned}$$

$\Delta ABCD$  is a parallelogram because they all have the same slope.

State the coordinates of point  $E$ , the midpoint of  $\overline{BC}$ .

[The use of the set of axes on the next page is optional.]

**Question 35 is continued on the next page.**

**Score 1:** The student correctly determined the coordinates of point  $D$ .

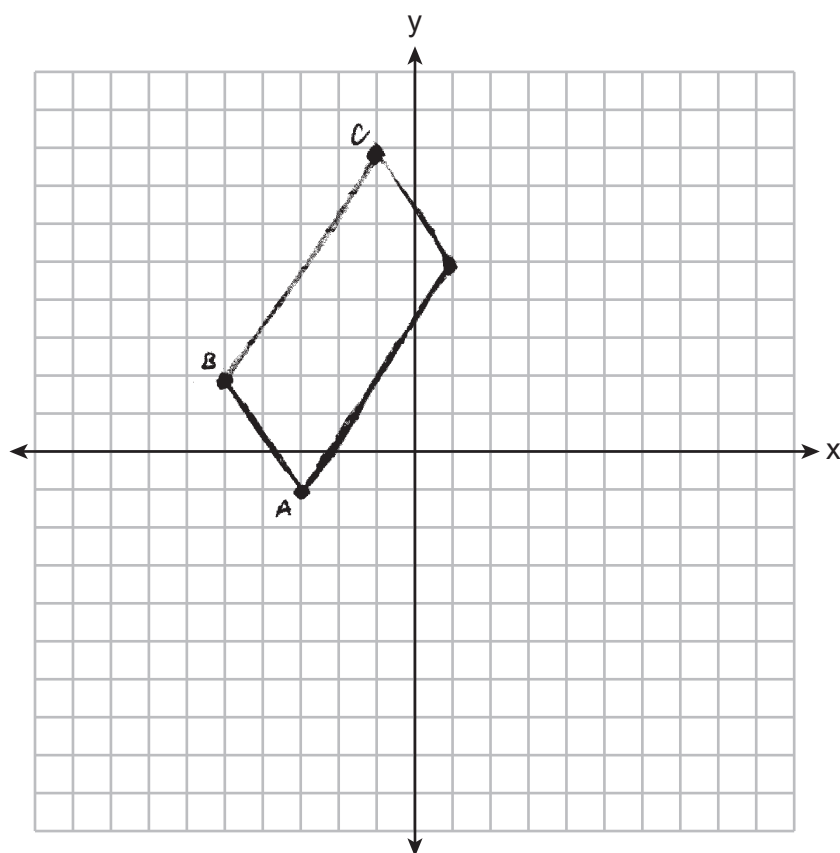
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**Question 35 continued.**

---

Prove  $\triangle ABE$  is an isosceles triangle.

[The use of the set of axes below is optional.]



### Question 35

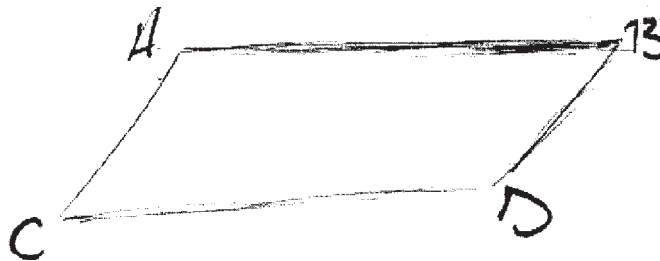
**35** Triangle  $ABC$  has vertices whose coordinates are  $A(-3, -1)$ ,  $B(-5, 2)$ , and  $C(-1, 8)$ .

State the coordinates of point  $D$  such that quadrilateral  $ABCD$  is a parallelogram.

[The use of the set of axes on the next page is optional.]

Prove  $ABCD$  is a parallelogram.

[The use of the set of axes on the next page is optional.]



State the coordinates of point  $E$ , the midpoint of  $\overline{BC}$ .

[The use of the set of axes on the next page is optional.]

$$m = \frac{x_2 - y_2}{2} + \frac{y_2 - x_1}{2}$$
$$m = \frac{8 - 2}{2} + \frac{-1 - 5}{2}$$
$$m = (3, -3)$$

Question 35 is continued on the next page.

**Score 0:** The student did not show enough correct relevant work to receive any credit.

Question 35 continued.

Prove  $\triangle ABE$  is an isosceles triangle.

[The use of the set of axes below is optional.]

$$a^2 + b^2 = c^2$$

$$2^2 + 2^2 = c^2$$

$$4 + 4 = c^2$$

$$\sqrt{8} = \sqrt{c^2}$$

$$8$$

$$a^2 + b^2 = c^2$$

$$2^2 + 2^2 = c^2$$

$$4 + 4 = c^2$$

$$\sqrt{8} = \sqrt{c^2}$$

$$8$$

$$A(-3, -1)$$

$$B(-5, 2)$$

$$a^2 + b^2 = c^2$$

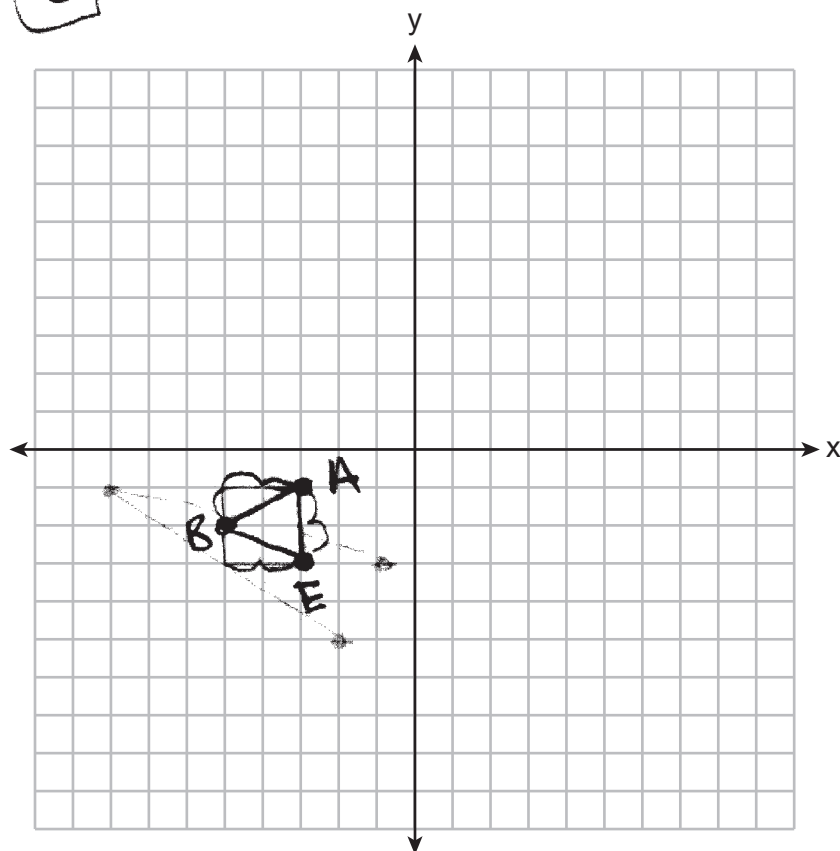
$$2^2 + 2^2 = c^2$$

$$4 + 4 = c^2$$

$$\sqrt{8} = \sqrt{c^2}$$

$$8$$

All sides are equal.



---

**Question 35**

---

**35** Triangle  $ABC$  has vertices whose coordinates are  $A(-3, -1)$ ,  $B(-5, 2)$ , and  $C(-1, 8)$ .

State the coordinates of point  $D$  such that quadrilateral  $ABCD$  is a parallelogram.

[The use of the set of axes on the next page is optional.]

$$D(-3, 5)$$

Prove  $ABCD$  is a parallelogram.

[The use of the set of axes on the next page is optional.]

$$\overline{BC} \parallel \overline{AB}$$

$$\overline{DE} \perp \overline{BA}$$

State the coordinates of point  $E$ , the midpoint of  $\overline{BC}$ .

[The use of the set of axes on the next page is optional.]

$$E(-4, 1)$$

**Question 35 is continued on the next page.**

**Score 0:** The student did not show enough correct relevant work to receive any credit.

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**Question 35 continued.**

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Prove  $\triangle ABE$  is an isosceles triangle.

[The use of the set of axes below is optional.]

